SYSTEMIC DOPPLER BASED AERIAL TARGET TRACKING

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ABSTRACT

For obvious reasons of cost, discretion and reliability costs, locating and tracking aerial targets under an electromagnetically completely passive paradigm, relying exclusively on illuminators of opportunity, is very appealing for military but also civilian tasks. Such a passive radar system could exploit signals emitted by existing commercial television or radio stations or even satellite signals, such as the ones belonging to the GPS. This paper focuses on target locating and tracking using a network of passive receivers and/or non-cooperative illuminators, forming a multi-static radar configuration. The central idea is to make use of the bi-static Doppler shift. To make tracking possible, more than one bi-static Doppler shift has to be measured and the information combined. It is the purpose of the paper to describe this kind of technique and to present its main issues and workarounds, using a gradual approach. Conclusions and perspectives are drawn in the end.

1. INTRODUCTION

Tracking in a passive radar system [1] [2] [6] usually relies on three kinds of measurements: TDOA (*Time Difference Of Arrival*), DOA (*Direction Of Arrival*), and Doppler shift [4]. The TDOA measurement is based on finding the time difference between the reflected signal's arrival and the direct signal's arrival. The instantaneous TDOA can be calculated with the equation:

$$\tau_k = \frac{a+b-L}{c} \tag{1}$$

where c is the speed of light, a is the distance from the receiver to the target, b is the distance from the transmitter to target, and L is the direct path distance between the transmitter and the receiver.

The instantaneous DOA for a constant velocity target is [4]:

$$\Phi[nT] = \tan^{-1}\left(\frac{x_0 + nT\dot{x}}{y_0 + nT\dot{y}}\right) \tag{2}$$

where (x_0, y_0) is the initial position of the target, T is the sampling period, n is the sample number, and $\dot{x}$ and $\dot{y}$ are the velocities in the x and y direction, respectively (assumed to be constant). The DOA measurement examines the change in angle as a function of time. At least two stationary receivers are needed for targets to be unambiguously tracked with DOA measurements.

The Doppler shift [4] can be expressed as:

$$F_d = -\frac{1}{\lambda}\left[\frac{da}{dt} + \frac{db}{dt}\right] \tag{3}$$

where λ denotes the radar wavelength. The Doppler shift relies on the rate of change in the sum of the transmitter-to-target and target-to-receiver path lengths.

Our goal is to investigate target tracking and association methods using only Doppler shift data, in both single and multi-target scenarios, similar to [5]. The objective is to determine whether Doppler data alone is sufficient to unambiguously locate targets with a single transmitter-receiver pair, in addition to developing and comparing different multi-target tracking and association schemes and techniques. While in [4], a L-M NLSE (Levenberg-Marquardt Nonlinear Least Squares Estimation) algorithm is used to to perform target tracking using both Doppler and DOA measurements, we will address the multi-target/multi-sensor association and tracking problem with only Doppler measurements. We make use of the L-M NLSE algorithm for estimating a target's state. To increase convergence, a preliminary initialization may be performed, by using a grid-based search technique.

2. DOPPLER SHIFT MEASUREMENT

Let us assume a planar motion for the aerial target (i.e. its altitude is constant). In this case, we will describe target's state by a set of two time-varying vectors:

$$\vec{p} = \begin{bmatrix} x \\ y \end{bmatrix}, \; and \;\; \dot{\vec{p}} = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \end{bmatrix} \tag{4}$$

The distances a and b may be written as:

$$\begin{aligned} a &= \|\mathbf{p}-\mathbf{p}_r\| = \sqrt{(x-x_r)^2 - (y-y_r)^2} \\ b &= \|\mathbf{p}-\mathbf{p}_t\| = \sqrt{(x-x_t)^2 - (y-y_t)^2} \end{aligned} \tag{5}$$

where $\mathbf{p}_r = [x_r \quad y_r]^T$ and $\mathbf{p}_t = [x_t \quad y_t]^T$ represent the receiver and the transmitter positions, respectively.

Thus:

$$\dot{a} = \frac{(\mathbf{p}-\mathbf{p}_r)^T \dot{\mathbf{p}}}{a}, \quad \dot{b} = \frac{(\mathbf{p}-\mathbf{p}_t)^T \dot{\mathbf{p}}}{b} \tag{6}$$

so that:

$$F_d = -\frac{1}{\lambda}\left\{\left[\frac{(\mathbf{p}-\mathbf{p}_r)^T \dot{\mathbf{p}}}{a}\right] + \left[\frac{(\mathbf{p}-\mathbf{p}_t)^T \dot{\mathbf{p}}}{b}\right]\right\}. \tag{7}$$

We may conclude that the Doppler shift bears information about target's position variation. However, this dependency is nonlinear, so we should carefully choose the estimation procedure. Since the classical LSE (Least Squares Estimation) is not appropriate for nonlinear problem, an iterative variant of it will be used, the L-M NLSE [4].

To somehow simplify the problem at hand, a number of assumptions will be made. Thus, we consider non-manoeuvring targets, of invariant vector speeds. This will help in parameter estimation.

3. ONE TARGET AND ONE RECEIVER

This elementary situation will allow us to illustrate the heart of the method, while exhibiting the main difficulties, specifically the ambiguity problem.

Without loss of generality, the coordinate system can be chosen to be centred at the midpoint of the line segment joining the transmitter and the receiver locations, with the Ox axis defined along this segment. We consider the unit vectors associated to the receiver and to the transmitter:

$$\mathbf{u}_r = \frac{\mathbf{p}-\mathbf{p}_r}{a} \tag{8}$$

$$\mathbf{u}_t = \frac{\mathbf{p}-\mathbf{p}_t}{b} \tag{9}$$

Thus, the Doppler shift may be expressed as:

$$\begin{aligned} F_d &= -\frac{1}{\lambda}\left[\mathbf{u}_r + \mathbf{u}_t\right]^T \dot{\mathbf{p}} = -\frac{1}{\lambda}\left[\mathbf{u}_r + \mathbf{u}_t\right] \bullet \mathbf{v} \\ &= -\frac{1}{\lambda}\left\|\mathbf{u}_r + \mathbf{u}_t\right\|\left\|\mathbf{v}\right\| \cos\theta \end{aligned} \tag{10}$$

where the vector $\mathbf{v} = \left[v_x \quad v_y\right]^T$ is the target's velocity vector and θ is the angle between vectors.

Consider now a vector defining target's state:

$$\mathbf{X}_1 = \left[x_0 \quad v_x \quad y_0 \quad v_y\right]^T \tag{11}$$

Because of the symmetry of equation (10), to each measured Doppler shift corresponding to a given target, another 3 target states will generate an identical measurement. These states will be seen as spurious solutions. Thus, locating and tracking are ambiguous (of order 4). The 3 ambiguous states express the symmetry with respect to the Ox, Oy and the origin O respectively:

$$\mathbf{X}_2 = \left[x_0 \quad v_x \quad -y_0 \quad -v_y\right]^T \tag{12}$$

$$\mathbf{X}_3 = \left[-x_0 \quad -v_x \quad y_0 \quad v_y\right]^T \tag{13}$$

$$\mathbf{X}_4 = \left[-x_0 \quad -v_x \quad -y_0 \quad -v_y\right]^T \tag{14}$$

Since 4 distinct initial states (trajectories) will produce identical Doppler shift responses, the initial target state cannot be uniquely determined from the Doppler shift response. This difficulty may be overcome if one additional receiver (or, alternatively, a transmitter) is considered, as long as this one does not lie on Ox or Oy, thus breaking the symmetry.

4. ONE TARGET AND TWO RECEIVERS

Here we illustrate the tracking ambiguity and the utility of a second receiver. First, when only one receiver is considered, 4 targets will likely generate the same Doppler shift. These are shown in Figure 1.

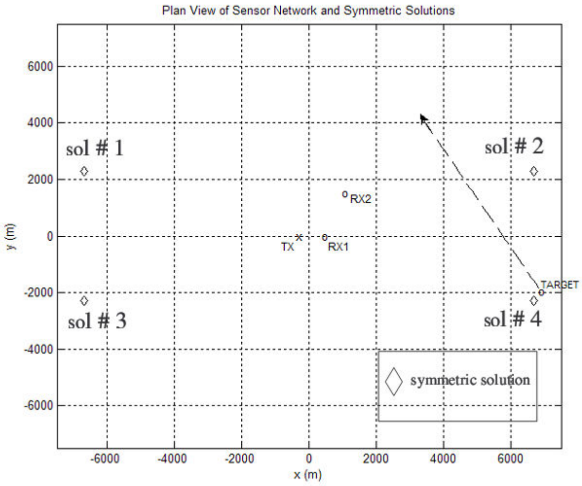

Figure 1: The real target and the ambiguous solutions

Since it is impossible to determine which solution represents the actual target using just a single receiver, a second receiver (RX2) is used to break the ambiguity. The Doppler shift for each of the four symmetric states (ambiguously determined using the received signal at RX1) is calculated at RX2. Then, the cost of each one is computed:

$$C_i = \sum_{k=0}^{N-1} \left\| m_2[k] - h_{2i}[k] \right\|^2 \tag{15}$$

where $m_2[k]$ is the actual measured Doppler shift at RX2 and $h_{2i}[k]$ is the estimated Doppler shift at RX2 produced by the i^{th} symmetric solution ($i = \overline{1,4}$) and N is the number of samples. The solution that gives the smallest of the four costs represents the actual target state. In our case, this is the solution number 4 (see Figure 2).

This simple example shows how is possible to break the ambiguity by adding a second receiver. This approach can be further generalized for single and multiple target tracking.

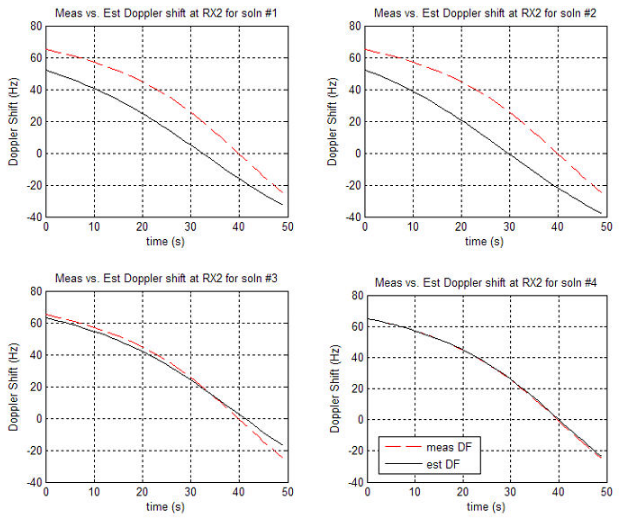

Figure 2: Measured and estimated Doppler shifts at RX2 for each symmetric solution

5. MULTIPLE TARGETS AND MULTIPLE RECEIVERS

It is conceivable that the previous approach, based on cost functions, is extendable to the general case, where a number of N targets are tracked using M receivers (a single transmitter is required). We denote by $\left(\mathbf{m}_{ij}\right)_{\substack{i=\overline{1,M}\\ j=\overline{1,N}}}$ the Doppler shift associated to the target j ,a s recorded by the receiver i (note that numbering targets is arbitrary at each receiver, as there is no clue to associate them). The vector $\mathbf{m}_{i,j}$ stands for the set of recorded Doppler shift values recorded at discrete measurement times, i.e. $\mathbf{m}_{i,j} = \left[m_{ij}[0] \quad m_{ij}[1] \quad ... \quad m_{ij}[K-1] \right]$, where K is the total number of samples.

The main aim of this method is to ensure that two different targets are not assigned to the same measured Doppler shift data. The procedure involves sequentially blocking out the remaining measured response and target that correspond to the lowest cost during the association process.

First, the Doppler shifts measured by RX1 are arbitrarily numbered, thus number identifiers being associated to each target $j = \overline{1,N}$, the corresponding Doppler shift being $\mathbf{m}_{1j}$.

Then, for each $\mathbf{m}_{1j}$, the set of four ambiguous states are computed, all of them being possible solutions. Generally speaking, only one of these four solutions denotes a real target. A number of $4N$ will be then examined.

In the third step, the Doppler shifts at RX2, as it would have been generated by each of the previously determined $4N$ solutions, are computed.

Next, the cost functions for each of the $4N$ solutions are computed, with respect to the RX2 receiver. Note that the $4N$ possible solutions have to be measured against the N Doppler shifts that RX2 actually records. So, a number of $4N^2$ costs will be computed, one for each combination.

Among them, the minimum cost will be identified as being a real target. This will unambiguously identify it, so that it may be dropped from the original set of N targets to be tracked. The corresponding three spurious solutions will be dropped, too.

Then the procedure starts once again, but this time with the analysis of only $4N^2 - 4N$ remaining costs, until the correct identification of all targets.

This technique is simple and quite fast to implement. However, it makes use of only two of the available receivers. Secondly, it could be further complicated by missing information (imagine that one ore more Doppler shifts are noisy or simply unavailable at RX2). Finally, there are 2 orders in which the information from RX1 and RX2 is examinable. To increase robustness, one should average over them.

One way to embed information from more sensors is to consider more couples of receivers: RX_i and RX_j. At each step, the coordinates of targets must be recomputed, since they are expressed in the original coordinate system, whose Ox axis is linked with RX1 and RX2.

Overall, the estimation algorithm should be applied 2^M times, each time a number of $4N^2$ costs being analyzed. A final average step concludes the method.

6. TOWARDS A SYSTEMIC APPROACH

In this section, we present a way to globally and simultaneously analyse the information provided by the available receivers. From a theoretical point of view, this is more a systemic-like approach to the given problem.

The same scenario as in the previous section is considered. For the $\left(\mathbf{m}_{ij}\right)_{\substack{i=\overline{1,M}\\ j=\overline{1,N}}}$ vector matrix, all possible combinations of Doppler shifts, for each receiver, are considered. This gives $\left(N!\right)^{M-1}$ combinations. Each combination is a permutation of the lower $M-1$ lines of the vector matrix $\left(\mathbf{m}_{ij}\right)_{\substack{i=\overline{1,M}\\ j=\overline{1,N}}}$ (the first line remains unchanged). For each combination, each column of the corresponding associated vector matrix is seen as the response of a possible real target.

The central question is to determine, among the all considered combinations, the one which is the true solution of the problem at hand. In fact, the solution will be the one exhibiting the minimum cost. Then, at each of the N targets, its real state $\mathbf{X}_j$ would be associated.

Since our problem is reformulated as an optimization problem, the global cost of Doppler shifts will be computed, for each combination. Then, the global optimization solution is given by:

$$\min_{\substack{states\ \mathbf{X}_j,\ j=\overline{1,N}\\ sol \in \{set\ of\ combinaisons\}}} C \qquad (16)$$

where $C = \sum_{j=1}^{N} C_j$ is the sum of all partial costs C_j .

The partial costs are given by:

$$C_j = C_j(\mathbf{X}_j) = \sum_{sol \in \{set\ of\ combinations\}} \left\| \mathbf{m}_{i,sol(i)} - \mathbf{h}_i(\mathbf{X}_j) \right\|^2 \quad (17)$$

so, the cost C_j is a function of the parameters (state) $\mathbf{X}_j$ of function j, as assumed by the current combination (in fact, all the quantities are also functions of the underlying sol combination).

In the equation above, $\mathbf{h}_i(\mathbf{X}_j)$ are the Doppler shifts as recorded by the receiver i, $i=\overline{1,M}$ (for the states $\mathbf{X}_j$). Again, the vector notations is a substitute for the temporal samples of the measured Doppler shift (its values are, in fact, instantaneous frequencies, not the original signal values), so that:

$$\mathbf{h}_i(\mathbf{X}_j) = \left[h_i(\mathbf{X}_j)[0] \quad h_i(\mathbf{X}_j)[1] \quad \ldots \quad h_i(\mathbf{X}_j)[K-1] \right] \quad (18)$$

where K is the total number of samples.

The minimization of cost C is not a trivial task. A large number of variables are involved and the cost function could exhibit a lot of local minima. One may be constrained to make use of specific optimization methods, such as genetic algorithms [3], to solve this task.

In the considered problem, it is possible to take advantage of the fact that costs C_j may be minimized independently since each one is function of only one state $\mathbf{X}_j$.

7. CONCLUSIONS AND PERSPECTIVES

The paper considered only a limited case, in which targets are assumed to have only a straight, uniform motion, while their vertical speed is assumed to be zero. Obviously, this is not the most realistic scenario. Manoeuvring targets, like fighters, largely overcome the limits of our assumptions.

A perspective work would consist, then, in generalizing our approach, by example by taking into account more complex kinds of target motion. In this case, the target could be described by a grown state vector:

$$\mathbf{X}_1 = \left[x_0, \xi_1, \xi_2, \ldots, \xi_Q, y_0, \eta_1, \ldots, \eta_Q \right]^T \quad (19)$$

where x_0 and y_0 are target coordinates at $t=0$, while $\xi_1,\ldots,\xi_Q$ and $\eta_1,\ldots,\eta_Q$ are their derivatives (accelerations), while Q is the degree of motion approximating polynomial:

$$x(t) = x_0 + \xi_1(t-t_0) + \frac{1}{2}\xi_2(t-t_0)^2 + \ldots \quad (20)$$

and

$$y(t) = y_0 + \eta_1(t-t_0) + \frac{1}{2}\eta_2(t-t_0)^2 + \ldots, \quad (21)$$

respectively.

These suggested improvements will turn the proposed method into a field deployable solution, thus confirming the possibilities open by the use of passive radars.

8. ACKNOWLEDGMENTS

This work was supported by the French Ministry of Foreign Affairs/EGIDE, under the grant ECO-NET 2007, and by the North Atlantic Treaty Organization, under the grant EAP.RIG.982933.

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