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Abstract.- We consider an OLG model with emissions arising from production and potential irreversible pollution. Pollution control goes through a system of permits and private agents can also maintain the environment. In this setting, we prove that there exist multiple equilibria. Due to the possible irreversibility, the economy can be dragged into both stationary and asymptotic poverty traps. First, we show that choosing a global quota on emissions at the lowest level beyond a critical threshold is a means to avoid the two types of traps. Next, we analyze the impact of a political reform on other equilibria. When the agents do not engage in maintenance, a fall in the quota implies a reduction of pollution but is detrimental to capital accumulation while, in the other case, it procures a double dividend.

Key words: overlapping generations, irreversible pollution, poverty trap, pollution permits

JEL codes: D91, D62, Q28, Q56, O11.

1 Introduction

The signature of the Kyoto Protocol (1997) for the reduction of greenhouse gas emissions, is a sign of the interest and the trust placed in the deployment of a pollution permits system. From a theoretical point of view, the guidelines for guaranteeing the functioning of this instrument are still a matter of debate. As far as the control of polluting emissions is concerned, the efficiency of this regulation model is not disputable. Indeed, since the work of Montgomery [1972] or Baumol and Oates [1988], we know that it is sufficient for the regulator, to choose a global emission quota (in function of some predefined objective) and let the market act, in order to guarantee an optimal allocation of permits between polluting firms. But among other discussion topics, the question of the impact of a regulation *via* permits on the economic development process remains, to a large extent, open.

We choose to approach this issue through an extension of the model of Prieur [2006]. In that model, we have proposed an original analysis the relationship between growth and the environment. More precisely, this model abandons the assumption of a constant-rate assimilation of pollution by the environment, which is systematically used by related papers such as John and Pecchenino[1994]. Instead, inspired in particular by Foster [1975] or Tahvonen and Withagen [1996], it is assumed that the waste assimilation capacity is limited. This assumption reflects the potential *irreversibility* of environmental damage. The original result is that a non-regulated growth process may lead a polluting economy into an *poverty trap*, both economic and ecological, despite an operating pollution abatement activity.

The existence of such a long-term state confirms the necessity for an intervention of the Public Authorities for the management of pollution problems, and leads us to study more deeply the means and the consequences of such an intervention. For this purpose, we develop further the model of Prieur [2006], assuming that the pollution proceeds from the production activity, and that it is controlled thanks to a pollution permit market. In this context, the question is first to know under which conditions this system of regulation allows to avoid the drift of an economy towards a poverty trap. Next, we seek to assess the effects of a reform of the pollution permit system on the growth perspectives of an economy.

There exists a large body of literature, dedicated to the analysis of the impact of a reform of the environmental policy on economic growth, based on the assumption that agents have an infinite lifespan. However, these papers concentrate exclusively on the tax instrument (see in particular Bovenberg and Smulders [1995], [1996] or Bovenberg and de Mooij [1997]). A feature common to all these papers is the introduction of an environmental externality in production, which translates the idea that the quality of the environment should improve the productivity of private inputs. Their essential conclusion is that a more ambitious policy (that is, a raise in the tax on polluting emissions) may provide a double dividend: a simultaneous increase of environmental quality and of the growth rate, provided that the environment has a strong positive impact on the technology.

On the contrary, few studies consider this problem for a regulation through permits, with the notable exceptions of Jouvet, Michel and Vidal [2002] and Ono [2002]. These papers also differ from the preceding ones by the formalism adopted. Indeed, their objective is to measure the macroeconomic consequences of a strengthening of the permit system (that is, a decrease in the global emission quota) in the setting of an *overlapping generations* model, by considering the pollution as a production factor. In Jouvet, Michel and Vidal [2002], the consequence of a more severe policy depends essentially on technological parameters. A decrease in the quota penalises (respectively, stimulates) the accumulation of capital when the inputs are complements (respectively, strong substitutes). For moderate values of the elasticity of substitution, the direction of the global impact remains undetermined. Ono [2002] shows that a decrease in the emission quota allocated to polluters may even have, in the long run, an effect contrary to what is expected, by provoking a decrease in the capital level and an increase in the pollution level. Our contribution also prolongates this study to the situation where there is a risk of irreversible pollution.

We first show that there exist multiple equilibria, among which some have the characteristics of a poverty trap. However, we show that, contrary to the conclusions of Prieur [2006], is is possible to guarantee the absence of traps by a proper setting of permits. Actually, a means to make the economy immune to a stabilization at such a state is to fix the global pollution quota *above* a certain threshold. In other words, it is necessary to allow firms to pollute sufficiently. Once in a situation where poverty traps are excluded, we evaluate the impact of a reform of the environmental policy on the properties of the other equilibria of the model. It turns out that the behavior depends on whether private agents engage into pollution abatement, or not. For the constrained equilibrium solution (the one where agents do not depollute), there exists a compromise between capital accumulation and pollution control. If lowering the pollution quota indeed allows to reduce pollution, it also generates a negative effect on capital accumulation. For the interior equilibrium solution (the one where agents do depollute), the key is the evolution of the balance between financial and environmental constraints imposed to the agents. In fact, at the locally stable equilibrium, we show that the economy enjoys a double dividend: lowering the quota allows the economy to reach a long-term state which is both richer and less polluted. The derivation of such a result does necessitate to resort to the controversial assumption of a positive environmental externality on production, as opposed to the literature on pollution tax reforms. Finally, we proceed with a dynamic analysis of the situation. We conclude from it that, to begin with, imposing a restrictive pollution quota is a way to prevent the exclusion of constrained equilibria which are desired states. Moreover, and if one excludes the hopeless situation where the environment is irreversibly degraded when the permit system is set up, this system allows the economy to follow a growth path which respects the environment. In summary, aside from the compromise associated with the constrained solution, the general recommendation that can be deduced from our analysis is to set the global emission quota as small as possible *above* a certain threshold, thereby excluding poverty traps.

The paper is organized as follows. Section 2 sets out the model. Section 3 derives the equilibrium and analyzes the impact of a political reform on the equilibrium properties. Section 4 performs some numerical simulations so as to outline the implications of a change in policy on the global dynamics and, notably, on the possibility to reach a safe and wealthy steady state. Finally, Section 5 concludes.

2 The model

We develop an overlapping generations model *à la* Allais [1947], Samuelson [1958] and Diamond [1965]. In a perfectly competitive world, the firms produce a single homogeneous good used both for consumption and investment. The production process generates harmful polluting emissions. Pollution control goes through the implementation of a policy consisting in both the definition of a global emission quota at each period $\bar{E}_t$ and the creation of an exchange market for pollution permits. More precisely, we assume that the quota imposes upon the economy in an exogenous manner. The level of emission to be respected, for instance, decided during international negotiations (like the Kyoto protocol (1997)) where all participants promise to reduce their emissions. The governement's role is thus limited to sell a volume of pollution permits corresponding to $\bar{E}_t$ to the polluting firms. It is also responsible for the distribution of the income obtained from the sale of permits (the environmental allowance) to households. In addition, following Ono [2002], we assume that the households can also engage in environmental maintenance.

2.1 Pollution dynamics

In the absence of human activity, pollution accumulation, for non negative levels of the stock P_t, is described by the following equation:

$$P_{t+1} = P_t - \Gamma(P_t) \tag{1}$$

where $\Gamma(P_t)$ corresponds to the natural decay function that gives the amount of pollution assimilated by nature each period. Nature's ability to absorb pollution depends on the level of pollutant concentration. More precisely, our aim is to express the idea that too high levels of pollution alter the environment's recovery process in an irreversible way. Therefore, following Forster [1975], Cesar and de Zeeuw [1994] and Tahvonen and Withagen [1996], we assume an inverted U-shape decay function (see fig.1) whose properties, summarized in the assumption below, give an account of the potential irreversibility of environmental damages caused by pollution:

Assumption 1. *The decay function $\Gamma(P) : \mathbb{R}^+ \to \mathbb{R}^+$ is $\mathcal{C}^2$ and concave ($\Gamma''(P) \leq 0$) over the interval $[0, P]$. It is first increasing from $\Gamma(0) = 0$ to a level $\tilde{P}$ then, decreasing until the pollution reaches the irreversibility threshold $\bar{P}$ ($\Gamma'(P) > 0\ \forall P \in [0, \tilde{P})$, $\Gamma'(P) < 0\ \forall P \in (\tilde{P}, \bar{P})$*

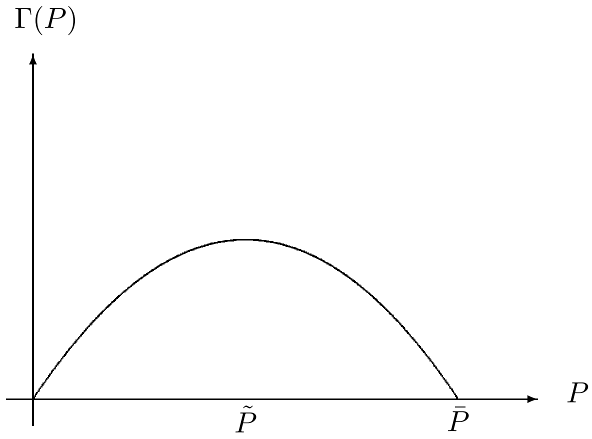

Figure 1: The assimilation function

with $\tilde{P} < \bar{P}$*). Beyond this value, assimilation is nil:* $\Gamma(P) = 0 \ \forall P \geq \bar{P}$*. We also assume that the amount of pollution assimilated at each period is lower than the stock i.e.* $\Gamma(P) < P \ \forall P > 0$.

For low pollution levels, the volume of pollution absorbed by nature is first growing with the stock until reaching a maximal absorbtion rate $\Gamma_{\max}$ at some $P = \tilde{P}$. Then, beyond the turning point $\tilde{P}$, the regeneration capacity starts to decline and assimilation decreases with the pollutant concentration. Finally, as soon as pollution reaches the critical level $\bar{P}$, the natural rate of decay is nill and pollution accumulation becomes irreversible. In other words, once the stock of pollution has achieved the critical threshold $\bar{P}$, the recovery process of nature is completely and permanently overwhelmed.

We now turn to the analysis of the private agents' choices and trade-off.

2.2 Production

Under perfect competition, the firms produce the final good Y_t with a constant returns to scale technology using labor L_t and capital K_t:

$$Y_t = \tilde{A} z_t K_t^{\nu} L_t^{1-\nu} \tag{2}$$

where $z_t \in [0, 1)$ is an index of the technology's intensity of pollution, $\tilde{A} > 0$ corresponds to a productivity scalar and $\nu \in (0, 1)$ is a constant parameter.

Production activity leads to polluting emissions. Following Stokey [1998], the flow of emissions writes:

$$E_t = z_t^{\zeta} Y_t \tag{3}$$

with $\zeta > 0$. Therefore, the emissions-output ratio is positively linked to the index z_t.

Finally, a combination of equations (2) and (3) allows us to express the production function as constant returns technology with respect to capital, labor and emissions:

$$Y_t = AK_t^{\alpha} L_t^{\beta} E_t^{1-\alpha-\beta} \tag{4}$$

with $A = \tilde{A}^{\frac{\zeta}{1+\zeta}}$, $\alpha = \frac{\nu\zeta}{1+\zeta}$, $\beta = \frac{(1-\nu)\zeta}{1+\zeta}$.

In Jouvet, Michel and Rotillon [2005], the authors highlight, in a wellfare maximization perspective, the superiority of a system of auctions over the principle of a free allocation of permits to firms by showing that the latter is a source of economic distorsions. On the basis of their results, we exclude grandfathering. We shall note that Ono [2002] assume, on the contrary, that a part of the quota is allocated freely to firms that can next participate to the permits market transactions. However his approach is *in fine* rigorously identical to ours since, in his model's equilibrium, remains an income $q_t\bar{E}_t$, exactly equal to the revenue coming from the sale of the whole quota, which is entirely taxed and paid back to the young households. Thus, firms are obliged to purchase, at the market price q_t, the amount of pollution permits that corresponds precisely to their own need E_t in order to be able to produce.

We assume that capital fully depreciates in one period. Firms maximise profits, taking the price of inputs as given:

$$\pi_t = AK_t^{\alpha} L_t^{\beta} E_t^{1-\alpha-\beta} - w_t L_t - R_t K_t - q_t E_t \tag{5}$$

where w_t represents the wage rate, r_t is the real rental rate of capital and q_t is the price of permits.

The first order conditions for profit maximization, expressed in terms of per capita variables with $k_t = K_t/L_t$ and $e_t = E_t/L_t$, write :

$$w_t = \beta A k_t^{\alpha} e_t^{1-\alpha-\beta} \tag{6}$$

$$R_t = \alpha A k_t^{\alpha-1} e_t^{1-\alpha-\beta} \tag{7}$$

$$q_t = (1-\alpha-\beta) A k_t^{\alpha} e_t^{-\alpha-\beta}. \tag{8}$$

2.3 The households

We consider an infinite horizon economy composed of finite-lived agents. A new generation is born at each period $t = 1, 2, ...$, and lives for two periods: youth and old age. There is no population growth and the size of a generation is normalized to one $N \equiv 1$. The young agent born at period t is endowed with one unit of labor which he (she) supplies to firms ineslatically for a real wage w_t. His (her) first period income is also composed of the revenue from the sale of a quantity E_t of permits, at the price q_t. This revenue corresponds to the environmental allowance distributed by the government. He (she) allocates this total income to savings s_t and environmental maintenance m_t.[1,2] When retired, the agent supplies his (her) savings to firms and earns the return of savings $R_{t+1}s_t$ (with $R_{t+1} = 1 + r_{t+1}$ the interest factor). His (her) income is entirely devoted to consumption c_{t+1}. The two budget constraints he (she) faces, in both periods of life, write respectively:

$$w_t + q_t E_t = s_t + m_t \tag{9}$$

$$c_{t+1} = R_{t+1} s_t. \tag{10}$$

Following Ono [2002], we assume that the pollution abatement activity, driven by households, remains effective despite the permits system. Therefore, the economy has two distinct means for fighting against pollution. If the environmental policy is above all intended to regulate emissions, it also affects, through the distribution of the environmental allowance, the households' depollution effort. One may note that this additional income also stimulates productive investment, through savings.

The preferences of the agent born at date t are defined over old age consumption and environmental quality. They are described by the following utility function $U(c_{t+1}, P_{t+1})$:

Assumption 2. *The utility function $U(c, P) : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}$ is $\mathcal{C}^2$. It is increasing and concave with respect to consumption but decreasing in pollution: $U_1 \geq 0$, $U_2 \leq 0$, $U_{11}, U_{22} \leq 0$. The cross derivative is negative $U_{12} \leq 0$.[3] We further assume that $\lim_{c \to 0} U_1(c, P) = +\infty$.*

Emissions contribute to the accumulation of the pollutant stock. It is also possible to control the periodic flux of emissions and to improve environmental quality through the abatement expenditures of the young m_t. Real emissions are simply represented by the following linear function: $\Theta_t = E_t - \gamma m_t$ with $0 \leq \gamma < 1$. In the presence of human activity, the law of evolution of pollution then becomes:

$$P_{t+1} = P_t - \Gamma(P_t) + E_t - \gamma m_t. \tag{11}$$

In this framework, households typically face an intergenerational externality. When the young chooses the amount of resources to devote to maintenance, he (she) only cares about the environment he (she) will enjoy in old age. But, the agent ignores future benefits of his (her) investment.

The representative agent born at date t shares his (her) first period income among savings (which determines the consumption of the final good) and abatement (which influences the "consumption" of the environmental public good) in order to maximise his (her) lifetime utility.

[1] It is possible to reinterpret m_t as a tax levied by a one period lived government in order to finance the abatement activity, for the benefit of agents living during its period of office (John *et al.* [1995]).

[2] We do not consider any first period consumption. This simplifying assumption allows us to focus on the crucial trade-off between final good and environmental good consumptions (see next page the representative agent problem). Anyway, adding a first period consumption would not change our qualitative results.

[3] Pollution exerts a "distaste" effect on consumption (Michel and Rotillon [1995]). The marginal utility of consumption is decreasing in P which means that the higher the pollution, the lesser the consumption.

Taking as given prices and pollution at the beginning of period t, the representative agent's problem writes:

$$\max_{s_t, m_t, c_{t+1}} U(c_{t+1}, P_{t+1})$$

subject to,

$$\begin{cases} w_t + q_t E_t = s_t + m_t \\ c_{t+1} = R_{t+1} s_t \\ P_{t+1} = P_t - \Gamma(P_t) + E_t - \gamma m_t \\ m_t \geq 0. \end{cases}$$

The first order condition reads:

$$-R_{t+1} U_1(c_{t+1}, P_{t+1}) - \gamma U_2(c_{t+1}, P_{t+1}) + \mu = 0 \tag{12}$$

with $\mu \geq 0$, the associated Lagrange multiplier that satisfies:

$$\mu m_t = 0. \tag{13}$$

Since there is a non negativity constraint on m_t, we have to distinguish the case where abatement is ineffective *i.e*, $m_t = 0$, from the one where the agents choose to engage in maintenance *i.e*, $m_t \geq 0$. Moreover, this study must also be divided in two sub-cases depending on whether or not, environmental quality has crossed the irreversibility threshold $\bar{P}$. Therefore, we have to characterize the competitive equilibrium by analyzing separately the four possible cases for which the model exhibits quite distinct economic and environmental dynamics.

In the following sections, our purpose is to evaluate the impact of environmental policy and the effect of a policy reform (*i.e.* a change in the global quota $\bar{E}_t$) on equilibrium properties both at steady state and on the transitional dynamics.

3 The competitive equilibrium

First, we focus on the properties (existence, unicity, stability) of each type of equilibrium. Next, we make comparative statics so as to measure the sensitivity of equilibrium levels of capital and pollution with respect to the quota. This analysis is successively conducted for the constraint ($m_t = 0$) and the interior ($m_t \geq 0$) solutions.

3.1 Zero maintenance equilibrium

A solution where the constraint $m_t = 0$ holds corresponds to the situation where the weight of environmental and financial constraints are such that households have not enough incentives to abate pollution. Thus, they devote the whole of their income Ω_t to savings:

$$\Omega_t = s_t \tag{14}$$

with,

$$\Omega_t = w_t + q_t E_t.$$

Now we define the intertemporal equilibrium with perfect foresight:

Definition 1 *Given the environmental policy $\{\bar{E}_t\}$, a corner equilibrium is a sequence of per capita variables $\{c_t, s_t\}$, a sequence of aggregate variables $\{L_t, K_t, E_t, P_t\}$ and a sequence of prices $\{R_t, w_t, q_t\}$ such that:*

i/ households and firms are at their optimum: condition (14) and the two conditions (6)-(7), for profit maximization, are satisfied,

ii/ all markets clear: $L_t = N = 1$, $K_{t+1} = s_t (= k_{t+1})$ and $E_t = \bar{E}_t (= e_t)$ on the permits market,

iii/ budget constraints (9) and (10) are satisfied,

iv/ the dynamics of pollution are given by (11).

From (6), (8) and the equilibrium condition for the permits market, we get the income as a function of k_t and $\bar{E}_t$:

$$\Omega_t = (1-\alpha) A k_t^{\alpha} \bar{E}_t^{1-\alpha-\beta}. \tag{15}$$

Equilibrium dynamics directly follow from the combination of (11), (14), (15) and the market clearing condition for capital:

$$\begin{cases} k_{t+1} = (1-\alpha) A k_t^{\alpha} \bar{E}_t^{1-\alpha-\beta} \\ P_{t+1} = P_t - \Gamma(P_t) + \bar{E}_t \end{cases} \tag{16}$$

and we shall note that stock variables dynamics are independent from each other. In addition, pollution accumulation is solely determined by the exogenous quota.

In this region of the $k-P$ space, a steady state solves:

$$\begin{cases} k = (1-\alpha) A k^{\alpha} \bar{E}^{1-\alpha-\beta} \\ \Gamma(P) = \bar{E} \end{cases} \tag{17}$$

where $\bar{E} = \lim_{t\to\infty} \bar{E}_t$ is assumed to exist.

The first equation admits a unique solution,[4] $\bar{k}(\bar{E})$ with,

$$\bar{k}(\bar{E}) = \left[(1-\alpha) A \bar{E}^{1-\alpha-\beta}\right]^{\frac{1}{1-\alpha}}$$

and the existence conditions are summarized in the following proposition:

[4] The subscript "c" (*resp.* "i") stands for the corner (*resp.* interior) solution. The second index "r" (*resp.* "i") will refer, in the remainder of the paper, to a reversible (*resp.* irreversible) level of pollution.

Proposition 1 *There is no steady state that exhibits irreversible pollution. There exists a steady state with a reversible level of pollution if and only if*

$$\max_{P\in[0,\bar{P}]}\{\Gamma(P)\} = \Gamma_{\max} \geq \bar{E}. \tag{18}$$

Furthermore, if $\Gamma_{\max} > \bar{E}$*, then there exist two distinct solutions.*

Proof. When pollution is irreversible, the second equation in (17) impose $\bar{E} = 0$. But, under our specifications of production and preferences (Assumption 2), we can exclude this limit case. If pollution is reversible, then the stationary level of pollution must solve, for a given quota $\bar{E}$: $\Gamma(P) = \bar{E}$. According to the inverted U shape of the function $\Gamma(P)$, it is clear that $\Gamma(P) = \bar{E}$ admits a solution $P^*_{cr}(\bar{E})$ iff $\Gamma_{\max} \geq \bar{E}$ where $\Gamma_{\max}$ is the maximal absorption level, reached for a given $\tilde{P}$. Moreover if the inequality in (18) is strict, then we get two positive steady state values for pollution. ■

Condition (18) was already used in Tahvonen et Withagen [1996] or Prieur [2006] and conveys the idea that the maximum potential of assimilation by nature is higher than the stationary emissions level. The latter precisely corresponds, in the zero maintenance space, to the global quota on emissions. It is worth noting that the necessary and sufficient condition (18) imposes an upper bound to the domain of variation of $\bar{E}$.

Once the existence condition is set, we focus on the effect of an environmental policy reform in the zero maintenance space. Our approach only makes sense if we restrict the analysis to the (locally) stable equilibrium. In the case where there exist two steady states, we easily prove that the only stable solution is the one associated with the lowest level of pollution (see appendix B.2). We then consider a strengthening in the permits system consisting in a decrease in the emission quota.

Proposition 2 *A decrease in the quota* $\bar{E}$ *implies a fall in both the levels of pollution and capital at the stable steady state.*

Proof. It is straighforward that $k^{*'}_{cr}(\bar{E}) > 0$. Now, if we refer to the inverted-U shape of the assimilation function $\Gamma(P)$, then it is clear that the fall in the quota $\bar{E}$ causes a fall in the level of stationary pollution at the low stable steady state: $P^{*'}_{cr}(\bar{E}) > 0$. ■

When only one of the two instruments for pollution control (permits) is effective, we generalize Jouvet, Michel et Vidal [2002]'s result, obtained for complementary inputs, to the case where they are substitutable (the elasticity of substitution is equal to one for a Cobb-Douglas technology).[5] In fact, we detect the existence of a dilemma between economic growth and envi-

[5]Note that the authors consider a framework which is quite distinct from ours. They assume that infinite lifetime permits belong to households that pass them on from generations to generations and rent them to firms. Thus comparing our results with theirs is a purely informal exercise but, is explained by the closeness of our problematics.

ronmental preservation: a stricter policy allows to reduce the level of stationary pollution but it is done to the detriment of capital accumulation and long run wealth. A reduction of $\bar{E}$ causes a drop in both emissions and the pollution accumulated at each period. However, it also generates a negative income effect since the wage and the environmental allowance decrease. Finally this effect acts as a brake upon savings and capital accumulation.

Now we turn to the analysis of the positive maintenance equilibrium.

3.2 Positive maintenance equilibrium

A solution with $m_t > 0$ represents the case where a slackening of the financial constraint and/or a reinforcement in the environmental constraint makes agents willing to engage in maintenance. In this case, the households' problem admits an interior solution and the FOC is:

$$-R_{t+1}U_1(c_{t+1}, P_{t+1}) - \gamma U_2(c_{t+1}, P_{t+1}) = 0. \tag{19}$$

The definition of equilibrium is modified and the variable m_t now plays an active role:

Definition 2 *A competitive interior equilibrium is a sequence of per capita variables $\{c_t, m_t, s_t\}$, a sequence of aggregate variables $\{L_t, K_t, E_t, P_t\}$ and a sequence of prices $\{R_t, w_t, \phi_t\}$ such that:*

i/ households and firms are at their optimum: the FOC (19) and the three conditions (6), (7) and (8), for profit maximization, are satisfied,

ii/ all markets clear: $L_t = N = 1$, $K_{t+1} = s_t(= k_{t+1})$ and $E_t = \bar{E}_t(= e_t)$,

iii/ budget constraints (9) and (10) are satisfied,

iv/ pollution evolves according to (11).

The equilibrium analysis consists in considering the system of equations (6)-(8), (11), (19) and the market clearing conditions. Combining these equations yields the expression of consumption and maintenance decisions as a function of the capital stock and the quota:

$$c_t = \alpha A k_t^{\alpha} \bar{E}_t^{1-\alpha-\beta} \tag{20}$$

$$m_t = (1-\alpha) A k_t^{\alpha} \bar{E}_t^{1-\alpha-\beta} - k_{t+1}. \tag{21}$$

By substituting expressions (7) and (20) into the FOC, equation (19) rewrites:

$$-R(k_{t+1}, \bar{E}_{t+1})U_1(c(k_{t+1}, \bar{E}_{t+1}), P_{t+1}) - \gamma U_2(c(k_{t+1}, \bar{E}_{t+1}), P_{t+1}) = 0. \tag{22}$$

This equation implicitly defines an equilibrium relation, valid for any t, between P_t, k_t and $\bar{E}_t$:

$$P_t = \Phi(k_t, \bar{E}_t) \tag{23}$$

that governs the dynamics in the whole positive maintenance space.

It is decreasing in k_t: $\Phi_1 < 0$.[6] A rise in k_t tends to reduce the cost of maintenance (first term in (22)) since it lowers the interest factor and increases the consumption ($R_1 < 0$, $c_1 > 0$, $U_{11} < 0$). In addition, due to the distaste effect exerted by pollution and the rise in consumption, it also goes with an increase in the benefits arising from maintenance ($c_1 > 0$, $U_{12} < 0$). Therefore, the higher the capital, the higher the incentive to maintain and the lower the pollution. According to this relation, at each period, capital stock is inversely linked to the pollution concentration. Note that the sign of Φ_2 is *a priori* indeterminate. If a rise in $\bar{E}_t$ increases the benefits to maintain (through the increase in $c()$), it is associated with two opposite effects on the cost of maintenance. In fact, it rises both the interest factor and the consumption. Now, if we assume that the intertemporal elasticity of substitution $\sigma = -U_1/(cU_{11})$ is lower than one, which means that savings is decreasing in the interest factor, the overall effect on the cost is negative. Finally, we have $\Phi_2 < 0$.[7]

Dynamics are then described by the following system of equations,

$$\begin{cases} P_{t+1} = \Phi(k_{t+1}, \bar{E}_{t+1}) \\ P_{t+1} = P_t - \Gamma(P_t) + \Theta(k_t, \bar{E}_t, k_{t+1}) \end{cases} \tag{24}$$

where $\Theta(k_t, \bar{E}_t, k_{t+1})$ represents the real emissions:

$$\Theta(k_t, \bar{E}_t, k_{t+1}) = \bar{E}_t - \gamma\left((1-\alpha)Ak_t^{\alpha}\bar{E}_t^{1-\alpha-\beta} - k_{t+1}\right)$$

In the next step of our analysis, we focus on the properties of the equilibrium depending on whether or not, the economy has achieved the critical threshold $\bar{P}$. We restrict our study to the interval $[0, \bar{k}(\bar{E})]$ with,

$$\bar{k}(\bar{E}) = \left[(1-\alpha)A\bar{E}^{1-\alpha-\beta}\right]^{\frac{1}{1-\alpha}} \tag{25}$$

on which stationary maintenance is necessarily non negative: $m(k, \bar{E}) \geq 0$ (see appendix $A.1$).

[6] Total differentiation of (22) with respect to k yields:

$$\frac{dP_t}{dk_t} = -\frac{R_1U_1 + Rc_1U_{11} + \gamma c_1U_{12}}{RU_{12} + \gamma U_{22}}$$

[7] The expression of Φ_2 is given by:

$$\frac{dP_t}{d\bar{E}_t} = -\frac{R_2U_1 + Rc_2U_{11} + \gamma c_2U_{12}}{RU_{12} + \gamma U_{22}}$$

The first derivative is positive under the assumptions on preferences and the Cobb-Douglas production function. The sign of the numerator in Φ_2 is unknown. But, we can rewrite:

$$R_2U_1 + Rc_2U_{11} = R_2U_1(1 - \frac{1}{\sigma})$$

now, imposing $\sigma < 1$ implies $\Phi_2 < 0$.

3.2.1 Steady state with irreversible pollution

When pollution is irreversible, the system (24), evaluated at steady state, writes:

$$\begin{cases} P = \Phi(k, \bar{E}) \\ \Theta(k, \bar{E}) = 0 \end{cases}$$

where $\bar{E}$ exists and is the limit value of the quota: $\bar{E} = \lim_{t\to\infty} \bar{E}_t$. For a given $\bar{E}$, we set the conditions under which the second equation admits a solution $k^*_{ii}(\bar{E})$.

Proposition 3 *There exists a steady state $(k^*_{ii}(\bar{E}), P^*_{ii}(\bar{E}))$ associated with a level of irreversible pollution if and only if the global quota is below a limit value $\bar{E}_L$, with:*

$$\bar{E}_L = \left(\gamma A(1-\alpha)^2\right)^{\frac{1-\alpha}{\beta}} (\alpha(1-\alpha)A)^{\frac{\alpha}{\beta}} . \tag{26}$$

Proof. See appendix $A.2$ ■

This type of solution corresponds to an ecological poverty trap since the level of stationary pollution is greater than the irreversibility threshold $\bar{P}$. Moreover, according to the relation (23), the level of capital is less than the one reached at any interior solution with reversible pollution. Therefore, it is also an economic poverty trap. However, contrary to Prieur [2006], it is possible to impose a condition that prevents from the occurence of such a long run state.

Corollary 1 *A necessary and sufficient condition to exclude the existence of poverty traps is to fix the quota on emissions to a sufficiently high level: $\bar{E} > \bar{E}_L$.*

Thus the environmental policy, enacted at the supranational scale, should autorize firms to emit a sufficient amount of pollution to avoid the economy possible stabilization in a poverty trap.

This *a priori* surprising result is explained by the fact that, in the positive maintenance space, the economy has two instruments that affect the level of polluting emissions. Now, the existence of a steady state supposes that real emissions are nill. In other words, depollution by households must exactly compensate the polluting emissions by firms. This situation precisely occurs when the exogenous quota is set below the critical value $\bar{E}_L$. So, the scope of this result must be relativized. In fact, by fixing $\bar{E} > \bar{E}_L$, one mechanically ensures the absence of traps. But it is highly likely that the economy, located in this region, finally suffers a perpetual increase in pollution associated with an continous erosion of the level of wealth. This kind of development trajectory is similar to a sort of asymptotic poverty trap (or also to a process of divergence). We will go back to this important point in section 4.2 devoted to the dynamic analysis. We will notably have to adress the following questions : Can an economy, that does not initially belong to the irreversible region,to reach it and diverge ? To what extend does the possible divergence depend on the choice of the quota ?

If one admits that one of the main objectives of economic policy in general, and environmental policy in particular, is precisely to protect the economy from being dragged down into the ecological and economic poverty trap, we impose, in the following, that $\bar{E} > \bar{E}_L$. In this context, we next focus on the properties of a solution that exhibits a reversible level of pollution.

3.2.2 Steady state with reversible pollution:

When stationary pollution stock is less than the threshold, the system to solve becomes:

$$\begin{cases} P = \Phi(k, \bar{E}) \\ \Gamma(P) = \Theta(k, \bar{E}). \end{cases}$$

Substituting (32) in the right-hand side of the second equation allows us to reduce the study to the analysis of the behaviour of two functions that only depend on capital. We then get the following result.

Proposition 4 *If*

$$\max_{P \in [0, \bar{P}]} \{\Gamma(P)\} \geq \max_{k \in]\underline{k}, \bar{k}(\bar{E})]} \{\Theta(k, \bar{E})\} \tag{27}$$

$$\bar{k}(\bar{E}) \geq \varphi^{-1}(\tilde{P}) \tag{28}$$

where $\varphi(k)$ is the rewriting of the equilibrium relation for a given $\bar{E}$: $\varphi(k) = \Phi(k, \bar{E})$, then there exists a steady state $(k^*_{ir}(\bar{E}), P^*_{ir}(\bar{E}))$ associated with reversible pollution.

Proof. See appendix A.1. ■

Condition (27) is similar to the necessary and sufficient condition (18) used to prove the existence of a corner steady state. In fact, in the interior space, emissions reach their maximum ($=$ $\bar{E}$) when depollution effort vanishes *i.e.* at the upper bound $k = \bar{k}(\bar{E})$. The additional condition (28) ensures some correspondance between the domains of variation of the stock variables k and P.

Note that under these two conditions, there exists one or at most two steady state(s). In case of unicity, we necessarily have $k^*_{ir}(\bar{E}) < \varphi^{-1}(\tilde{P})$ (which implies $P^*_{ir}(\bar{E}) > \tilde{P}$). When there are two solutions,[8] pollutions levels are located on both sides of this critical value: $P^{*h}_{ir}(\bar{E}) < \tilde{P} < P^{*l}_{ir}(\bar{E})$.

Before studying equilibrium sensitivity to a change in the quota, we consider the most interesting case where there exist two steady states (SS). Local stability analysis is conducted in the

[8] Since, at equilibrium, pollution and capital are inversely linked, we simplify the designation of solutions by calling high (resp. low) equilibrium the one characterized by an important (resp. low) level of wealth. Hence, the high SS is also associated with a weak pollutant concentration.

appendix $B.2$. As already mentioned, it is possible to deduce from conditions (27) and (28) the location of stationary pollution levels with respect to $\tilde{P}$. Therefore, we know that only the high equilibrium $(k_{ir}^{*h}(\bar{E}), P_{ir}^{*h}(\bar{E}))$ respects the sufficient condition for local stability $\Gamma'(P) > 0$. Since in OLG models, when there are two solutions, one is locally stable while the other is unstable, there is a strong presumption that the stable solution will be the one that exhibits the highest capital stock and the less pollution (our simulations in section 4.2 will confirm this intuition).

Anyway, the analysis of a permit system reform leads to the following conclusions.

Proposition 5 *At the stable steady state* $(k_{ir}^{*}(\bar{E}), P_{ir}^{*}(\bar{E}))$*,*

- *if*

$$\bar{E} \geq (\gamma(1-\alpha-\beta))^{(1-\alpha)/\beta}(A(1-\alpha))^{1/\beta} \tag{29}$$

then $k_{ir}^{*\prime}(\bar{E}) < 0$*,*

- *if, in addition,*

$$\frac{\Theta_1(k_{ir}^{*}(\bar{E}), \bar{E})}{\Theta_2(k_{ir}^{*}(\bar{E}), \bar{E})} < \frac{c_1(k_{ir}^{*}(\bar{E}), \bar{E})}{c_2(k_{ir}^{*}(\bar{E}), \bar{E})} \tag{30}$$

then $P_{ir}^{*\prime}(\bar{E}) > 0$.

Proof. See appendix C. ■

Increasing $\bar{E}$ has two opposite effects on real emissions. First, it entails a rise in polluting emissions by firms. But, it also stimulates maintenance, through the positive income effect, which in turn tends to reduce emissions. Now, under condition (29), the net effect is positive, that is, real emissions are increasing in $\bar{E}$ at equilibrium.

This condition is sufficient to show that a reduction of the quota causes a rise in the stock of capital at the stable steady state. Let us decompose the effect of a fall in the quota on stationary variables. This decrease is first associated with a negative income effect (see the budget or financial constraint (9)). It leads to a drop in the wage and the environmental allowance which implies that the agent has relatively less resources to devote to savings and depollution (tightening of the financial constraint). It also results in a substitution effect due to the fall in emissions and the pollution accumulated at ech period (see the dynamics given by (1)). *Ceteris paribus*, with the reduction of the quota, the affected generation can allocate a lower amount of resources to depollution to maintain environmental quality which will be enjoyed in second period of life (slackening of environmental constraint). It also allows the households to save a bigger share of their income which favours capital accumulation.

Now, we have to address the question to know why the latter effect dominates at the stable (high) steady state. At the high SS, before the reform, the economy is endowed with an important capital stock. Moreover, pollution level is less than the threshold $\tilde{P}$ and goes hand in hand with an assimilation function that is increasing in the stock of pollutant. The fall in the quota causes a decrease in the income which is *a priori* unfavourable to both savings and depollution spending. But, this tightening of the financial constraint remains quite moderate since the economy owns a sizeable level of wealth. The reduction of the amount of permits sold to firms also implies a slackening of the environmental constraint that was already not very stringent. Therefore, the agent has some latitude to absorb the repercussions of the income decrease on capital accumulation. Here the substitution effect is entirely applicable: depollution serves as an adjustment variable in such a way that the fall in income does not penalize savings. Finally, the level of capital raises.

The second condition (30) concerns the direct and indirect effects of a change in $\bar{E}$ on both real emissions and consumption. An increase in $\bar{E}$ rises consumption through its (direct) positive effect on the interest factor. However, it causes a drop in capital (since $k_{ir}^{*\prime}(\bar{E}) < 0$) that, on the contrary, lowers consumption (indirect negative effect). The same reasoning applies for real emissions. If a higher quota means higher emissions (direct positive effect), it is also associated with a lesser capital and, consequently, lesser emissions (indirect negative effect).

This condition finally states that the ratio of the effects on consumption exceeds the corresponding ratio for emissions. This inequality holds if, for instance, the global impact of a rise in $\bar{E}$ on consumption is negative while it is positive for emissions (see appendix C) and we have $P_{ir}^{*\prime}(\bar{E}) > 0$.[9] In this case, a reduction of the quota first implies a fall in emissions that tends to reduce the level of pollution at the stable SS. Second, it results in an increase in equilibrium consumption that goes with a decrease in pollution because of the distate effect exerted by pollution.

Therefore, the reform procures a double dividend since a more restricting quota allows the economy to reach a steady state where both the level of wealth is higher and the pollution is lesser. The impact of a fall in $\bar{E}$ presents some similarities with the conclusions of the litterature on tax reform (See among others, Bovenberg and Smulders [1995], [1996] or Bovenberg and de Mooij [1997]). We shall also note that our result, contrary to the aforementioned papers, is not conditioned by the controversial assumption of the existence of a positive environmental externality in production.

In this section, we have shown the existence of multiple equilibria with very different properties. As in Prieur [2006], some of them are similar to ecological and economic poverty traps. However, in our setting, it appears that it is possible to prevent the economy from stabilizing at such a state provided that the quota is set at a sufficiently high level. The analysis of the impact of an environmental policy reform (tightening) on different equilibria properties reveals two important features. First, in the absence of depollution by households, a stricter policy allows to reach a less polluted long run state but is detrimental to capital accumulation. Second, in the interior space, it brings a double dividend.

[9]Note that if, following Ono [2002], we consider the specific class of separable utility functions that are logaritmic in consumption, we necessarily have $P_{ir}^{*\prime}(\bar{E}) > 0$ under condition (29) alone.

In the following section, we turn to the dynamic analysis by first studying the localisation and the evolution of the frontiers (delimiting notably the interior region from the zero maintenance space) with respect to the quota. Then, we deal with the issue of the admissibility of the different equilibria. Finally, we go back to the problem of divergence in a numerical example. It is worthnoting that the second and third steps of the analysis we be conducted under the use of specific utility and assimilation functions. This approach allows us not only to explicitly study the admissibility of steady states but also to compute the global dynamics so as to perform the simulations.

4 Effect of permits on dynamics

4.1 The frontier case

As explained in Prieur [2006], the study of the frontier case is central for analyzing the admissibility of steady states. In order to simplify this analysis and simply draw a tendency concerning the frontiers' behaviour relatively to the quota, we assume a constant policy *i.e.* $\bar{E}_t = \bar{E}\ \forall t$.

Definition 3 *In the $k - P$ space, the first frontier, delimiting irreversible pollution levels from reversible ones, corresponds to the irreversibility threshold: $P_t = \bar{P}$. The second frontier, hereafter called the indifference frontier, represents the set of points (k_t, P_t) where the agents are indifferent whether or not they invest in depollution. Let $P_t = f(k_t, \bar{E})$ be this frontier. When the system is located in the region above the frontier (resp. below), maintenance is non negative: $m_t \geq 0$ (resp. $m_t = 0$).*

The location of the steady states with respect to these frontiers is crucial when we are concerned with the question of admissibility. In fact, we have studied the four dynamic systems corresponding to each possible region in the $k - P$ space. We have next established the existence of solutions for two of these subspaces. But, it is possible that, during the convergence toward a stable solution of a determined zone, the equilibrium trajectory crosses one or the other frontier before reaching the steady state. Now, as soon as the trajectory goes through one of the two frontiers, the dynamics are governed by a new system totally different from the one valid in the previous region.

Let us focus on the properties of the indifference frontier. This frontier is implicitly given by the FOC (12) in which we set $m_t = \mu = 0$. It defines the pollution as a monotonic decreasing function of both the capital stock and the quota:[10] $P_t = f(\underset{-}{k_t}, \underset{-}{\bar{E}})$. The richer the economy, the

[10] Total differentiation of the FOC gives:

$$\frac{dP_t}{dk_t} = -\Omega_1 \frac{R_1 U_1 + (RU_{11} + \gamma U_{12})(R_1 \Omega + R)}{(1 - \Gamma')(RU_{12} + \gamma U_{22})}$$

lower the level of pollution from which the decision to depollute is taken.[11] Moreover, we note that the frontier falls, in the space of states variables, when the quota increases.

In fact, starting from an initial state in the corner zone, the economy will engage all the faster in depollution since the global quota is high. This feature has non trivial implications on the agents' behaviour. In fact, emiting an important amount of permits will hasten the moment when the agents will be willing to devote resources to depollution. This property is directly due to the tightening of the environmental constraint: more permits allocated to firms means more harmfull emissions at each period. The explanation is also based on the role played by the redistributive facet of environmental policy: a rise in $\bar{E}$ tends to increase the agents' income and implies that the weight of the financial constraint diminishes in their tradeoff. Finally, they have a greater incentive to depollute. On the contrary, fixing the quota to a lower level provokes a shifting of the frontier toward the top of the $k-P$ space and delays the instant when the depollution becomes effective.

4.2 Admissibility analysis

One may ask the question to know if whether or not it is better to set a high quota, so to reach the frontier rapidly but with a potentially important level of pollution, or to resort to a severe policy with a low quota (which implies a more distant frontier and a slower accumulation of pollution). Part of the answer is provided by the analysis of the different equilibria admissibility.

From now on, we consider specific forms of the utility and the assimilation functions. More precisely, we use the following logistic specification of the assimilation,

$$\Gamma(P_t) = \begin{cases} \theta P_t(\bar{P} - P_t) & \forall P_t < \bar{P} \\ 0 & P_t \geq \bar{P} \end{cases}$$

and we assume an additive utility function:

$$U(c_{t+1}, P_{t+1}) = \log c_{t+1} - \phi \frac{P_{t+1}^2}{2}.$$

Let us define the values $\check{k}(\bar{E})$ and $\hat{k}(\bar{E})$ as follows:

$$\check{k}(\bar{E}) = \left(\frac{1}{\gamma\phi(\bar{P} + \bar{E})(1-\alpha)A\bar{E}^{1-\alpha-\beta}} \right)^{\frac{1}{\alpha}}$$

$$\hat{k}(\bar{E}) = \left(\frac{1}{\gamma\phi\bar{E}(1-\alpha)A\bar{E}^{1-\alpha-\beta}} \right)^{\frac{1}{\alpha}}.$$

$$\frac{dP_t}{d\bar{E}} = -\frac{(R_1\Omega_2 + R_2)U_1 + (RU_{11} + \gamma U_{12})((R_1\Omega_2 + R_2)\Omega + R\Omega_2)}{(1-\Gamma')(RU_{12} + \gamma U_{22})}$$

and, under our assumptions on preferences, the Cobb-Douglas technology and the condition on the elasticity of substitution, these derivatives are negative.

[11] Which explains once again by the evolution of the balance of power between environmental and financial constraints.

The indifference frontier, for the example considered here, is given by (see appendix D.1):

$$f(k_t, \bar{E}) = \begin{cases} g(k_t, \bar{E}) & \text{for any } k_t \in [0, \check{k}(\bar{E})] \\ \frac{1}{2\theta}\left(-(1-\theta\bar{P}) + \sqrt{(1-\theta\bar{P})^2 + 4\theta g(k_t, \bar{E})}\right) & \text{for any } k_t \in (\check{k}(\bar{E}), \hat{k}(\bar{E})] \\ 0 & \text{for any } k_t > \hat{k}(\bar{E}) \end{cases} \quad (31)$$

with,

$$g(k_t, \bar{E}) = \frac{1}{\gamma\phi(1-\alpha)Ak_t^{\alpha}\bar{E}^{1-\alpha-\beta}} - \bar{E}.$$

Now we can focus on the admissibility of the different steady states. By definition, both corner and interior reversible solutions satisfy the first admissibility condition, concerning their location with respect to the irreversibility threshold. The study of the location of the two types of SS towards the other frontier is explained in appendix D.2. The respect of the second admissibility condition, simultaneously by corner and interior solutions, imposes that the quota belongs to a specific range: $\bar{E} \in [\bar{E}_i, \bar{E}_s]$. The upper bound refers to the location of stationary capital, for a corner solution, with regard to the striking value $\hat{k}(\bar{E})$. If the quota exceeds the amount $\bar{E}_s$, then the frontier cuts the abscissa axis ($P_t = 0$) before the level $\bar{k}(\bar{E})$. Therefore, the economy finds unable to reach the constraint solutions since these last are finally located in the interior space and consequently are not admissible. Thus the inequality $k^*_{cr}(\bar{E}) \leq \hat{k}(\bar{E})$, that can be rewritten $\bar{E} \leq \bar{E}_s$, is a necessary condition for admissibility of corner SS.[12]

It is worth noting that the ranking between the bounds of the range $[\bar{E}_i, \bar{E}_s]$ and $\bar{E}_L$ (defined by (26)) is *a priori* unknown. If $\bar{E}_L < \bar{E}_s$, then there exists a non empty range $(\bar{E}_L, \bar{E}_s)$ on which it is possible to fix a quota that avoids poverty traps and satisfies the necessary condition for admissibility of constraint SS. If, however, $\bar{E}_L \geq \bar{E}_s$, then we face the following tradeoff: imposing a quota $\bar{E} > \bar{E}_L$ ensures the absence of traps but is also translated into the impossibility of reaching the corner solutions. On the contrary, by choosing a quota such that $\bar{E} < \bar{E}_s$, the economy would potentially converge to the better or the worse SS. For interior solutions, the same reasoning applies, in a symmetrical way, when we consider the ranking between $\bar{E}_i$ and $\bar{E}_L$.

Now we proceed to simulations so as to measure the global dynamics' sensivity with respect to a change in the quota.

4.3 Numerical Example

The study of the interior equilibrium with irreversible pollution has shown that it is possible to exclude poverty traps provided that the exogenous quota is higher than the critical level $\bar{E}_L$ (see section 3.2.1). However, nothing garantees that the growth path of the economy, located in this region, is not divergent. The concept of divergence refers to the fact that the polluting economy

[12]The other inequality $\bar{E} \geq \bar{E}_i$ is only a sufficient admissibility condition for interior solutions (see appendix D.2).

suffers a perpetual rise in emissions which in turn is goes with a stage of economic recession (see figure 2 for an illustration of this kind of trajectory).[13] Note that divergence can also be interpreted as the convergence toward an asymptotic poverty trap.

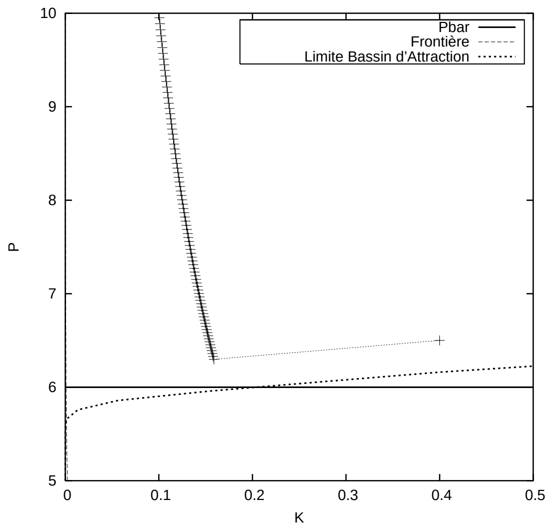

Figure 2: The divergence trajectory

Starting the analysis by assuming that the quota equals a level $\bar{E} > \bar{E}_L$, we wonder what is the impact of the choice of $\bar{E}$ on the possibility, for the economy, to reach the irreversibility space[14] knowing that it does not originally belong to it.

In order to deal with this issue, we compute the global dynamics of the model for the specifications given in 4.2 (see appendix E) and make some simulations for the following set of parameters:

$$\left\{A, \alpha, \beta, \theta, \gamma, \phi, \bar{P}\right\} = \{1.9, 0.3, 0.6, 0.15, 1, 1, 6\}$$

More precisely, we focus on the dynamics in the interior region and compare the evolution of the stable reversible solution's basin of attraction[15] with the one of the divergence region. Since

[13]The intuition behind the existence of this kind of development path is the following. In this region, the pollutant concentration is such that, on the one hand, nature does not assimilate pollution any more and, on the other hand, households suffer from the damages caused by pollution. In order to remedy to these damages, they have no other option than to devote a sizeable share of their resources to the depollution activity. But this decision goes against savings and consumption (that must remain positive according to preferences). Therefore, it translated into a break in capital accumulation. Moreover, this effort reveals unsufficient, on the duration, to compensate for polluting emissions by firms and to stop the rise in pollution. Even if, between the first and second periods, pollution decreases, we next see a fall in capital stock associated with an increase in pollution. This impoverishment process will inexorably reoccur, from periods to periods. The trajectory finally meets the equilibrium relation.

[14]The belonging to this region is a determining factor to explain the process of divergence.

[15]which is unique here and corresponds to the high SS defined in section 3.2.2.

the irreversibility threshold equals $\bar{E}_L = 0.58$, the simulations are made for two distinct value of the global quota: $\bar{E}_1 = 0.6$ and $\bar{E}_2 = 1.2$. Graphical representations of the basins of attraction, for both of these values, is displayed in figures 3 and 4.

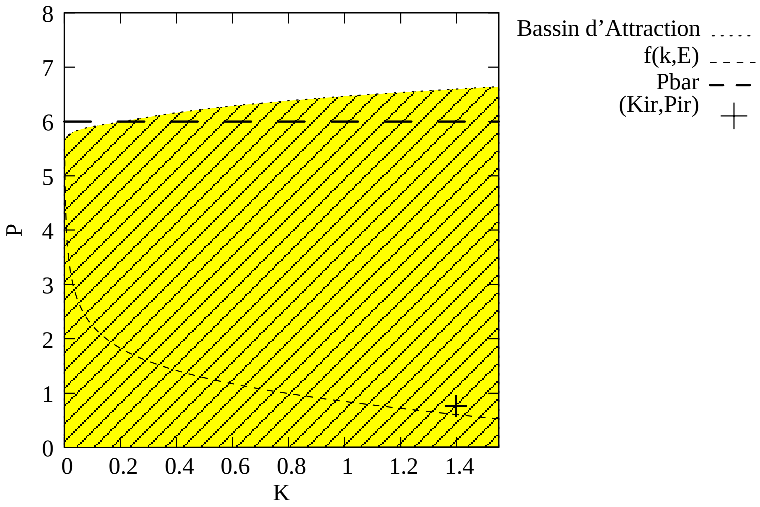

Figure 3: Basin of attraction and divergence region when $\bar{E} = 0.6$

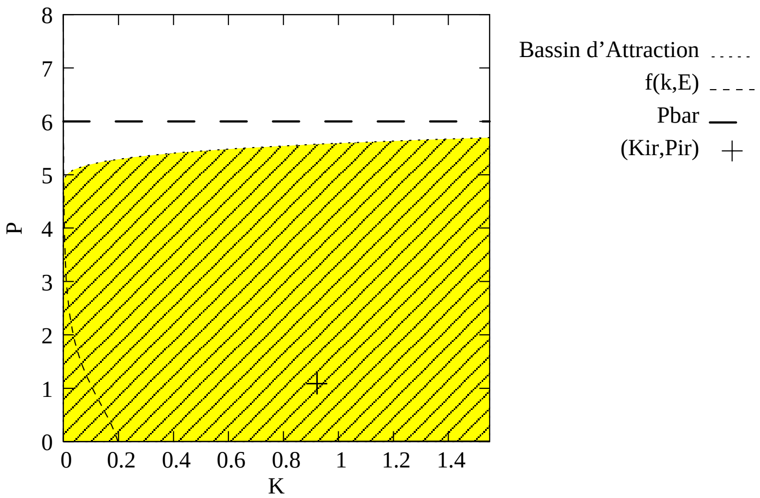

Figure 4: Basin of attraction and divergence region when $\bar{E} = 1.2$

The comparison betwen these two graphics clearly reveals that the "frontier" delimiting the basin of attraction from the divergence region goes down, in the $k - P$ space, when the quota raises. Whereas diverging suppose to be initially situated in the irreversible pollution region when the quota is low (except for very low capital levels), we see that the set of initial conditions from which the economy experiments (asymptotic) divergence exhibits pollution levels less than the irreversibility threshold $\bar{P}$ once the quota is relatively important. In other words, choosing the strictest quota minimizes, indeed rules out, the risk, for an economy that does not initially suffer from the irreversibility of environmental damages, to follow a development trajectory characterized by the impoverishment in environmental and physical capitals.

This property seems quite natural insofar as a high quota of permits contributes to reinforce the environmental constraint weight. The rhythm of pollution accumulation is more sustained and consequently the recovery process of nature is saturated faster. In turn, agents react by giving the priority to depollution expenditures to the detriment of wealth accumulation. The impoverishment mechanism decribed above will finally arise for lower pollution levels (and less than the irreversibility threshold).

From this numerical example, we then confirm the results obtained, for interior solutions, during the stationary analysis since the observations tend to recommend to announce the lowest quota provided that it is greather than the critical threshold $\bar{E}_L$.

Before ending this discussion, we have to express the following remark. The fact to emit a quota $\bar{E} = \bar{E}_L + \varepsilon$, with $\varepsilon > 0$ infinitesimal, protects the economy not only from a convergence toward a poverty trap but also from a process of divergence as soon as its environment, at the initial period, is safe.[16] However, it remains the hypothetical cas where initial pollution is already irreversible. Considering this extreme situation would logically induce us to review appreciably our conclusions. In this case, one can expect that public autorities will have to set the quota to a very low level, and less than $\bar{E}_L$, in order to allow the economy to stabilize at a stationary poverty trap. The convergence toward these states may finally constitute a lesser evil with regard to the perpetual impoverishment that goes with the process of divergence.

5 Conclusion

In an OLG model with irreversible pollution, Prieur [2006] has shown that a possible outcome of the development process, without pollution control, is the convergence toward an economic and ecological poverty trap. This paper first addresses the question to know whether or not, a pollution regulation through the implementation of a permits system is a mean to prevent the economy from reaching a trap. In this framework, the economy can potentially face two traps of different nature. The first one is a steady state, with an irreversible level of pollution and a low level of wealth, in which the economy can stabilize in the long run. The second one is similar to an "asymptotic" poverty trap in the sense that it corresponds to a growth path associated with a perpetual erosion in both economic and environmental resources. The analysis reveals the existence of a critical threshold for polluting emissions. Now, choosing an emission quota above

[16]This is the case of study that *a priori* makes the more sense. In fact, the idea that we get about the role of a system of pollution regulation is precisely to intervene before facing an irreparable situation.

this level is a mean to avoid the "stationary" trap. Moreover, fixing the quota at the lowest level beyond this threshold is also sufficient to protect the economy, that is not initially endowed with an irreversible level of pollution, against the recession going with the asymptotic trap.

In the context of the absence of traps, we next turn to the analysis of an environmental policy reform that consists in a reduction of the global quota on emissions. Its repercussions are widely dependent on the type of equilibrium considered. In fact, the equilibria with reversible pollution are only distinguished by the fact that private agents engage, or not, in maintenance. Therefore, at the corner solution (no maintenance), the fall in the quota effectively causes a decrease in the level of stationary pollution. But, this effort is detrimental to capital accumulation. In other words, there exists a dilemma between pollution control and economic growth. On the contrary, at the interior equilibrium (positive maintenance), we show that a tightening of environmental policy goes with both a fall in pollution and a rise in capital at steady state. Thus, an environmentally ambitious reform of the permits system brings a double dividend to the economy. This striking result echoes the conclusions of the literature on tax reform (see notably Bovenberg and Smulders [1995], [1996] and Bovenberg and de Mooij [1997]). However, in contrast with these papers, it does not rest on the controversial assumption of the existence of a strong environmental externality in production.

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Appendix

A. Existence conditions of interior equilibria

A.1 Properties of $m(k, \bar{E})$:

For a given $\bar{E} > 0$, we have:

- First, $\exists! k \in [0, +\infty[$ such that $m(k, \bar{E}) = 0$. Let $\bar{k}(\bar{E})$ be this value. We restrict the remainder of the study to the range $[0, \bar{k}(\bar{E})]$ on which we have $m(k, \bar{E}) \geq 0$ with:

$$\bar{k}(\bar{E}) = \left[(1-\alpha)A\bar{E}^{1-\alpha-\beta}\right]^{\frac{1}{1-\alpha}}$$

and we see that $\bar{k}'(\bar{E}) > 0$: the upper bound of the interval is increasing in the quota.[17] It is worthnoting that this upper bound exactly corresponds to the level of capital $k_c^*(\bar{E})$ reached at the constraint SS.

- Second, $\exists! k \in [0, \bar{k}(\bar{E})]$ such $m_1(k, \bar{E}) = 0$. Let $\tilde{k}(\bar{E})$ be this value. We know $m_1(k, \bar{E}) \geq 0$ $\forall k \leq \tilde{k}(\bar{E})$ with:

$$\tilde{k}(\bar{E}) = \left[\alpha(1-\alpha)A\bar{E}^{1-\alpha-\beta}\right]^{\frac{1}{1-\alpha}}$$

note that $\tilde{k}'(\bar{E}) > 0$ $\forall \bar{E} > 0$.

- Finally, we get $m_{11}(k, \bar{E}) < 0$, $m(0, \bar{E}) = m(\bar{k}(\bar{E}), \bar{E}) = 0$ $\forall \bar{E} > 0$ and

$$m(\tilde{k}(\bar{E}), \bar{E}) = A(1-\alpha)^2 \left[\alpha(1-\alpha)A\right]^{\frac{\alpha}{1-\alpha}} \bar{E}^{\frac{1-\alpha-\beta}{1-\alpha}} > 0 \ \forall \bar{E} > 0.$$

Once these properties are known, we can focus on the existence conditions

A.2 Existence of an irreversible SS (proposition 3):

The capital, at SS, solves the following equation $\bar{E} = \gamma m(k, \bar{E})$. This equation admits a solution if and only if:

$$\gamma \left(\max_{k \in [0, \bar{k}(\bar{E})]} m(k, \bar{E}) \right) \geq \bar{E}$$

that is, $\gamma m(\tilde{k}(\bar{E}), \bar{E}) \geq \bar{E}$.

Direct computations reveal that the relation $\gamma m(\tilde{k}(\bar{E}), \bar{E}) = \bar{E}$ has a unique fixed point $\bar{E}_l$ with:

$$\bar{E}_l = \left(\gamma A(1-\alpha)^2\right)^{\frac{1-\alpha}{\beta}} \left(\alpha(1-\alpha)A\right)^{\frac{\alpha}{\beta}}$$

and it follows that $\gamma m(\tilde{k}(\bar{E}), \bar{E}) \geq \bar{E} \leftrightarrow \bar{E} \leq \bar{E}_l$.

[17] Moreover, we compute the partial derivatives with respect to $\bar{E}$:

$$\begin{aligned} m_2(k, \bar{E}) &= (1-\alpha-\beta)(1-\alpha)Ak^{\alpha}\bar{E}^{-\alpha-\beta} > 0 \\ m_{22}(k, \bar{E}) &= -(\alpha+\beta)(1-\alpha-\beta)(1-\alpha)Ak^{\alpha}\bar{E}^{-\alpha-\beta-1} < 0 \end{aligned}$$

A.3 Existence of a reversible SS (proposition 3):

First of all, it is worth noting that the study (existence, stability) of interior steady states will be performed for a given $\bar{E}$. Thus, it is possible to rewrite the equilibrium relation as follows:

$$\Phi(k, \bar{E}) = \varphi(k) \tag{32}$$

with $\varphi'(k) < 0$.

The admissibility of a reversible SS requires $P < \bar{P}$. According to the equilibrium relation (32), this is equivalent to $k > \underline{k} = \varphi^{-1}(\bar{P})$. So, in order to deals with the issue of existence, the range $]\underline{k}, \bar{k}(\bar{E})]$ must be non empty.

Then, the study boils down to compare the behaviour of two functions of k, for a given $\bar{E}$. We restrict this study to the case where $\bar{E} > \bar{E}_l$, the condition that ensures the absence of poverty traps.

- the first function $\Theta(k)$ gives the amount of stationary emissions: $\Theta(k) = \bar{E} - \gamma m(k, \bar{E})$. Its behaviour derives from the properties of $m(k, \bar{E})$ and from the condition $\bar{E} > \bar{E}_l$: $\Theta(k) > 0$ $\forall k \in]\underline{k}, \bar{k}(\bar{E})]$, $\Theta'(k) = -\gamma m_1(k, \bar{E})$, $\Theta(0) = \Theta(\bar{k}(\bar{E})) = \bar{E}$. It is first decreasing until $\tilde{k}(\bar{E})$, then increasing until $\bar{k}(\bar{E})$. It is convex: $\Theta''(k) > 0$. Thus, $\Theta(k)$ has a U shape.

- by substituting the expression of P given by(32) in the assimilation function, we get the second function $\Lambda(k)$: $\Lambda(k) = \Gamma(\varphi(k))$. The sign of its derivative, $\Lambda'(k) = \varphi'(k)\Gamma'(\varphi(k))$, follows from the properties of $\Gamma()$ since $\varphi'(k) < 0$: it is negative when $\varphi(k) \in [0, \tilde{P}] \leftrightarrow k \in [\varphi^{-1}(\tilde{P}), \varphi^{-1}(0)]$ while it is positive for any $k \in [\varphi^{-1}(\bar{P}), \varphi^{-1}(\tilde{P})]$. Thus, $\Lambda(k)$ inherits from the inverted-U shape of $\Gamma()$. Moreover, we have $\Lambda(\underline{k}) = 0$ and $\Lambda(\varphi^{-1}(\tilde{P})) = \Gamma(\tilde{P}) = \Gamma_{\max}$.

From these two functions' properties, we can deduce the ranking at the lower bound of the interval: $\Theta(\underline{k}) > \Lambda(\underline{k}) = 0$. Following Prieur (2006), we now set the two following conditions:

The first condition,

$$\max_{k \in]\underline{k}, \bar{k}(\bar{E})]} \{\Lambda(k)\} \geq \max_{k \in]\underline{k}, \bar{k}(\bar{E})]} \{\Theta(k)\}$$

is a rewriting of the necessary and sufficent condition (18) for the existence of a constraint SS since it is equivalent to $\Gamma_{\max} \geq \bar{E}$. It implies that the two curves intersect on the interval $]\underline{k}, \varphi^{-1}(\tilde{P})]$. Then, we impose a technical condition

$$\bar{k}(\bar{E}) \geq \varphi^{-1}(\tilde{P}) \leftrightarrow \tilde{P} \leq \varphi(\bar{k}(\bar{E}))$$

so as to ensure not only the non-emptiness of the studied range $]\underline{k}, \bar{k}(\bar{E})]$ but also that the intersection point is reached before the upper bound $\bar{k}(\bar{E})$.

Therefore, under these conditions, there exist(s) at least one (and at most two) steady state(s) with positive maintenance and reversible pollution. In case of unicity, we necessarily have $k^*_{ir} < \varphi^{-1}(\tilde{P})$ and, consequently, $P^*_{ir} > \tilde{P}$. When there are two solutions, their respective levels of pollution are located from both sides of the striking value $\tilde{P}$. Note that the additional condition

$\Theta(\bar{k}(\bar{E})) > \Lambda(\bar{k}(\bar{E}))$, by fixing the ranking at the upper bound $\bar{k}(\bar{E})$, is sufficient to garantee the existence of two solutions.

B. Local Dynamics

B.1 The constraint equilibrium:

By linearizing the system (16) around a steady state $(k^*_{cr}(\bar{E}), P^*_{cr}(\bar{E}))$, we get:

$$\begin{cases} dk_{t+1} = \Omega_1(k^*_{cr}(\bar{E}), \bar{E})dk_t \\ dP_{t+1} = (1 - \Gamma'(P^*_{cr}(\bar{E})))dP_t \end{cases}$$

The conditions $\Omega_1(k^*_{cr}(\bar{E}), \bar{E}) < 1$ and $\Gamma'(P^*_{cr}(\bar{E})) > 0$ are sufficient to prove local stability. For any $\bar{E} > 0$, there exits a unique solution $k^*_{cr}(\bar{E})$ and we have $\Omega_1(k^*_{cr}(\bar{E}), \bar{E}) < 1$. Thus, the property of stability derives from the location of the level of stationary pollution with respect to the value $\tilde{P}$ such that $\Gamma'(\tilde{P}) = 0$. When there is only one solution, it precisely corresponds to this value, $P^*_{cr}(\bar{E}) = \tilde{P}$, which implies that the second condition is not satisfied since $\Gamma'(P^*_{cr}(\bar{E})) = 0$. When the inequality in (18) is strict, the second equation in (17) admits two solutions that are located from both sides of $\tilde{P}$: $P^*_{cr}(\bar{E})^- < \tilde{P} < P^*_{cr}(\bar{E})^+ \leftrightarrow \Gamma'(P^*_{cr}(\bar{E})^+) < 0 < \Gamma'(P^*_{cr}(\bar{E})^-)$. Hence, we can conclude that the "low" SS is locally stable while the other is unstable.

Note that the first condition (resp. the second) reflects the system's ability to assimilate a shock on capital (resp. on pollution).

B.2 The interior reversible SS:

For a reversible SS $(k^*_{ir}(\bar{E}), P^*_{ir}(\bar{E}))$, the linearization of (24) gives the Jacobian matrix (J):

$$\begin{pmatrix} dk_{t+1} \\ dP_{t+1} \end{pmatrix} = \begin{pmatrix} -\frac{\gamma\Omega_1(k^*_{ir}(\bar{E}),\bar{E})}{\varphi'(k^*_{ir}(\bar{E}))-\gamma} & \frac{1-\Gamma'(P^*_{ir}(\bar{E}))}{\varphi'(k^*_{ir}(\bar{E}))-\gamma} \\ -\frac{\gamma\Omega_1(k^*_{ir}(\bar{E}),\bar{E})\varphi'(k^*_{ir}(\bar{E}))}{\varphi'(k^*_{ir}(\bar{E}))-\gamma} & \frac{(1-\Gamma'(P^*_{ir}(\bar{E})))\varphi'(k^*_{ir}(\bar{E}))}{\varphi'(k^*_{ir}(\bar{E}))-\gamma} \end{pmatrix} \begin{pmatrix} dk_t \\ dP_t \end{pmatrix}$$

Now, it is clear that $\det(J) = 0$. In fact, due to the equilibrium relation (32), the system reduces to a one dimensional dynamics. Thus, the two eigenvalues are: $\lambda_1 = 0$ and $\lambda_2 =$tra(J) with

$$tra(J) = \frac{-\gamma\Omega_1(k^*_{ir}(\bar{E}), \bar{E}) + (1 - \Gamma'(P^*_{ir}(\bar{E})))\varphi'(k^*_{ir}(\bar{E}))}{\varphi'(k^*_{ir}(\bar{E})) - \gamma}.$$

Knowing that $\Omega_1() > 0$ and $\varphi'() < 0$, this expression is positive since the assumption $\Gamma(P) < P \ \forall P$ implies $\Gamma'(P) < 1 \ \forall P$.

Therefore, the following condition is sufficient for local stability:

$$\gamma(1 - \Omega_1(k^*_{ir}(\bar{E}), \bar{E})) > \Gamma'(P^*_{ir}(\bar{E}))\varphi'(k^*_{ir}(\bar{E}))$$

and we shall note that the two conditions set in $B.1$ $(\Omega_1(k^*_{ir}(\bar{E}), \bar{E}) < 1$ and $\Gamma'(P^*_{ir}(\bar{E})) > 0)$ ensure that this inequality holds.

C. Proof of proposition 5

In appendix $A.3$, we have analyzed the existence of an interior reversible SS by comparing the behaviour of two functions of k for a given quota $\bar{E}$. Now, we consider the impact of a change in $\bar{E}$ on the equilibrium outcome $(k_{ir}^*(\bar{E}), P_{ir}^*(\bar{E}))$.

The steady state solves the following system:

$$\begin{cases} P_{ir}^* = \Phi(k_{ir}^*, \bar{E}) \\ \Gamma(P_{ir}^*) = \Theta(k_{ir}^*, \bar{E}) \end{cases}$$

By substituting the equilibrium relation in the second equation, we get:

$$\Gamma(\Phi(k_{ir}^*, \bar{E})) = \Theta(k_{ir}^*, \bar{E}).$$

This equation implicitly defines k_{ir}^* as a function of $\bar{E}$: $k_{ir}^* = k_{ir}^*(\bar{E})$ with,

$$\frac{dk_{ir}^*}{d\bar{E}} = \frac{\Theta_2 - \Phi_2\Gamma'}{\Phi_1\Gamma' - \Theta_1}.$$

Provided that our analysis only makes sense for the stable SS, we refer to the two sufficient conditions fo local stability: $\Omega_1(k_{ir}^*(\bar{E}), \bar{E}) < 1$ and $\Gamma'(P_{ir}^*(\bar{E})) > 0$. The sign of the partial derivatives Φ_1 and Φ_2 being known, it remains to determine the sign of Θ_2. The emissions function writes $\Theta(k, \bar{E}) = \bar{E} - \gamma m(k, \bar{E})$. For any k, its derivative with respect to $\bar{E}$ is: $\Theta_2(k, \bar{E}) = 1 - \gamma\Omega_2(k, \bar{E})$. Since $\Omega_{12}(k, \bar{E}) > 0$, Ω_2 is increasing in k. Computing its value at the upper bound $\bar{k}(\bar{E})$ yields:

$$\Omega_2(\bar{k}(\bar{E}), \bar{E}) = (1-\alpha-\beta)(A(1-\alpha))^{1/(1-\alpha)}\bar{E}^{-\beta/(1-\alpha)}.$$

Now, it appears that $\Omega_2(\bar{k}(\bar{E}), \bar{E}) \leq 1/\gamma$ is equivalent to:

$$\bar{E} \geq (\gamma(1-\alpha-\beta))^{(1-\alpha)/\beta}(A(1-\alpha))^{1/\beta}.$$

Let $\bar{E}_c$ be this lower bound. For any $\bar{E} \geq \bar{E}_c$, we have $\Theta'(k, \bar{E}) \geq 0$ $\forall k \in]\underline{k}, \bar{k}(\bar{E})]$.[18] Therefore, if this inequality holds (condition (29) in prop. 5), then it appears that $k_{ir}^{*\prime}(\bar{E}) < 0$

Next, we replace k_{ir}^* with $k_{ir}^*(\bar{E})$ in the equilibrium relation so as to compute the derivative of $P_{ir}^*(\bar{E})$. We get

$$P^{*\prime}(\bar{E}) = \frac{\Phi_1\Theta_2 - \Phi_2\Theta_1}{\Phi_1\Gamma' - \Theta_1}$$

[18] Since we restrict the analysis to quotas that are greater than the threshold $\bar{E}_l$, imposing $\bar{E}_l \geq \bar{E}_c$ is suficient to conclude. More precisely, $\bar{E}_l \geq \bar{E}_c \leftrightarrow$

$$\beta \geq (1-\alpha)(1-\alpha^{\alpha/(1-\alpha)})$$

This bound is not very restrictive. If we suppose that the share of labour in production $1-\nu$ belongs the range $(0.6, 0.7)$ (which is the common range for the estimations of this parameter), then this inequality is satisfied, for instance, for $\zeta = 1$.

and, this expression is equivalent to:

$$P^{*\prime}(\bar{E}) = \frac{U_1(\Theta_1 R_2 - \Theta_2 R_1) + (RU_{11} + \gamma U_{12})(\Theta_1 c_2 - \Theta_2 c_1)}{(\Phi_1 \Gamma' - \Theta_1)(RU_{12} + \gamma U_{22})}.$$

The denominator and the first term in the numerator are positive. Since $RU_{11} + \gamma U_{12} < 0$, imposing $\Theta_1 c_2 - \Theta_2 c_1 < 0$ (second condition in prop. 5) ensures $P_{ir}^{*\prime}(\bar{E}) > 0$. This condition rewrites:

$$\frac{\Theta_1(k_{ir}^*(\bar{E}), \bar{E})}{\Theta_2(k_{ir}^*(\bar{E}), \bar{E})} < \frac{c_1(k_{ir}^*(\bar{E}), \bar{E})}{c_2(k_{ir}^*(\bar{E}), \bar{E})}.$$

Now, note that consumption and emissions, at steady state, express as follows:

$$c(k_{ir}^*(\bar{E}), \bar{E}) = R(k_{ir}^*(\bar{E}), \bar{E}) = c_{ir}(\bar{E})$$

$$\Theta(k_{ir}^*(\bar{E}), \bar{E}) = \bar{E} - \gamma m(k_{ir}^*(\bar{E}), \bar{E}) = \Theta_{ir}(\bar{E})$$

and their derivative with respect to $\bar{E}$ reads respectively:

$$c_{ir}'(\bar{E}) = c_1 k_{ir}^{*\prime}(\bar{E}) + c_2$$

$$\Theta_{ir}'(\bar{E}) = \Theta_1 k_{ir}^{*\prime}(\bar{E}) + \Theta_2.$$

Thus imposing $c_{ir}'(\bar{E}) < 0$ and $\Theta_{ir}'(\bar{E}) > 0$ implies:

$$-\frac{\Theta_1(k_{ir}^*(\bar{E}), \bar{E}) k_{ir}^{*\prime}(\bar{E})}{\Theta_2(k_{ir}^*(\bar{E}), \bar{E})} < 1 < -\frac{c_1(k_{ir}^*(\bar{E}), \bar{E}) k_{ir}^{*\prime}(\bar{E})}{c_2(k_{ir}^*(\bar{E}), \bar{E})}$$

and the condition (30) follows from this ranking.

D. admissibility Analysis

D.1 The indifference Frontier:

First of all, under our specifications for $U(c, P)$ and $\Gamma(P)$, note that the FOC (and the equilibrium relation) reads:

$$P_t = \frac{1}{\gamma \phi k_t}.$$

The expression of the frontier $f(k_t, \bar{E})$ then follows from the FOC in which we set $m_t = \mu = 0$:

$$-\frac{1}{\Omega(k_t, \bar{E})} + \gamma \phi (P_t - \Gamma(P_t) + \bar{E}) = 0. \tag{33}$$

Its particular shape is dependent on the level of pollution (reversible or not):

- when pollution is irreversible, the frontier is given by:

$$P_t = f^1(k_t, \bar{E}) = \frac{1}{\gamma \phi (1-\alpha) A k_t^{\alpha} \bar{E}^{1-\alpha-\beta}} - \bar{E} \tag{34}$$

and this expression is valid as long as $P_t \geq \bar{P}$, or:

$$k_t \leq \check{k}(\bar{E}) = \left(\frac{1}{\gamma\phi(\bar{P}+\bar{E})(1-\alpha)A\bar{E}^{1-\alpha-\beta}}\right)^{\frac{1}{\alpha}}.$$

If $\check{k}(\bar{E}) > \bar{k}(\bar{E})$, then it is always located above the irreversibility threshold.

Assume now that $\check{k}(\bar{E}) < \bar{k}(\bar{E})$,[19] for any $k_t > \check{k}(\bar{E})$, the computation of the frontier requires to solve the following polynomial:

$$P_t(1-\theta(\bar{P}-P_t)) - \left(\frac{1}{\gamma\phi(1-\alpha)Ak_t^{\alpha}\bar{E}^{1-\alpha-\beta}} - \bar{E}\right) = 0.$$

Let us suppose that the discriminant is positive,

$$\Delta^f = (1-\theta\bar{P})^2 + 4\theta\left(\frac{1}{\gamma\phi(1-\alpha)Ak_t^{\alpha}\bar{E}^{1-\alpha-\beta}} - \bar{E}\right) > 0.$$

Direct calculations reveal that the first root is always negative and can be excluded. Thus, we focus on the second root that is *a priori* associated with positive and lower than the threshold $\bar{P}$ values of pollution

$$P_t = \frac{-(1-\theta\bar{P}) + \sqrt{\Delta^f}}{2\theta}. \tag{35}$$

The function in (35) is monotonic decreasing in k. So, there existes a unique value $\hat{k}(\bar{E})$ such that it crosses the abscissa ($P = 0$) with,

$$\hat{k}(\bar{E}) = \left(\frac{1}{\gamma\phi\bar{E}(1-\alpha)A\bar{E}^{1-\alpha-\beta}}\right)^{\frac{1}{\alpha}}$$

and it appears that, for any $k_t \leq \hat{k}(\bar{E})$, the square root in (35) is non negative. Moreover, we confirm that Δ^f is stricly positive on $[0, \hat{k}(\bar{E})]$.

Now, provided that we only consider non negative levels of pollution, the expression of the frontier, when pollution is reversible, is:

$$P_t = f^2(k_t, \bar{E}) = \begin{cases} \frac{-(1-\theta\bar{P})+\sqrt{\Delta^f}}{2\theta} & \text{for } k_t \leq \hat{k}(\bar{E}) \\ 0 & \text{otherwise} \end{cases} \tag{36}$$

Beyond the level $\hat{k}(\bar{E})$, it precisely meets the abscissa. It means that the agents engage in maintenance regardless of the level of pollution.

D.2 Admissibility of reversible SS:

[19] This condition boils down to impose, for instance,

$$A \geq \frac{1}{(\gamma\phi(\bar{P}+\bar{E}))^{1-\alpha}(1-\alpha)\bar{E}^{1-\alpha-\beta}}.$$

For a constraint solution, it is straighforward that $P^*_{cr}(\bar{E}) < \bar{P}$. The admissibility also requires $P^*_{cr}(\bar{E}) < f(k^*_{cr}(\bar{E}), \bar{E})$. If $k^*_{cr}(\bar{E}) = \bar{k}(\bar{E}) \leq \check{k}(\bar{E})$, then the frontier is always located above the threshold $\bar{P}$, on the range $[0, \check{k}(\bar{E})]$. Thus the solution is admissible since $P^*_{cr}(\bar{E}) < \bar{P}$. Otherwise $(\bar{k}(\bar{E}) > \check{k}(\bar{E}))$, two cases possibly arise:

- if $\bar{k}(\bar{E}) \leq \hat{k}(\bar{E})$, we have to set a condition on parameters to ensure $P^*_{cr}(\bar{E}) < f(k^*_{cr}(\bar{E}), \bar{E})$.
- if $\bar{k}(\bar{E}) > \hat{k}(\bar{E})$, then the solution is inadmissible because $P^*_{cr}(\bar{E}) > f(k^*_{cr}(\bar{E}), \bar{E}) = 0$.

For the positive maintenance SS, by construction we have $k^*_{ir}(\bar{E}) > \varphi^{-1}(\bar{P}) \leftrightarrow P^*_{ir}(\bar{E}) < \bar{P}$. It remains to study its location with respect to the indifference frontier. Assume first that $k^*_{ir}(\bar{E}) \leq \check{k}(\bar{E})$. On the range $[0, \check{k}(\bar{E})]$, the frontier is above $\bar{P}$, so we have $f(k^*_{ir}(\bar{E}), \bar{E}) > \bar{P} > P^*_{ir}(\bar{E})$: the solution is not admissible since it is finally located in the constraint region. Assume next that $k^*_{ir}(\bar{E}) > \check{k}(\bar{E})$, in this case, it is always admissible since:

- if $k^*_{ir}(\bar{E}) \in]\check{k}(\bar{E}), \hat{k}(\bar{E})]$, then direct calculations show that $P^*_{ir}(\bar{E}) > f(k^*_{ir}(\bar{E}), \bar{E}) \leftrightarrow \gamma m(k^*_{ir}(\bar{E}), \bar{E}) > 0$ and this inequality is satisfied.
- if $k^*_{ir}(\bar{E}) > \hat{k}(\bar{E})$, we necessarily have $P^*_{ir}(\bar{E}) > f(k^*_{ir}(\bar{E}), \bar{E}) = 0$.

To summarize, the following double inequality: $\check{k}(\bar{E}) < \bar{k}(\bar{E}) \leq \hat{k}(\bar{E})$ is a necessary condition under which the two types of solutions are simultaneously admissible. In fact, if $\bar{k}(\bar{E}) \leq \check{k}(\bar{E})$ then the constraint solution alone is admissible while if, on the contrary, $\bar{k}(\bar{E}) > \hat{k}(\bar{E})$, only the interior SS is potentially admissible.

The condition $\bar{k}(\bar{E}) \leq \hat{k}(\bar{E})$, for a constraint SS, can be rewritten as follows: $\bar{E} \leq \bar{E}_s$ with

$$\bar{E}_s = \left(\frac{1}{A(1-\alpha)(\gamma\phi)^{1-\alpha}}\right)^{1/(2(1-\alpha))-\beta}.$$

The inequality $\check{k}(\bar{E}) < \bar{k}(\bar{E})$ is equivalent to:

$$((A(1-\alpha)\bar{E}^{1-\alpha-\beta})^{1/(1-\alpha)}\gamma\phi(\bar{E}+\bar{P}) > 1$$

we cannot express it in terms of a condition on $\bar{E}$. But, if we note that $\bar{E} < \bar{P}$, the right-hand side is greater than

$$2((A(1-\alpha)\bar{E}^{1-\alpha-\beta})^{1/(1-\alpha)}\gamma\phi\bar{E}$$

and, imposing that this expression is greater than one, which is equivalent to,

$$\bar{E} \geq \bar{E}_i = \left(\frac{1}{2A(1-\alpha)(\gamma\phi)^{1-\alpha}}\right)^{1/(2(1-\alpha))-\beta}$$

is sufficient to have $\bar{k}(\bar{E}) \leq \hat{k}(\bar{E})$.

E. Global Dynamics:

- In the constraint region, dynamics are given by:

$$k_{t+1} = (1-\alpha)Ak_t^{\alpha}\bar{E}^{1-\alpha-\beta}$$

$$P_{t+1} = P_t(1 - \theta(\bar{P} - P_t)) + \bar{E}$$

if pollution is reversible. Otherwise, the dynamics for pollution write:

$$P_{t+1} = P_t + \bar{E}$$

- Dynamics, when maintenance is positive, become:

$$k_{t+1} = \frac{1}{\gamma\phi P_{t+1}}$$

$$P_{t+1} = \frac{x(k_t, P_t) + \sqrt{x(k_t, P_t)^2 + 4/\phi}}{2} \geq 0$$

with,

$$x(k_t, P_t) = P_t + \bar{E} - \gamma(1 - \alpha)Ak_t^{\alpha}\bar{E}^{1-\alpha-\beta}$$

when pollution is irreversible and, if not:

$$x(k_t, P_t) = P_t(1 - \theta(\bar{P} - P_t)) + \bar{E} - \gamma(1 - \alpha)Ak_t^{\alpha}\bar{E}^{1-\alpha-\beta}.$$