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# The Synthesis toolbox

## From modal automata to controller synthesis

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# Chapter 1

# Introduction

Synthesis is a tool used to synthesize controllers for discrete event systems. Synthesis operates in a wider framework than the one used Ramadge and Wonham[6]. Here we consider infinite behaviours as well as finite ones; moreover, the user can specify some kind of dynamic controllability.

Synthesis is an implementation of algorithms proposed by several authors[3, 8]. We have implemented operations presented in [2] allowing the synthesis of decentralized controllers; the case of partial observability is not yet handled.

The tool considers the synthesis problem as a workflow which transforms the couple (*system*,*specification*) into several objects : a multi-parity game, a parity game, a winning strategy and, finally, the controller. Since Synthesis has been developped as an experimental tool, it does not implement the workflow in a one-step operation; on the contrary the workflow must be executed step by step. The user has the opportunity to study generated objects all along the Synthesis workflow.

Synthesis is a command-line oriented program. It is developped on a Linux system but works also on MacOS platforms. Future works will tackle other Unix-like platforms.

In this primer, we present formal aspects of the synthesis workflow as well as Synthesis in use. Appendices present the commands allowed by the Synthesis shell and the syntaxes of files supported by the program.

The first version of this toolbox was implemented by students under the direction of A. Arnold in 2002 and 2003: A. Cailley, S. Canto, H. Desheraud, A.-C. Froment, R. Gobard, S. Labrune, F. Mendes, S. Muhr and F. Zucconi.

The image used for the cover page of this report is an engraving called “Adam and Eve” made in 1504 by Albrecht Dürer.

Remarks and comments might be sent to the architects of Synthesis:

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# Chapter 2

# Starting Synthesis

Synthesis runs as a command interpreter offering history and file completion if it has been compiled with the GNU Readline library[7]. If the program can be used interactively like a UNIX shell, it can be used also in a batch mode in order to execute existing script files.

## 2.1 Getting started

When the program is started a synthesis prompt appears:

```
point@localhost: ~/tmp
synthesis
synthesis>
```

From this prompt the user can enter commands or leaves the interpreter. All the commands allowed by Synthesis are described in Appendix A page 27.

```
synthesis>print "hello world"
hello world
synthesis>exit
point@localhost: ~/tmp
```

To leave the program one can use an EOF character (CTRL-D on most systems) or the exit command; in both cases the history of the commands entered while the session are stored into the file .synthesis_history.syn in the current directory; this file is reloaded the next time Synthesis is started from this directory.

```
point@localhost: ~/tmp
cat .synthesis_history.syn
print "hello world"
exit
point@localhost: ~/tmp
```

The program might be executed in batch mode. This mode consists in calling Synthesis with file names; the program loads (or execute for scripts) each file and terminates. If the command-line option -i is used then Synthesis does not terminate i.e. after the reading of files the interactive mode is started. Example:

```
point@localhost: ~/tmp
cat t1.fam test.syn
name A;
x = nu -> <a>x.<b>y;
y = nu -> <a>y;
<initial={x}>.
```

```
print "this is a test"
point@localhost: ~/tmp
synthesis t1.fam test.syn
this is a test
point@localhost: ~/tmp
synthesis -i t1.fam test.syn
this is a test
synthesis>
```

## 2.2 Redirection

Synthesis uses a similar notation (but less powerfull) than standard UNIX shells to redirect output streams into files. The output of a command can be used to create a file (using >) or appended at the end of an existing file (using >>). If the output file does not exist >> has the same effect than >. Example:

```
point@localhost: ~/tmp
synthesis -i t1.fam
synthesis>print "// the multi-parity automaton A is translated in a simple" > result.fam
synthesis>print "// parity automaton" >> result.fam
synthesis>parity A >> result.fam
synthesis>exit
point@localhost: ~/tmp
cat result.fam
// the multi-parity automaton A is translated in a simple
// parity automaton
name $$;
T = 2 -> [a, b]T;
y = 2 -> <a>y.[b]T;
x = 2 -> <a>x.<b>y;
<initial={x}>.
point@localhost: ~/tmp
```

In the current version of Synthesis, just > and >> are implemented; there is no pipe mechanism and the error stream can not be redirected.

## 2.3 Input file formats

Synthesis supports several types of files but uses only one command to read them: load. The format of a file is identified by its extension:

.syn is associated with the script file format. These files contain lists of Synthesis commands. For instance this is the file format of the history file .synthesis_history.syn seen above.

.mec is associated with Mec 4[1] files restricted to simple transition systems (i.e. not obtained by a synchronized product).

.fam is associated with files describing modal automata (see section 4.1).

.game is associated with files describing parity games (see also section 4.1).

Transition systems, modal automata and games can not be entered directly from the Synthesis prompt.



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## 2.4 Displaying objects

Any object manipulated by `Synthesis` can be displayed using the `show` command. Each object is displayed using its native format; some translators exist (see Appendix A). In order to determine the objects in use by the program, one uses `show` with a type name: `processes`, `automata` or `games`. Example (2.2 continued):

```
synthesis>point@localhost: ~/tmp
syn -i t1.fam
synthesis>show processes
synthesis>show automata
A
synthesis>show games
synthesis>show A
name A;
T = nu -> [a, b]T;
y = nu -> <a>y.[b]T;
x = nu -> <a>x.<b>y;
<initial={x}>.
synthesis>
```

## 2.5 Obtaining help

The user can use the `help` command in order to obtain some explanation about `Synthesis` commands. `help` without argument lists all commands supported by the program and `help` *`cmd`* gives informations about the *`cmd`* command:

```
synthesis>help help
help usage :
Show help about commands. Type 'help command-name' in order to obtain help
about the command 'command-name'
synthesis>
```



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# Chapter 3

# Synthesis in practice

The purpose of this chapter is to show `Synthesis` in use. All along these pages we focus on a "small" problem: to prevent the collision of two trains. The system considered in this chapter is composed of two trains, $T_1$ and $T_2$, sharing the same railroad. In order to prevent the struck of trains, a deviation track is added along the main road. The trains can choose to continue on the main track or to follow the deviation. This system is depicted on the figure 3.1.

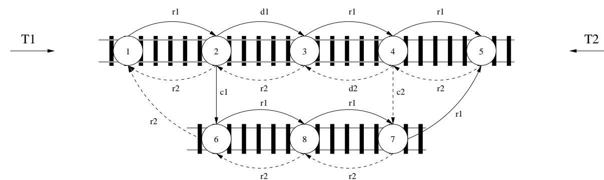


Figure 3.1: *$T_1$ comes from the left (at location 1) and $T_2$ comes from the right (at location 5). The possible moves for $T_1$ are drawn with filled arrows and those for $T_2$ with dashed arrows. Each train can move to the deviation track (from location 2 for $T_1$ and from 4 for $T_2$).*

Each train is modelled using a transition system. The movements of $T_i$ are modelled with the events $r_i$, $c_i$ and $d_i$: $c_i$ and $d_i$ are used in place of $r_i$ to indicate the change of way of the train. We make this replacement for two reasons:

- We use two distinct events because `Synthesis` allows only deterministic processes.
- We distinguish these events from $r_i$ because in the next section, $c_i$ and $d_i$ will be considered as controllable events while $r_i$ remains uncontrollable.

Each train is modelled by a transition system which looks like the system on figure 3.1 except that each state is equiped with loops labelled with the actions of the other train; the figure 3.2 depicts the model of the train $T_1$. These loops indicate that a movement of a train does not affect the state of the other.

The model of the whole system has 64 states and 112 transitions. `Synthesis` do not accept composite transition systems so this model has been obtained by hand:

- Load the models of trains $T_1$ and $T_2$ into `Synthesis`.
- Synchonize the two transition systems and generate a Mec 4 file containing this synchronized product.
- Edit the file :
  - Identify the 8 states where trains are struck and mark them with the property `danger`.
  - Remove useless transitions originating for `danger` states.

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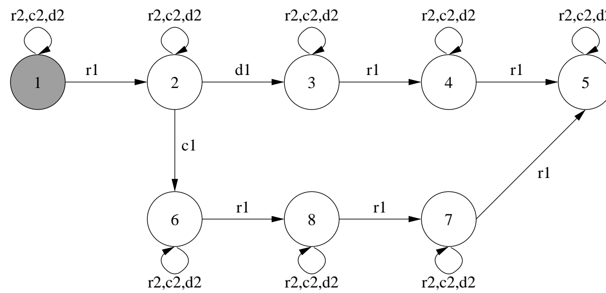


Figure 3.2: *The transition system modelling the train $T_1$. An action of $T_2$ does not change the state of $T_1$. The initial state is drawn in gray.*

- Identify the final state (in this state no more danger exists) and mark it with the property `final`.
- Save the file and go back into `Synthesis`.

Now, our goal is to prevent the trains to strike themselves. In order to achieve this objective we are looking for a controller which leads the trains into the `final` state and prohibits the paths to a `danger` state. Thus, the states of the supervised system (i.e. the system synchronized with the controller) must be a model of the following modal automaton:

```
name sys_spec;
x = mu -> ~danger.~final.(<r1>x.[r2,c1,c2,d1,d2]x
                         +<r2>x.[r1,c1,c2,d1,d2]x
                         +<c1>x.[r1,r2,c2,d1,d2]x
                         +<d1>x.[r1,r2,c1,c2,d2]x
                         +<d2>x.[r1,r2,c1,c2,d1]x
                         +<c2>x.[r1,r2,c1,d1,d2]x)
                  + final.([r1,r2,c1,c2,d1,d2]T);
<initial={x}>.
```

This automaton can be interpreted as follows:

- Either the system is in the final state (i.e there is no more danger for the trains);
- Either the system is in a state which is neither the final nor a danger state and from this state there must exist a transition leading the system into a state having the same property or being the final state.

The reader should have noted the `T` in the second member of the equation defining `x`. This `T` denotes an implicit state (called *top*) of the automaton. This state is defined by the following equation : `T = nu [*]T` where $[*]$ is a shorcut replacing all the events of the automaton.

## 3.1 A centralized controller

In the previous section we have defined the specification (`sys_spec`) that must be satisfied by the controlled system. In order to obtain the constraint that must be fulfilled by the controller, we compute the quotient of the specification by the system:

```
synthesis>load sys.mec spec.fam
Loading file 'spec.fam'
Loading file 'sys.mec'
synthesis>show processes
sys
synthesis>show automata
```

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```
sys_spec
synthesis>controller_spec := quotient sys_spec sys
```

The last line computes a new modal automaton, `controller_spec`, modelling the expected behaviours for the controller. Now we are able to compute a correct controller but, with no more specifications, the result would be a controller that is allowed to control any action of the trains. If the controller is seen as an automatic switch between tracks then it can't act on movement actions of the trains (i.e $r_i$); thus, the events $r_1$ and $r_2$ of the system must be specified uncontrollable:

```
name controller_additional_spec;
x = nu -> <r1,r2>x.[c1,c2,d1,d2]x;
<initial={x}>.
```

This automaton specifies that for each state of the controller there must exist at least two transitions, one labelled with $r_1$ and the other with $r_2$. In order to obtain one controller it remains:

- to compute the product `final_spec` of `controller_spec` with `controller_additional_spec`;
- to generate the parity game `G` computed from `final_spec`;
- to compute a winning strategy `S` in the game `G`;
- to generate the controller `C` from `S`;
- to synchronize the system and the controller to get the supervised system.

All these steps are done as follows:

```
synthesis>final_spec := product controller_spec controller_additional_spec
synthesis>G := game final_spec
synthesis>S := strategy G
synthesis>C := minimize(unmark(control S))
synthesis>
synthesis>Csys := sync sys C
synthesis>dot C > C.dot
synthesis>dot Csys >> Csys.dot
```

The controller is generated using the `control` command. To obtain `C` we apply two other operations to the result of `control`:

`minimize` This command reduces the number of states of the process. This is essentially an algorithm reducing the state space of an automaton; here we consider that all states are accepting but they are distinguished according to the properties labelling them (e.g. `danger` or `final` in our example).

`unmark` This command removes properties which label the states of a process. This is useful to reduce futher more the number of states of the controller when calling the `minimize` command.

The last lines computes the controlled system and outputs the controller and the controlled system into `dot` file format[5]. These results are shown on figure 3.3.

## 3.2 The decentralized case

In the previous section, the system was managed by a global controller which could act on controllable actions of both trains; the controller was considered as a program managing switches located at position 2 and 4.

Now we consider that there is two independant programs controlling switches. The controller `C1` (resp. `C2`) at location 2 (resp. 4) can control only the events $c_1$ and $d_1$ (resp. $c_2$ and $d_2$). While the constraint for the controlled system remains the same than in the centralized case, the additional specifications for the new controllers indicate that only $c_i$ and $d_i$ are controllables:

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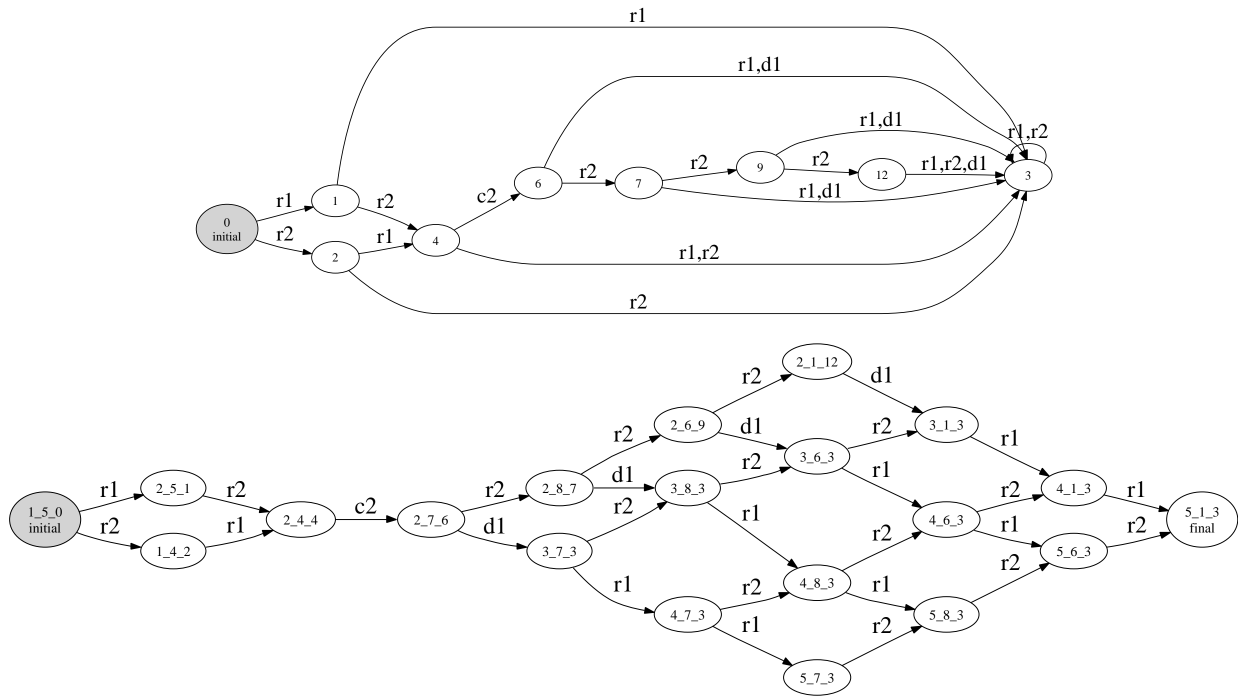


Figure 3.3: *The controller and the supervised system generated by* `Synthesis`*.*

```
name controller_at_2_additional_spec;
x = nu -> <r1,r2,c2,d2>x.[c1,d1]x;
<initial={x}>.
```

```
name controller_at_4_additional_spec;
x = nu -> <r1,r2,c1,d1>x.[c2,d2]x;
<initial={x}>.
```

In order to obtain a controller `C1` for the switch at location 2, we have to proceed as explained in Chapter 4: the specifications of the system `sys_spec` have to be divided by those of `C2` (i.e. `controller_at_4_additional_spec`) and by the process `sys`. The resulting quotient is multiplied by the specifications of `C1`.

```
synthesis>q := quotient sys_spec controller_at_4_additional_spec sys
synthesis>C1_spec := product q controller_at_2_additional_spec
synthesis>C1 := minimize(unmark(control (strategy (game C1_spec))))
synthesis>dot C1 > C1.dot
```

The figure 3.4 depicts `C1` and the system under its control. One should note that if `C1` is the only controller in use there remains critical scenarii. For instance `C1` allows the sequence `(r2`, `r1`, `d2`, `r2)` which leads both trains in the same place `2`.

Finally, the controller `C2` (drawn on figure 3.5) is computed using the following commands:

```
synthesis>q := quotient sys_spec (sync sys C1)
synthesis>C2_spec := product q controller_at_4_additional_spec
synthesis>C2 := minimize(unmark(control (strategy (game C2_spec))))
synthesis>Csys12 := sync (sync sys C1) C2
synthesis>dot Csys12 > Csys12.dot
```

The last commands compute the whole controlled system. The result is depicted on the figure 3.6. One can verify that the controlled system is safe: there is no `danger` state. Thanks to `Synthesis` we get a system for which the trains can not strike themselves.

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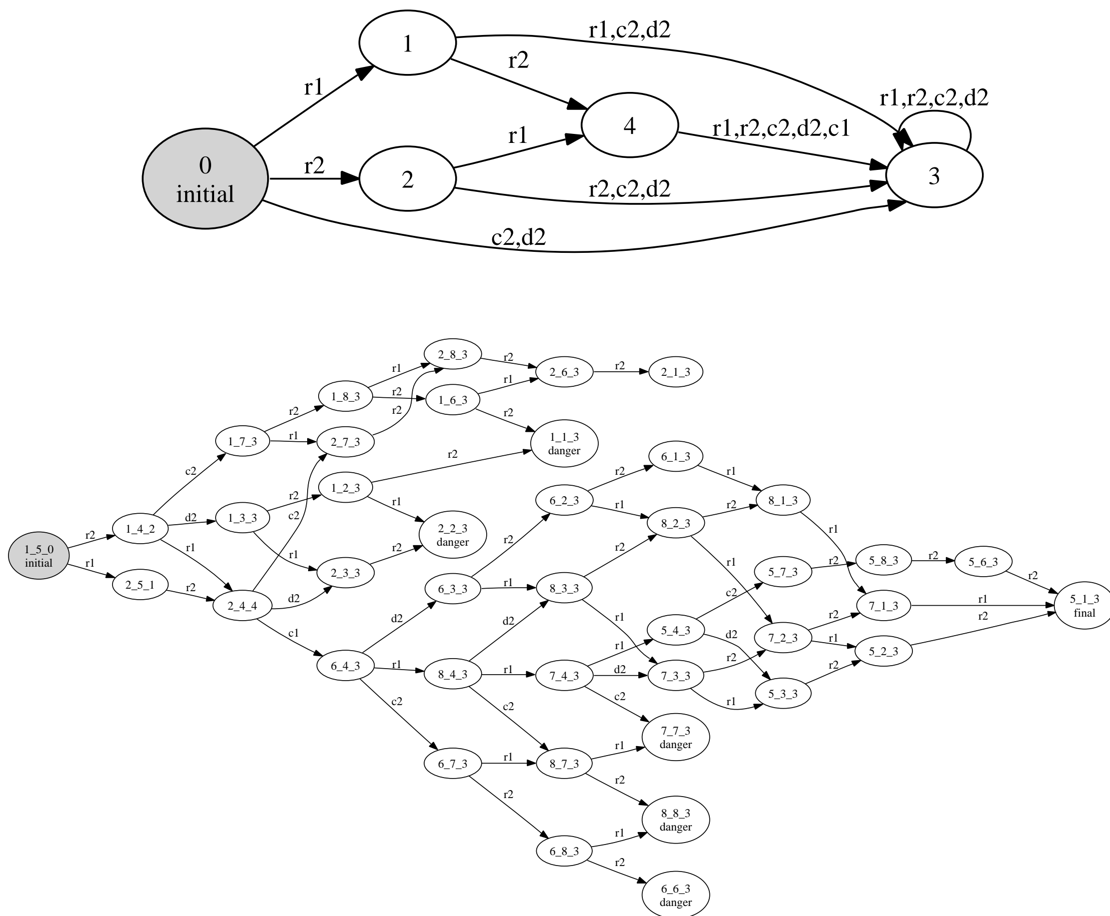


Figure 3.4: *The controller* `C1` *(upper graph) and the system under its control. One can note that all the states of the controller are complete with respect to uncontrollable events (i.e* $r_1$, $r_2$, $c_2$, $d_2$*).*

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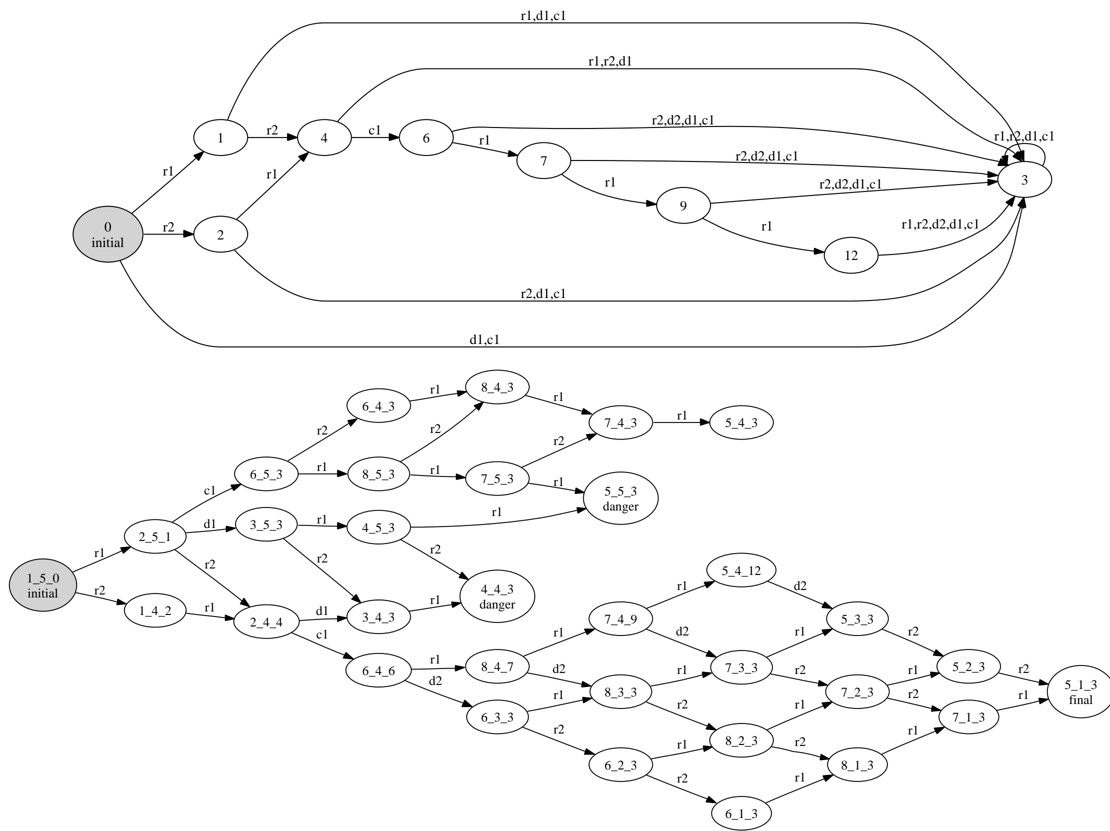


Figure 3.5: *The controller* `C2` *(upper graph) and the system under its control. States are complete w.r.t* $r_1$, $r_2$, $c_1$, $d_1$.

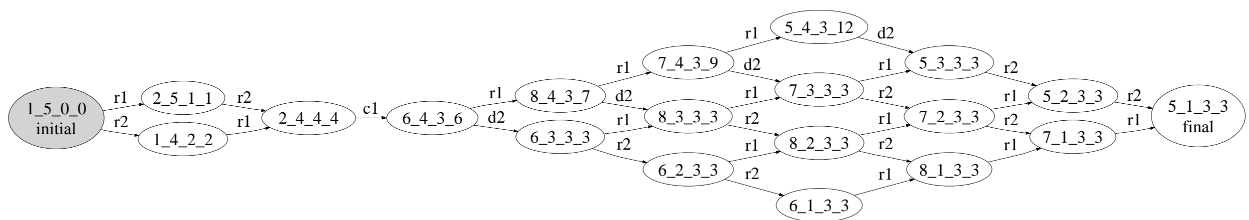


Figure 3.6: *The system supervised by the controllers* `C1` *and* `C2`.

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# Chapter 4

# Theoretical aspects of Synthesis

From now we consider finite and deterministic processes having finite or infinite behaviours. We denote by $P \models \Phi$ the fact that a process $P$ satisfies a property $\Phi$. Two processes $P_1$ and $P_2$ over the same set of actions might be synchronized on their common actions; this synchronization is denoted $P_1 \times P_2$.

From this setup, the *controller synthesis problem* can be expressed as follows:

($*$) Given a process $P$ and $n+1$ properties $\Phi$ and $\Psi_i$ for $i = 1 \ldots n$, the problem is to find $n$ processes $C_i$ such that $P \times C_1 \times \cdots \times C_n \models \Phi$ and, for all $i = 1 \ldots n$, $C_i \models \Psi_i$. $P \times C_1 \times \cdots \times C_n$ is called the *controlled* or *supervised system*. If $n > 1$ we say that the control is *decentralized*.

In the standard Ramadge and Wonham framework[6] we only require that all the prefixes of controlled behaviours belong in the language defined by $\Phi$; furthermore, Ramadge and Wonham's framework assumes a predefined set of *controllable* and *uncontrollable* events.

In the context of `Synthesis` we are also interested by infinite behaviours of the supervised system and there is no predefined controllable sets. The requirements, that the controlled system must fulfill, are expressed in terms of $\mu$-calculus formulas. The controllability of events is also expressed by this mean; this allows the user to describe dynamic controllability.

In [8] this problem is addressed and extended for partial observability in [2]. The authors define *division* operations ($/$) such that the controllers $C_i$ in the problem ($*$) verify the following properties:

$$
\begin{array}{l}
C_1 \models \Phi/\Psi_n/\ldots/\Psi_2/P \wedge \Psi_1 \\
C_2 \models \Phi/\Psi_n/\ldots/\Psi_3/(P \times C_1) \wedge \Psi_2 \\
\qquad\vdots \\
C_n \models \Phi/(P \times C_1 \times \cdots \times C_{n-1}) \wedge \Psi_n
\end{array}
$$

Now, in order to synthesize expected controllers, we have to find models of $\mu$-calculus formulas. The problem of finding a model of a $\mu$-calculus formula $F$ is addressed from the game theory point of view: finding a model for $F$ consists in finding a winning strategy in a parity game $G(F)$ built from $F$.

`Synthesis` implements all operations required to find the controllers $C_i$'s as expected in the controller synthesis problem ($*$). The production of a controller can be seen as a workflow transforming a process and $\mu$-calculus formulas (see figure 4.1). In this chapter we formalize the algorithms used to implement this workflow.

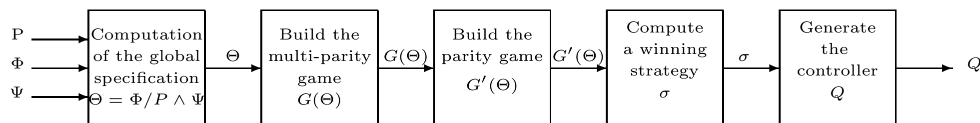


Figure 4.1: The `Synthesis` workflow

## 4.1 Processes, modal automata, games and strategies

A *process* is a vector $P = \langle A, \Lambda, S, \delta, \lambda, s_0 \rangle$ where $A$ is its set of actions, $S$ is its set of states, $s_0$ is the initial state and $\delta$ is the transition function defined from $S \times A$ to $S$. $\Lambda$ is a set of atomic propositions and $\lambda : S \to 2^\Lambda$ is the labelling function associating to each state a set of propositions supposed true in the state.

Given two processes $P_i = \langle A_i, \Lambda_i, S_i, \delta_i, \lambda_i, s_{0i} \rangle$ for $i = 1, 2$, the *product* of $P_1$ and $P_2$ is the process $P_1 \times P_2 = \langle A_1 \cup A_2, \Lambda_1 \cup \Lambda_2, S, \delta, \lambda, (s_{01}, s_{02}) \rangle$ where:

- $S = \{(s_1, s_2) \in S_1 \times S_2 \mid \forall p \in \Lambda_1 \cap \Lambda_2, p \in \lambda_1(s_1) \iff p \in \lambda_2(s_2)\}$
- $\delta((s_1, s_2), a) = (\delta_1(s_1, a), \delta_2(s_2, a))$ if $\delta_1(s_1, a)$ and $\delta_2(s_2, a)$ are defined
- $\lambda((s_1, s_2)) = \lambda_1(s_1) \cup \lambda_2(s_2)$.

Given an alphabet $A$ and a set of atomic propositions $\Lambda$, a *modal automaton* is a set of equations of the form:

$$x_i \overset{\rho(x)}{=} \bigvee_{j \in J_i} \left( \bigwedge_{p \in K_{i,j} \subseteq \Lambda} p \wedge \bigwedge_{q \in L_{i,j} \subseteq \Lambda} \neg q \wedge \bigwedge_{a \in A} (a) x_{a,i,j} \right)$$

where $x_i$, $x_{a,i,j}$ are states of the automaton, $p$ and $q$ are atomic propositions and $(a)$ denotes $\langle a \rangle$ or $[a]$. $\rho(x)$ assigns to each equation a vector of natural numbers called *ranks*.

More formally a modal automaton is a vector $\mathcal{A} = \langle A, \Lambda, X, R, \Delta, x_0, \rho \rangle$ where $A$ is an alphabet of actions, $X$ is its state space and $x_0$ is the initial state, $\Lambda$ is a set of atomic propositions, and:

- $\Delta : X \to 2^R$ assigns to each state a set of *rules* taken from the set $R \subseteq 2^\Lambda \times 2^\Lambda \times 2^A \times X^A$. A *rule* $r \in R$ represents one conjunction of the equation system; $r$ is a vector $\langle P, N, E, \sigma \rangle$ such that:
  - $P \subseteq \Lambda$ (resp. $N \subseteq \Lambda$) is a set of positive (resp. negative) atomic propositions and $P \cap N = \emptyset$;
  - $E$ is the set of actions that are existentially quantified;
  - $\sigma : A \to X$ is a mapping assigning to each action the equation that must be satisfied after the given action.
- $\rho : X \to \mathbb{N}^n$ is a mapping assigning to each state its *rank*. If $n = 1$ we say that $\rho$ is a *parity* condition otherwise it is a *multi-parity* condition.

Sometimes we use a particular state $\top$ (*top*). For this state we set: $\Delta(\top) = \langle \emptyset, \emptyset, \emptyset, A \times \{\top\} \rangle$ and $\rho(\top) = \langle 0, \dots, 0 \rangle$.

A modal automaton accepts processes. A process $P$ is accepted by a modal automaton $\mathcal{A}$ is denoted by $P \models \mathcal{A}$. This acceptance relation is defined in terms of winning strategies in games built from $\mathcal{A}$ and the accepted processes.

A *multi-parity game* is a vector $G = \langle V, V_0, V_1, v_0, E, \Omega \rangle$ where $V = V_0 \uplus V_1$ is the set of positions and $v_0$ is the starting position, $E \subseteq V \times V$ denotes allowed moves between positions (if $(v, v') \in E$ then $v'$ is called a *successor* of $v$) and $\Omega : V \to \mathbb{N}^n$ is a function assigning to each state a vector of natural numbers called *ranks* (or *priorities*); if $n = 1$ the game is called a *parity* game.

We consider games with two players 0 and 1, pushing a token over positions in $V$. A *play* is a sequence of positions starting at $v_0$. A turn proceeds in the following way: if the token is in a position $v \in V_0$ then the player 0 chooses $v'$ a successor of $v$, otherwise the player 1 makes the choice. Once the token is moved onto $v'$ a new turn is started. If a player can not move the token (i.e there is no edge outgoing from $v$ in $E$) he looses the play. Infinite plays are allowed; in this case, for a play $v_0 v_1 \dots$, the player 0 wins if the greatest rank encountered infinitely often in the sequence is even (in the case of multi-parity games, the condition must hold for each component of $\Omega(v_i)$).

A play is *maximal* if it is infinite or terminates in a position without outgoing edge. So, a maximal play is *winning* for player 0 if it satisfies the parity condition or ends in a vertex belonging to $V_1$ without successor.



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For every sequence of vertices $\vec{v}$ ending in a vertex $v$ from $V_0$, a *strategy* $\sigma$ for player 0 is a function such that $\sigma(\vec{v}) \in V$. We require $\sigma(\vec{v})$ to be defined if $v$ has a successor.

A play *respects* a strategy $\sigma$ if it is a finite or infinite sequence of positions $v_0 v_1 \ldots$ such that $v_{i+1} = \sigma(v_0 \ldots v_i)$ for every $i$ with $v_i \in V_0$.

A strategy for player 0 is *winning from a vertex* $v$ if and only if every maximal play starting in $v$ and respecting the strategy is winning for player 0. We say that a strategy for player 0 is *winning* if it is winning from every position from which there exists a winning strategy for player 0.

## 4.2 Product and quotient operations

### 4.2.1 Product of modal automata

We consider two modal automata $\mathcal{A}_i = \langle A, \Lambda, X_i, R_i, \Delta_i, x_{0_i}, \rho_i : X_i \to \mathbb{N}^{n_i} \rangle$ for $i = 1, 2$ and we build the automaton $\mathcal{A}_1 \times \mathcal{A}_2$ which verifies: $P \models \mathcal{A}_1 \times \mathcal{A}_2$ if and only if $P \models \mathcal{A}_1$ and $P \models \mathcal{A}_2$.

$\mathcal{A}_1 \times \mathcal{A}_2$ is the vector $\langle A, \Lambda, X, R, \Delta, x_0, \rho \rangle$ defined by:

- $X = X_1 \times X_2$ and $x_0 = (x_{0_1}, x_{0_2})$
- $\Delta((x_1, x_2)) = \{r_1 \times r_2 \mid r_i = \langle P_i, N_i, E_i, \sigma_i \rangle \in \Delta_i(x_i) \text{ for } i = 1, 2\}$ where $r_1 \times r_2$ is the product of rules $r_1$ and $r_2$ which is defined if $P_1 \cap N_2 = \emptyset$ and $P_2 \cap N_1 = \emptyset$ by the vector $\langle P, N, E, \sigma \rangle$ with $P = P_1 \cup P_2$, $N = N_1 \cup N_2$, $E = E_1 \cup E_2$ and $\sigma(a) = (\sigma(x_1), \sigma(x_2))$.

  The transformation of modalities can be seen as a table:

  | $r_2 \backslash r_1$ | $\langle . \rangle$ | $[.]$ |
  |---|---|---|
  | $\langle . \rangle$ | $\langle . \rangle$ | $\langle . \rangle$ |
  | $[.]$ | $\langle . \rangle$ | $[.]$ |

- $\rho$ is defined from $X$ into $\mathbb{N}^{n_1+n_2}$ by the concatenation of ranks from $\mathcal{A}_1$ and $\mathcal{A}_2$: $\rho((x_1, x_2)) = \rho_1(x_1).\rho_2(x_2)$.

### 4.2.2 Quotient $\mathcal{A}/P$

Given a modal automaton $\mathcal{A} = \langle A, \Lambda, X, R, \Delta, x_0, \rho \rangle$ and a process $P = \langle A, \Lambda, S, \delta, \lambda, s_0 \rangle$, we compute a modal automaton, noted $\mathcal{A}/P$, such that for any process $Q$: $P \times Q \models \mathcal{A} \iff Q \models \mathcal{A}/P$. $\mathcal{A}/P$ is the vector $\langle A, \Lambda, X_/, R_/, \Delta_/, x_{0/}, \rho_/ \rangle$ such that:

- $X_/ = (X \times S) \cup \{\top\}$.
- $\Delta((x, s)) = \{r/s \mid r \in \Delta(x)\}$. If $r = \langle P, N, E, \sigma \rangle$ then $r/s$ is defined if and only if $P \cap (\Lambda \setminus \lambda(s)) = \emptyset$, $N \cap \lambda(s) = \emptyset$ and forall $a \in E$, $\delta(s, a)$ is defined. If $r/s$ exists then it is the vector $r/s = \langle P_/, N_/, E_/, \sigma_/ \rangle$ defined by:
  - $P_/ = P \cup \lambda(s)$
  - $N_/ = N \cup (\Lambda \setminus \lambda(s))$
  - $E_/ = E$
  - $\sigma_/(a) = \begin{cases} (\sigma(x), \delta(s, a)) & \text{if } \delta(s, a) \text{ is defined} \\ \top & \text{otherwise} \end{cases}$
- $x_{0/} = (x_0, s_0)$
- $\rho_/((x, s)) = \rho(x)$



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### 4.2.3 Quotient $\mathcal{A}_1/\mathcal{A}_2$

Given two modal automata $\mathcal{A}_i = \langle A, \Lambda, X_i, R_i, \Delta_i, x_{0_i}, \rho_i : X_i \to \mathbb{N}^{n_i}\rangle$ we compute a modal automaton, noted $\mathcal{A}_1/\mathcal{A}_2$, such that for any process $Q$, $Q \models \mathcal{A}_1/\mathcal{A}_2 \iff \exists P, P \models \mathcal{A}_2 \wedge P \times Q \models \mathcal{A}_1$. $\mathcal{A}_1/A_2$ is the vector $\langle A, \Lambda, X_/, R_/, \Delta_/, x_{0/}, \rho_/\rangle$ such that:

- $X_/ = (X_1 \times X_2) \cup \{\top\}$.
- $\Delta((x_1, x_2)) = \{r_1/r_2 \mid r_1 \in \Delta_1(x_1) \wedge r_2 \in \Delta_2(x_2)\}$. If $r_i = \langle P_i, N_i, E_i, \sigma_i\rangle$ then $r_1/r_2$ is defined if and only if $P_1 \cap N_2 = \emptyset$ and $N_1 \cap P_2 = \emptyset$. If $r_1/r_2$ exists then it is the vector $r_1/r_2 = \langle P_/, N_/, E_/, \sigma_/\rangle$ defined by:
  - $P_/ = P_1 \cup P_2$
  - $N_/ = N_1 \cup N_2$
  - $E_/ = E_1$
  - $\sigma_/(a) = \begin{cases} (\sigma_1(x_1), \sigma_2(x_2)) \text{ if } a \in E_1 \cup E_2 \\ \top \text{ otherwise} \end{cases}$

  The transformation of modalities can be seen as a table:

  | $r_2\backslash r_1$ | $\langle . \rangle$ | $[.]$ |
  |---|---|---|
  | $\langle . \rangle$ | $\langle . \rangle$ | $[.]$ |
  | $[.]$ | $\langle . \rangle$ | $\top$ |

- $x_{0/} = (x_{0_1}, x_{0_2})$
- $\rho_/ : X \to \mathbb{N}^{n_1+n_2}$ is the concatenation of ranks from $\mathcal{A}_1$ and $\mathcal{A}_2$: $\rho((x_1, x_2)) = \rho_1(x_1).\rho_2(x_2)$.

## 4.3 From modal automaton to game

Let $\mathcal{A} = \langle A, \Lambda, X, R, \Delta, x_0, \rho : X \to \mathbb{N}^n\rangle$ be a modal automaton. In this section we construct a game $G(\mathcal{A})$ such that $\mathcal{A}$ is satisfiable (i.e. there exists a process $P$ such that $P \models \mathcal{A}$) if and only if there exists a winning strategy from the initial position of $G(\mathcal{A})$.

The game $G(\mathcal{A}) = \langle V, V_0, V_1, v_0, E, \Omega : V \to \mathbb{N}^n\rangle$ is defined as follows:

- $V_0 = X$ and $V_1 = R$
- $v_0 = x_0$
- $E \subseteq V \times V$ is defined by:
  - for all $x \in X$ and for all $r \in \Delta(x)$, $(x, r) \in E$
  - for all $r =< p, n, e, \sigma >\in R$ and for all $x \in \sigma(e)$, $(r, x) \in E$
- And the ranks of positions is defined as follows:
  - for all $v \in V_0$, $\Omega(v) = \rho(v)$
  - for all $v \in V_1$, $\Omega(v) = \langle 0, \dots, 0\rangle$



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## 4.4 From multi-parity to parity

Let $G = \langle V, V_0, V_1, v_0, E, \Omega : V \to \mathbb{N}^n \rangle$ be a game with a multi-parity acceptance condition and with a starting position $v_0$. Let $r = v_0 v_1 \dots$ be an infinite play with its associated sequence of ranks $\Omega(r) = \Omega(v_0)\Omega(v_1) \dots$. This play is winning for player 0 if, for each $i = 1 \dots n$, $\lim_{j\to\infty} sup(\Omega(v_j)[i])$ is even (where $\Omega(v_j)[i]$ denotes the $i$th component of $\Omega(v_j)$.

In this section we present an algorithm transforming the multi-parity game $G$ into an equivalent parity game $G'$. This algorithm is derived from [4] and is proposed by A. Arnold. Let us define the game $G'$.

Let $max_i = max\{\Omega(v)[i] \mid v \in V\}$ be the maximal rank reached by each component of $\Omega$; we shall note $L = \Sigma max_i$. We consider the set of shuffles $M_{\sqcup\sqcup} \subseteq (\mathbb{N} \times \{1, \dots, n\})^\star$:

$$M_{\sqcup\sqcup} = (max_1, 1)(max_1 - 1, 1) \dots (1, 1) \sqcup\sqcup \dots \sqcup\sqcup (max_n, n)(max_n - 1, n) \dots (1, n)$$

The word $(max_1, 1) \dots (1, 1) \dots (max_n, n) \dots (1, n) \in M_{\sqcup\sqcup}$ is noted $M_0$.

For example, if $n = 2$, $max_1 = 3$ and $max_2 = 2$ then $M_{\sqcup\sqcup} = 3^1 2^1 1^1 \sqcup\sqcup 2^2 1^2 = \{ 2^2 1^2 3^1 2^1 1^1, 2^2 3^1 1^2 2^1 1^1, 2^2 3^1 2^1 1^2 1^1, 2^2 3^1 2^1 1^1 1^2, 3^1 2^2 1^2 2^1 1^1, 3^1 2^2 2^1 1^2 1^1, 3^1 2^2 2^1 1^1 1^2, 3^1 2^1 2^2 1^2 1^1, 3^1 2^1 2^2 1^1 1^2, 3^1 2^1 1^1 2^2 1^2 \}$ (we use superscripts $x^y$ in place of $(x, y)$). Each element of $M_{\sqcup\sqcup}$ can be considered as a mapping from $[1, L]$ into $\mathbb{N} \times \{1, \dots, n\}$.

Given a word $w \in M_{\sqcup\sqcup}$ and a vector of parities $\vec{p} = \langle p_1, \dots, p_n \rangle \in \mathbb{N}^n$ we define:

- $pos(w, \vec{p}) = min\{i \in [1, L] \mid \exists j, w(i) = (p_j, j)\}$ is the index of the first $p_j$ occuring in $w$.
- $\rho(w, \vec{p}) = w(pos(w, \vec{p}))$
- $succ(w, \vec{p}) = w'$ is the shuffle such that if $\rho(w, \vec{p}) = (p, k)$ then:
  - $w' = w$ if $p$ is odd;
  - $w'$ starts like $w$ without letters $(p', k)$ such that $p' \leq p$, and terminates with $(p, k)(p-1, k) \dots (1, k)$ if $p$ is even.
- $l(w, \vec{p}) = L - pos(w, \vec{p}) + 1$ is the length of the suffix of $w$ starting at position $pos(w, \vec{p})$

For example, if $w = 3^1 3^2 2^2 2^1 1^2 1^1$ and $\vec{p} = \langle 2, 2 \rangle$ then $pos(w, \vec{p}) = 3$, $\rho(w, \vec{p}) = (2, 2)$ and $succ(w, \vec{p}) = 3^1 3^2 2^1 1^1 2^2 1^2$.

The parity game $G' = \langle V', V'_0, V'_1, E' \subseteq V' \times V', \Omega' : V' \to \mathbb{N} \rangle, v'_0 \rangle$ is built from $G$ as follows. Each position from $V'$ is a triple from $V \times M_{\sqcup\sqcup} \times \mathbb{N}$. Starting from the initial position $v'_0 = (v_0, succ(M_0, \Omega(v_0)), l(M_0, \Omega(v_0)))$,we add an edge from $(v, m, l)$ to $(v', m', l')$ if $(v, v') \in E$, $m' = succ(m, \Omega(v'))$ and $l' = l(m, \Omega(v'))$. $\Omega'$ is defined by:

$$\Omega'((v, succ(m, \Omega(v)), l)) = 2l + \begin{cases} 2 & \text{if } \rho(m, \Omega(v)) = (p, k) \text{ and } p \text{ is even} \\ 1 & \text{if } p \text{ is odd} \end{cases}$$

**Theorem 1** *Let $r = v_0 \dots$ be a maximal play of $G$. $r$ is winning for player 0 in $G$ iff its corresponding play in $G'$ is winning for player 0.*

**Proof** The finite case is straightforward. Let's consider the infinite case.

Let $r' = (v_0, s_0, l_0)(v_1, s_1, l_1) \dots$ be the play in $G'$ corresponding to $r$. $s_0 = succ(M_0, \Omega(v_0))$ and for all $i > 0$, $s_i$ denotes the shuffle word $succ(s_{i-1}, \Omega(v_i))$. For $i = 1 \dots n$, we note $m_i = \lim_{j\to\infty} sup(\Omega(v_j)[i])$ the maximal rank of the $i$th component of $\Omega$ for the positions appearing infinitely often along $r$ (i.e. $r$ is winning iff all $w_i$'s are even).

$r'$ is infinite so, there exists a position, say $(v_k, s_k, l_k)$, such that the corresponding suffix of $r'$, $(v_k, s_k, l_k)$ $(v_{k+1}, s_{k+1}, l_{k+1}) \dots$, contains only the positions repeated infinitely often.

⇒ Here we assume that $r$ is winning.

One can remark that, due to the definition of *succ* and to the shuffle structure, all the couples $(p, i)$ such that $p \leq m_i$ are "accumulated" at the end of the memory words when $m_i$ is even. Thus, there



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exists $k' \geq k$ such that for all $j \geq k'$, $s_j = s.w_j$ where $s$ is a constant word ($s$ contains all the letters $(p,i)$ with $p > m_i$) and $w_j$ is a shuffle of the $n$ words $(m_i,i)(m_i-1,i)\ldots(1,i)$ and, for all $(v_j, s_j, l_j)$, we have $l_j \leq \Sigma m_i$. Due to the shuffle structure, each $w_j$ starts with some $m_l$ which is repeated infinitely often; so there exists a position where $(m_l, l)$ is selected (infinitely often) and for which $l_j$ takes its maximal value $\Sigma m_i$. $m_l$ is even so the rank associated to the position is the maximal value $2\Sigma m_i + 2$; we can concluded that $r'$ is winning.

$\Leftarrow$ Now we assume that $r$ is not winning i.e. there exists $i_0$ such that $m_{i_0}$ is odd. Every $m_i$ is repeated infinitly often and odd $m_i$'s do not modify memory words. Thus, for some odd parity $m_k$, there exists a position from which the memory words have the form $s_j = s.(m_k,k).w_j$ where $s$ contains no $m_i$ and remains constant along $r'$ and $w_j$ is modified by even $m_i$'s. Since $s.(m_k,k)$ is never modified and $m_k$ occurs infinitely often, there exists an edge between two positions $(v_{c-1}, s.(m_k,k).w_{c-1}, l_{c-1})$ and $(v_c, s.(m_k,k).w_{c-1}, l_c)$ such that $\Omega(v_c) = \langle \ldots, m_k, \ldots \rangle$.

The rank associated to the position $(v_c, s.(m_k,k).w_{c-1}, l_c)$ is equal to $2l_c+1$ which is odd and repeated infinitely often. This ranks is maximal since $l_c$ is maximal thus $r'$ is not winning for player 0.

**Encoding $M_{\sqcup\sqcup}$:** The multi-parity game is unfolded using elements of $M_{\sqcup\sqcup}$. The length of each shuffle generated by the algorithm is $L = \Sigma max_i$ the sum of maximal ranks of the game. This length can be reduced to $L/2$ by replacing each parity $p$ by $\lceil \frac{p}{2} \rceil$ in all the algorithm.

One can remark that all words generated from $M_0$ using the *succ* operation have the following property: if $w = w_1(2p,i)(2p-1,i)w_2 \in M_{\sqcup\sqcup}$ then for all $\vec{p} \in \mathbb{N}^n$, $succ(w,\vec{p}) = w'_1(2p,i)(2p-1,i)w'_2$. In other words, *succ* preserves the consecutivity of $(2p,i)$ and $(2p-1,i)$. Since we start with the word $M_0$ (which has the property), any word $w$ generated by the algorithm is such that, if $1 \leq 2p \leq max_i$ then $(2p,i)(2p-1,i)$ is a factor of $w$. We call $M'_{\sqcup\sqcup} \subseteq M_{\sqcup\sqcup}$ the set of words generated from $M_0$.

Each word of $M'_{\sqcup\sqcup}$ can be encoded using half of its letters by replacing each couple $(2p,i)(2p-1,i)$ by $(p,i)$. In fact, instead of starting with $M_0$ we start with $\lceil \frac{M_0}{2} \rceil = (\lceil \frac{max_1}{2} \rceil, 1)\ldots(1,1)\ldots(\lceil \frac{max_n}{2} \rceil, n)\ldots(1,n)$ and each multi-parity $\vec{p} = \langle p_1, \ldots, p_n \rangle$ of $G$ is translated into $\lceil \frac{\vec{p}}{2} \rceil = \langle \lceil \frac{p_1}{2} \rceil, \ldots, \lceil \frac{p_n}{2} \rceil \rangle$. Then, instead of looking for some $(p_j, j)$ into a word of $M'_{\sqcup\sqcup}$, we look for $(\lceil \frac{p_j}{2} \rceil, j)$ into a successor of $\lceil \frac{M_0}{2} \rceil$. In order to compute the successors of $\lceil \frac{M_0}{2} \rceil$ we just have to ensure that moves of $(\lceil \frac{p_j}{2} \rceil, j)$ are the same than those of $(p_j,j)(p_j-1,j)$ and that the good parity is assigned to each position. In order to ensure this condition we just have to test the parity of $p$ in place of $\lceil \frac{p}{2} \rceil$ in the definitions of *succ* and $\Omega'$:

- $succ(w,\vec{p}) = w'$ is the shuffle such that if $\rho(w,\vec{p}) = (\lceil \frac{p}{2} \rceil, k)$ then:
  - $w' = w$ if $p$ is odd;
  - $w'$ starts like $w$ without letters $(p',k)$ such that $p' \leq \lceil \frac{p}{2} \rceil$, and terminates with $(\lceil \frac{p}{2} \rceil, k)(\lceil \frac{p}{2} \rceil - 1, k)\ldots(1,k)$ if $p$ is even.
- $\Omega'((v, succ(m,\Omega(v)), l)) = 2l + \begin{cases} 2 & \text{if } \rho(m,\Omega(v)) = (\lceil \frac{p}{2} \rceil, k) \text{ and } p \text{ is even} \\ 1 & \text{if } p \text{ is odd} \end{cases}$

## 4.5 Computation of a winning strategies

Given a parity game $G(\mathcal{A})$ we compute winning strategies in $G$ using algorithms similar to those presented in [3]. The algorithms are not exactly the same because, in the framework of `Synthesis`, acceptance conditions for infinite plays are defined using the maximal parity occuring infinitely often. In this section we present the main ideas of the (adapted) algorithms taken from [3].

### 4.5.1 Memories over odd priorities

Given a parity game $G = \langle V, V_0, V_1, v_0, E, \Omega \rangle$ we denote by $d$ its maximal priority that we assume to be odd (if $d$ is even we consider $d+1$). For each odd priority $p \in \{1, \ldots, d\}$ we denote $n_p$ the size of the set



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$\{v \in V \mid \Omega(v) = p\}$. Now we define the set of memories:

$$M(G) = \{\bot, \top\} \cup \prod_{1 \leq p \leq d, p \ odd} \{0, \ldots, n_p\}$$

In the sequel we will consider the vectors $\vec{0} = (0, \ldots, 0)$ and $\vec{n} = (n_1, \ldots, n_d)$. In [3] the game $G$ is said to be $\vec{n}$*-bounded*.

$M(G)$ is ordered by the relation $\prec$ defined for all $\vec{m} = (m_1, \ldots, m_d)$, $\vec{m'} = (m'_1, \ldots, m'_d) \in M(G) \setminus \{\bot, \top\}$ by:

- $\bot \prec \vec{m} \prec \top$
- $\vec{m} \prec \vec{m'} \iff \exists i \in \{1, \ldots, d\}, m_i < m'_i \wedge \forall j > i, m_j = m'_j$

Now we define some functions defining transformations of $\vec{m} = (m_1, \ldots, m_p) \in M(G)$ for a given priority $p$:

- $up(\vec{m}, p) = \begin{cases} \top & \text{if } \vec{m} = \top \\ inc(\vec{m}, p) & \text{if } p \text{ is odd} \\ zero(\vec{m}, p) & \text{if } p \text{ is even} \end{cases}$

  where:

  - $zero(\vec{m}, p) = (0, \ldots, 0, m_{p+1}, \ldots, m_d)$
  - $inc(\vec{m}, p) = \begin{cases} (0, \ldots, 0, m_p + 1, m_{p+2}, \ldots, m_d) & \text{if } p \leq d \text{ and } m_p < n_p \\ inc(\vec{m}, p+2) & \text{if } m_p = n_p \text{ and } p < d \\ \top & \text{otherwise} \end{cases}$

- $down(\vec{m}, p) = \begin{cases} \bot & \text{if } \vec{m} = \bot \\ dec(\vec{m}, p) & \text{if } p \text{ is odd} \\ max(\vec{m}, p) & \text{if } p \text{ is even} \end{cases}$

  where:

  - $max(\vec{m}, p) = (n_1, \ldots, n_{p-1}, m_{p+1}, \ldots, m_d)$
  - $dec(\vec{m}, p) = \begin{cases} (n_1, \ldots, n_{p-2}, m_p - 1, m_{p+2}, \ldots, m_d) & \text{if } p \leq d \text{ and } 0 < m_p \\ dec(\vec{m}, p+2) & \text{if } m_p = 0 \text{ and } p < d \\ \bot & \text{otherwise} \end{cases}$

### 4.5.2 Winning strategies and controller synthesis

In order to get winning strategies in a parity game $G = \langle V, V_0, V_1, v_0, E, \Omega\rangle$ we need to compute a mapping $Mmax : V \rightarrow M(G)$. We obtain $Mmax$ using the following algorithm:

1. For each $v \in V$, set $Mmax(v)$ to $\vec{n}$
2. Find a position $v$ such that $down(\vec{m'}, \Omega(v)) \prec Mmax(v)$ where

$$\vec{m'} = \begin{cases} \max\{Mmax(v') \mid (v, v') \in E\} \text{ if } v \in V_0 \\ \min\{Mmax(v') \mid (v, v') \in E\} \text{ if } v \in V_1 \end{cases}$$

3. If $v$ does not exist then the algorithm returns.
4. If $v$ exists then we set $Mmax(v)$ to $down(\vec{m'}, \Omega(v))$
5. Repeat from step 2.



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For every $v \in V$ we define $W(v) = \{v' \mid (v, v') \in E \wedge up(Mmax(v), \Omega(v)) \preceq Mmax(v')\}$.

The mapping $W$ allows us to build memoryless strategies which are winning from $v_0$: given a position $v \in V_0$ a winning strategy consists to select a successor into $W(v)$. Of course, if no such successor exists for $v_0$ then no winning strategy has been found and we can not generate a valid controller.

Every memoryless strategy $\tau : V \to V$ such that $\tau(v) \in W(v)$ allows us to generate one controller $C_\tau$. In particular one might be interested by two classes of controllers corresponding to two classes of strategies. These strategies select the positions which, respectively, minimize or maximize the mapping $Mmax$ in the set $W(v)$:

- $\tau_{\min}(v) \in \{v' \in W(v) \mid \forall v'' \in W(v), Mmax(v') \preceq Mmax(v'')\}$
- $\tau_{\max}(v) \in \{v' \in W(v) \mid \forall v'' \in W(v), Mmax(v'') \preceq Mmax(v')\}$

Controllers generated with a $\tau_{\max}$ strategy are the most permissive ones while those generated with $\tau_{\min}$ are the most despotic.

**Controller synthesis** Now we are able to built a model for a modal automaton $\mathcal{A} = \langle A, \Lambda, X, R, \Delta, x_0, \rho \rangle$. As specified in Section 4.3, $\mathcal{A}$ is transformed into a parity game $G(\mathcal{A}) = \langle V, V_0, V_1, v_0, E, \Omega \rangle$ where $V_0 = X$ correspond to the states of the automaton, the initial position is the initial state $v_0 = x_0$ and $V_1 = R$ is the set of its rules. A memoryless strategy for player 0 in $G(\mathcal{A})$ is thus a mapping from $X$ into $R$.

For each memoryless strategy $\tau : X \to R$ one can create a valid controller (without labelling) $C_\tau = \langle A, \emptyset, X, \delta, \emptyset, x_0 \rangle$ where the transition function $\delta : X \times A \to X$ is defined as follows: for all $x \in X$, $\delta(x, a) = \sigma(a)$ where $\sigma$ is the successor function of the rule $\tau(x) = \langle P, N, E, \sigma : A \to X \rangle \in R$.



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# Appendix A

# Commands

This appendix lists the current commands implemented into `Synthesis`. From the shell prompt the user might execute the `help` command in order to get some informative message about allowed commands. The reader can find the BNF syntax of the `Synthesis` script at the page 33.

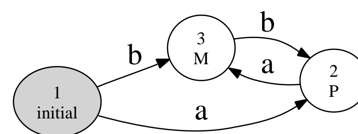
## I/O commands

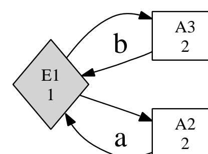
In general the commands of `Synthesis` print some message to the user. By default a command prints its output on the standard output of the program (i.e. the console). Using the same notation than UNIX shells, the outputs of `Synthesis` commands might be redirected (`>`) into a file or appended at the end (`>>`). Indeed, this redirection mechanism is the only way proposed by `Synthesis` to save objects.

`dot` $object_1 \ldots object_n$**:** This command displays its arguments into the `dot`[5] file format. Its arguments must be graph-based objects i.e. processes and games. Each type of object has its own layout. The figure below depicts the result generated by `dot` applied to the following process and game.

```
transition_system P;
1 |- a -> 2, b -> 3;
2 |- a -> 3;
3 |- b -> 2;
<initial={1};P={2};M={3}>.

game GP;
E1@1 -> A2, -> A3;
A2@2 |- a -> E1;
A3@2 |- b -> E1;
<initial={E1}>.
```




`load` $filename_1 \ldots filename_n$**:** With this command `Synthesis` reads the files specified by $filename_i$ (no wildcard substitution is implemented in the current version); existing objects are replaced by those readed from files. For each $filename_i$ the file format is identified using the filename extension. The extension recognized by `Synthesis` are:

`.syn` for synthesis scripts;

`.mec` **or** `.mec4` for `Mec 4` transition systems;

`.fam` for automaton specifications;

`.game` for parity games.

`mecv` $object_1 \dots object_n$: This command translates processes and modal automata into `Mec V`[9] file format. Processes are translated into AltaRica nodes. Automata translation is not yet implemented but they will be translated into the equation systems supported by `Mec V`.

`print` $arg_1 \dots arg_n$: This command is usefull only when `Synthesis` is used in batch mode. It prints its arguments as strings of characters; when printed, the arguments are separated by newlines. Example:

```
synthesis>print hello world
hello
world
synthesis>print "hello world"
hello world
synthesis>
```

`show` $object_1 \dots object_n$: This command prints its arguments in their native format (see the `load` command). Of course, if objects with different types are `show`ed into a same file, the user will not be able to reload it later.

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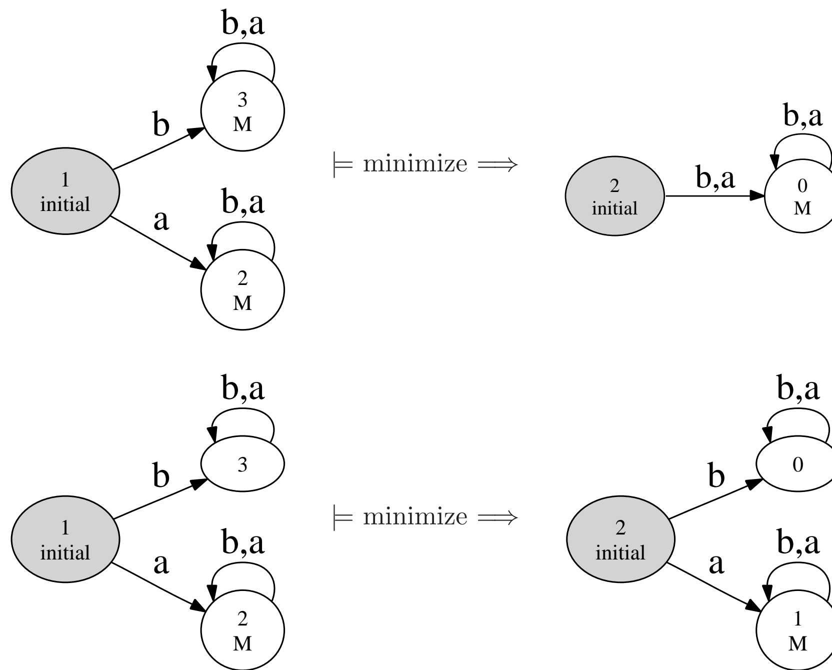
## Synthesis pipeline

`control` $g$: This command generates a process (a transition system) from the game $g$. We assume that the game argument is the result of a computation made with the command `strategy` i.e. the computation of winning strategies in the game. The resulting process is computed with the algorithm presented in Section 4.5.2.

`game` $a$: It computes a parity game from the automaton $a$ (see Section 4.1). The acceptance condition used by $a$ might be a parity or multi-parity condition; in both cases the acceptance condition is translated into a parity condition.

`minimize` $p$: This command implements an algorithm for minimizing the number of states in a word-automaton. The initial partition of the state space is the intersection of atomic properties of the process $p$; we consider that all states are accepting. The following figures show the result of `minimize` on a process with different sets of properties; in the first case both states 2 and 3 have the property $m$ while in the second case, only 3 has it:




`parity` $a$**:** This command translates the model-automaton $a$ doted with a multi-parity condition into an automaton with parity condition. This operation is implicitly done by the command `game`. This command implements the algorithm presented in Section 4.4.

`product` $a_1$ $a_2$**:** It computes the intersection of the modal automata $a_1$ and $a_2$. The acceptance condition of the resulting automaton is a multi-parity condition. The algorithm is presented in Section 4.2.1.

`quotient` $a$ $a|p$**:** This command implements the quotient of a modal automaton by a process or by another automaton. The acceptance condition of the resulting automaton is a multi-parity condition. The algorithms are presented in Sections 4.2.2 and 4.2.3.

`strategy` $g$**:** This command computes winning strategies for the game $g$. We apply the algorithm of Bernet, Janin and Walukiewicz presented in Section 4.5. The result of this command is a game (which is a subgraph of $g$) representing the strategies computed by the algorithm. With this result the user can generate a process with the command `control`.

`sync` $p_1$ $p_2$**:** This function computes the synchronized product of two processes $p_1$ and $p_2$. The function checks the consistency of state sets (*marks*) of processes. It might emit a warning message if it fails to synchronize two states because of inconsistency in their sets of marks. This is shown on the following example:

```
synthesis>show P
// # states = 3
// # transitions = 4
transition_system P;
3 |- b -> 2;
2 |- a -> 3;
1 |- a -> 2,
     b -> 3;
<initial = {1}; P = {2}; M = {3}>.
synthesis>show PP
// # states = 3
// # transitions = 2
transition_system PP;
1 |- a -> 3;
3 |- a -> 2;
<initial={1};P={3};M={}>.
synthesis>
synthesis>sync P PP
warning : states '3' and '2' are \
 inconsistent w.r.t their properties:
A: -P M
B: -P -M
```



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```
// # states = 2
// # transitions = 1
transition_system $$;
1_1 |- a -> 2_3;
2_3 |- ;
<P = {2_3}; M = {}; initial = {1_1}>.
synthesis>
```

`unmark` $p|a$: This command removes propositional constants from its argument. The following lines show the `unmark`ing of a process:

```
synthesis>show P
// # states = 3
// # transitions = 4
transition_system P;
3 |- b -> 2;
2 |- a -> 3;
1 |- a -> 2,
     b -> 3;
<initial = {1}; P = {2}; M = {3}>.
synthesis>unmark P
// # states = 3
// # transitions = 4
transition_system $$;
0 |- b -> 1;
1 |- a -> 0;
2 |- a -> 1,
     b -> 0;
<initial = {2}>.
synthesis>
```

# Memory management

`collect`: Some type of objects used by `Synthesis` are allocated by pages. This command is used to decrease the memory consumption; it tells `Synthesis` to free each page containing only unused objects. The user might evaluate the memory usage using the `pool` command.

`pools`: This command displays some statistics about the memory usage. This command is useful when lots of objects have been freed using the `remove` command; in this case the user can `collect` the memory not actually used by `Synthesis`.

Some type of objects are allocated by pages. The `pools` command displays informations about this pages. The output of `pools` is something like the following table:

```
synthesis>pools
compute pools
                       # P     # O/P   b/O     free    mem
Strategy PREDs  :      1       1024    8       1024    8.00kb
Game AE Edges   :      3       690     12      1354    24.26kb
Game EA Edges   :      1       1024    8       847     8.00kb
Game Positions  :      3       349     24      737     24.54kb
Automaton Edges :      8       690     12      600     64.69kb
Automaton Rules :      2       690     12      560     16.17kb
Automaton States:      2       421     20      330     16.45kb
SID Dico        :      1       1024    8       807     8.00kb

table of symbols size = 217
```

Each line of the previous table concerns a type of objects; it indicates: the number of currently allocated pages (`#P`), the number of objects by page (`#O/P`), the number of bytes used by one object (`b/O`), the number of currently unused objects (`free`) and the memory required to store all this objects (`mem`). Of course, when `free` $\gg$ `#P`$\times$`#O/P`, a call to `collect` will have a significant effect on the memory consumption of `Synthesis`.

`remove` $objid_1 \ldots objid_n$: An object remains in the memory space of `Synthesis` until its replacement (any identifier might be assigned a new object). The user has the choice to explicitly `remove` objects from memory using this command. The reader should note that the memory used by an object is not freed; the object is just marked as reusable. In order to actually liberate the objects the user must call the `collect` command.



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## Leaving the program

`exit`: This command might be used in place of `EOF` (i.e. `CTRL-D` on many systems) in order to terminate `Synthesis`. When the program terminates it saves the history of commands (if the program has been compiled with the `GNU readline` library) into the file `.synthesis_history.syn` in the current directory; this history is reloaded when `Synthesis` is restarted (from the same directory).

If this command is used into a script then the `Synthesis` interpreter don't execute commands after the `exit`. Example:

```
point@raoul: ~/tmp
cat script.syn
print "the first 'print' command is executed"
exit
print "the second 'print' command is not executed"
point@raoul: ~/tmp
synthesis script.syn
the first 'print' command is executed
point@raoul: ~/tmp
```

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# Appendix B

# Syntaxes

## Synthesis scripts

| | | |
|---|---|---|
| *script-file* | ::= | *list-of-commands* |
| *list-of-commands* | ::= | *redirected-command-or-assignment* |
| | ::= | *redirected-command-or-assignment* ; *command-list* |
| *redirected-command-or-assignment* | ::= | *command-or-assignment* `>` *string* |
| | ::= | *command-or-assignment* `>>` *string* |
| | ::= | *command-or-assignment* |
| *command-or-assignment* | ::= | *command* |
| | ::= | *assignment* |
| *command* | ::= | *identifier parameter-list* |
| *assignment* | ::= | *identifier* **:=** *command* |
| *parameter-list* | ::= | *parameter parameter-list* |
| | ::= | |
| *parameter* | ::= | *identifier* |
| | ::= | *integer* |
| | ::= | *string* |
| | ::= | ( *command* ) |
| *integer* | ::= | `[0-9][0-9]*` |
| *identifier* | ::= | `[a-zA-Z_][a-zA-Z0-9_]*` |
| *string* | ::= | `"[^"]*\"` |

## MEC4 File Format

| | | |
|---|---|---|
| *mec4-file* | ::= | *list-of-transition-systems* |
| *list-of-transition-systems* | ::= | *transition-system list-of-transition-systems* |
| | ::= | |
| *transition-system* | ::= | **transition_system** *ident attributes* ; *transition-system-body* |
| *transition-system-body* | ::= | *list-of-states sets-of-states* |
| *list-of-states* | ::= | *list-of-states* ; *state* |
| | ::= | *state* |

*state* ::= *state-name attributes* `|-` *transitions*
*transitions* ::= *list-of-transitions*
::=
*list-of-transitions* ::= *transition* , *list-of-transitions*
::= *transition*
*transition* ::= *event-name* `->` *state-name attributes*
*event-name* ::= *name*
*sets-of-states* ::= ; < list-of-set-of-states > .
::= .
*list-of-set-of-states* ::= *set-of-states*
::= *set-of-states* ; *list-of-set-of-states*
*attributes* ::= < *list-of-attributes* >
::=
*list-of-attributes* ::= *attributes* ; *list-of-attributes*
::= *attributes*
*attribute* ::= *ident* = *id-int*
::= *ident*
*set-of-states* ::= *ident* = { *list-of-state-name* }
::= *ident* = { }
*list-of-state-name* ::= *state-name* , *list-of-state-name*
::= *state-name*
*state-name* ::= *name*
*name* ::= *ident*
::= *integer*
*id-int* ::= *ident*
::= *integer*
*ident* ::= `[0-9_]*[A-Za-z_\^][A-Za-z0-9_\^]*`
*integer* ::= `[0-9]+`

# FAM File Format

*fam-file* ::= *list-of-automata*
*list-of-automata* ::= *automaton list-of-automata*
::=
*automaton* ::= **name** *ident automaton-width* ; *states properties*
*automaton-width* ::= < **width** = *integer* >
::=
*states* ::= *state-list*
*state-list* ::= *state*
::= *state state-list*
*state* ::= *kind-of-state* = *ranks* `->` *rules*
::= *kind-of-state* = *ranks* `->` ;
*ranks* ::= *ranks1*
::= *ranks2*



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| | | |
|---:|---|---|
| *ranks1* | ::= | **mu** |
| | ::= | **nu** |
| *ranks2* | ::= | < *integer-list* > |
| *integer-list* | ::= | *integer* **,** *integer-list* |
| | ::= | *integer* |
| *rules* | ::= | *rule-list* **;** |
| *rule-list* | ::= | *simple-rule* **+** *rule-list* |
| | ::= | *simple-rule* |
| *simple-rule* | ::= | *labels* **(** *members* **)** |
| | ::= | *labels conjunction* |
| | ::= | *conjunction* |
| *labels* | ::= | *label-list* |
| *label-list* | ::= | *label* **.** *label-list* |
| | ::= | *label* **.** |
| *label* | ::= | *ident* |
| | ::= | ˜ *ident* |
| *members* | ::= | *member* |
| *member* | ::= | *conjunction* **+** *member* |
| | ::= | *conjunction* |
| *conjunction* | ::= | *transitions* |
| *transitions* | ::= | *transition* **.** *transitions* |
| | ::= | *transition* |
| *transition* | ::= | < *ident-list* > *any-kind-of-state* |
| | ::= | [ *ident-list* ] *any-kind-of-state* |
| *ident-list* | ::= | *id-int* |
| | ::= | *id-int* **,** *ident-list* |
| *properties* | ::= | < *property-list* > **.** |
| *property-list* | ::= | *property* **;** *property-list* |
| | ::= | *property* |
| *property* | ::= | *ident* **=** **{** *list-of-states* **}** |
| *list-of-states* | ::= | *any-kind-of-state* **,** *list-of-states* |
| | ::= | *any-kind-of-state* |
| *any-kind-of-state* | ::= | *sort-of-state* |
| | ::= | **T** |
| *kind-of-state* | ::= | *id-int* |
| *id-int* | ::= | *integer* |
| | ::= | *ident* |
| *ident* | ::= | `[A-Za-z_][A-Za-z0-9_]*` |
| *integer* | ::= | `[0-9]+` |

## GAME File Format

| | | |
|---:|---|---|
| *game-file* | ::= | *list-of-games* |

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*list-of-games* ::= *game list-of-games*
::=

*game* ::= **game** *ident* **;** *game-edges initial-position*

*game-edges* ::= *game-edge* **;**
::= *game-edge* **;** *game-edges*

*game-edge* ::= *eve-edges*
::= *adam-edges*

*eve-edges* ::= *ident* **@** *number eve-target-list*
::= *ident* **@** *number*

*eve-target-list* ::= `->` *ident*
::= `->` *ident* **,** *eve-target-list*

*adam-edges* ::= *ident* **@** *number* `|-` *adam-target-list*
::= *ident* **@** *number* `|-`

*adam-target-list* ::= *ident* **->** *ident*
::= *ident* **->** *ident* **,** *adam-target-list*

*initial-position* ::= **< initial = {** *ident* **}** **> .**

*ident* ::= `[A-Za-z_][A-Za-z0-9_]*`

*number* ::= `[0-9]*`