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On Seat Congestion, Passenger Comfort and Route Choice in Urban Transit: a Network Equilibrium Assignment Model with Application to Paris

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Abstract

In network assignment models of urban transit, traffic congestion has been modelled either as vehicle congestion along the route track, or by reducing the service frequency with respect to excess flow of passenger arrivals. A third type of congestion has been modelled by Leurent (*1*), (*2*): that of seat congestion, because being seated or standing make distinct on-board states for a transit rider, resulting in distinct discomfort costs, with potential influence on route choice on the transit network.

The paper has a twofold objective of, first, providing a concise statement of the seat congestion model, and second, reporting on its application to the Paris metropolitan area – a problem of very large size. The model makes explicit the residual set capacity and its evolution at any stage along a service line; the priority rules amongst riders depending on their order of arrival in the competition to get a seat; and the randomness in leg cost to the rider. Line algorithms and consistent network algorithms are provided.

Keywords

Seat congestion. Seated capacity. Capacity constraint. Passenger comfort. Transit traffic Equilibrium. Transit network assignment. Paris transit network.

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1. INTRODUCTION

In densely populated urban areas, public transit is the transport mode specifically purported to carry large passenger flows along medium to long distances that require mechanical transport. The higher the passenger flows, the larger the scale economies that can be achieved, and the heavier and more efficient transit mode that can be implemented: from the bus to the train, passing by the tram and the metro. This yields benefit not only to the service operator, but also to the transit rider because of increased travel speed and also increased frequency (the Mohring effect). This is also beneficial to the environment since transit is safer than the car, spends less energy and emits fewer pollutants with respect to total distance travelled by trip-makers.

This has provided considerable impetus to develop transit in urban areas, particularly so in the perspective of sustainable development. The planning of transit networks has taken a classical, rationalized form of mixed technical and economic analysis: a transport project or scheme is simulated on the basis of a transit network assignment model (*3*), (*4*), which yields estimates of traffic and revenues, to be balanced against the project costs. Out of a set of alternative schemes, the most attractive one with respect to a set of appraisal criteria is recommended by the transportation planner to the political decision-maker.

This methodology closely resembles that for road network planning, which has caused multimodal planning to emerge in the 1980s. However, road network assignment has motivated much more modelling effort than transit assignment, notably so as regards congestion phenomena: the more traffic along a route, the longer it takes to travel along it, thus the more costly is the route to the trip-maker who decides on which route to use in a cost-minimizing manner. It is well known to transportation planners that the time savings, in other words the congestion gains, of a road transport scheme may represent up to 80% of its simulated benefits. This may boost the return-on-investment indicator in the economic appraisal of the project utility to the society. Oddly enough, little if any account has been taken of the congestion that pertains to public transport, despite the fact that transit makes the by-choice arena for congestion owing to the magnitude of its flows! Of course, the packing of many passengers into a given transit vehicle splits the issue of transit congestion in two parts that pertain to the vehicle or the passenger, respectively. Congestion in vehicle flow along the roadway was addressed by Spiess and Florian (*5*) in a static way same as in road network assignment, on the basis of a speed-flow relationship at the link level. The next step was to relate the passenger flow incoming at a station to board a transit line, to the line frequency, thus linking the vehicle congestion to the passenger flow: see De Cea and Fernandez (*6*) and Comminetti *et al* (*7*), (*8*). Alternatively, Marcotte and Nguyen (*9*) suggested to bound the boarding flow to the capacity available at that stop; this line was continued by Shimamoto *et al* (*10*) who considered the probability of accessing the transit service at each stop.

Recently Leurent (*1*), (*2*) has developed a model of seat congestion, which makes a third piece for congestion in the transit assignment toolkit: the approach is to identify the residual seat capacity at any stage along the line, and to share it amongst standing riders under priority rules. As seating or standing make distinct states for the rider, these induce distinct travel costs, which influence the route choice. In a Stated Preference survey ordered by Stif, the Paris transit regulatory body (*11*), the unit cost of standing time was evaluated at 1.6 times that of seated time, plus an additional 0.3 times if the in-vehicle standing areas are densely crowded.

The objective of the paper is to state that model of seat congestion in a simplified, qualitative way and to report on its application to the transit network in the Paris metropolitan area. This application was conducted by the authors in partnership with Ratp, the major transit operator in the Paris area. Amongst the outcomes, let us quote that on central metro lines at the morning peak period, significant changes of up to 30% were observed in the link flows as compared to results from the previous, standard model; furthermore, the mean generalized cost of a transit trip was increased by 15%.

The body of the paper is organized into three main parts and a conclusion. Section 2 states the model in a qualitative way by providing the physical and behavioural assumptions about individual trips faced to given traffic states along a transit line: this involves notions of comfort states, travel behaviour and priority rules. Section 3 provides the mathematical treatment, from line loading and costing algorithms to the problem of transit network traffic equilibrium and the network assignment algorithms; *notice that this is a technical section which may be omitted at first reading*. Section 4 describes the application to the Paris transit network, in terms of application data, simulation tool and computation of equilibrium, assignment results with comparison to those based on the previous model taken as reference. Lastly, Section 5 provides some concluding comments and points to research topics.

2. THE MODEL OF SEATED CAPACITY

Our modelling assumptions pertain to comfort states and their cost (subsection 2.1), and also travel behaviour and priority rules across users (subsection 2.2). They lead us to define the notion of a service mode (subsection 2.3), which is a specific way of using a line “leg” from access station to egress station. As the users compete for the residual seat capacity, there is randomness in getting a seat, which makes the leg cost a random variable (subsection 2.4).

2.1 Comfort states and their cost

In a public transport service, it frequently occurs that several categories of places with distinct attributes be provided to the users. In urban transit by bus, tram, metro or train, the distinction between being seated and standing is particularly significant: a seated rider is less inconvenienced by the vehicle’s acceleration and deceleration and by the slopes and curves in the vehicle’s trajectory; he may invest his travel time into a complementary activity such as reading, listening to music, relaxing or working; moreover, he is much less submitted to crowding.

Let us assume that on a transit line segment from a station to the next, there are two riding states namely seating and standing, with associated costs that reflect the users’ preference to have a seat. Let a denote a line segment, $\underline{c}_a$ its discomfort cost to a rider at seating, and $\overline{c}_a$ its discomfort cost to a rider at standing: the basic assumption is that $\underline{c}_a \leq \overline{c}_a$ whatever the traffic load.

2.2 Travel behaviour and priority rules

It is assumed that every rider is a cost-minimizing individual decision-maker, striving to reduce his travel cost. Then a rider who is standing tries to get a seat as soon as one becomes available. As there is a limited number of available seats, say κ for capacity, and also a number x of riders that would like to sit, it may be the case where $\kappa < x$, meaning that capacity is less than demand. In this case only a proportion κ / x of riders may sit. The issue of which riders would get a seat is addressed here in a simple way, assuming that all of them have an equal probability to sit, i.e. neglecting the individual attributes of age and physical needs, eagerness-to-sit, planned egress station etc.

The time at which seats become available and competition occurs is important: standing riders that stay on board have an advantage over the incoming riders, which is modelled by assuming two successive competitions: the first one among "through" riders, the other one among incoming riders. Thus, in our basic model of seat congestion, *two priority rules* are assumed: (1) that standing riders with same level of priority have equal chance of getting a vacant seat; and (2) that standing passengers going through a transit stop obtain access to vacant seats prior to riders boarding at that stop.

2.3 Services modes on a transit leg

On a "*leg*" from an access station to an egress station along a transit line, the user is provided service in a particular way, depending on which comfort states he gets on each line segment in the leg.

Let us define a "*service mode*" as the sequence of segment comfort states along the leg. In a leg made up of N segments, if there are two comfort states associated to each segment, there could be as much as 2^N service modes associated to the leg, but in fact the users' behaviour reduces the number of alternative service modes to $N+1$: after getting a seat a user is assumed not to release it until arrival at his egress station.

Thus a service mode is fully described by the station $i+m$ at which the user gets a seat, with index $m \in \{0,1,2..N\}$: by convention, getting a seat on exiting at $i+N$ means standing all over the leg.

The cost of service mode m from access station i to egress station $j = i + N$ is:

$$c_{ij}^{m} = [\textstyle\sum_{k=i}^{i+m-1} \overline{c}_{a\approx(k,k+1)}] + [\textstyle\sum_{k=i+m}^{j-1} \underline{c}_{a\approx(k,k+1)}] \qquad (1)$$

2.4 Leg cost as a random variable

The attribution of a given service mode does not depend solely on the user, because his preference to be seated rather than to stand may get into competition with others' preferences, resulting in a collective allocation process of seats rather than in an individual choice of a service mode.

The seat allocation process at each station may be summarized by two sitting probabilities: the first one say p_i^o for through riders, and the other one say p_i^+ for incoming riders.

The probability of keeping standing from i to $i+N$ is

$$\pi_{i,i+N}^{N} = (1-p_i^{+}).[\prod_{k=1}^{N-1}(1-p_{i+k}^{o})] \tag{2a}$$

The probability of sitting at station $i+m$ is:

$$\pi_{i,i+N}^{0} = p_i^{+} \text{ at } m=0\text{, then} \tag{2b}$$

$$\pi_{i,i+N}^{m} = (1-p_i^{+}).[\prod_{k=1}^{m-1}(1-p_{i+k}^{o})].p_{i+m}^{o} \text{ at } m\in\{1,2..N-1\} \tag{2c}$$

The probabilities $(\pi_{i,i+N}^{m})_{m=0..N}$ are associated with the service modes and describe the random process of getting a service mode. Service mode *m* has probability $\pi_{i,i+N}^{m}$ stated in (2) and cost $c_{i,i+N}^{m}$ stated in (1).

Thus, to the user the leg cost is a random variable $c_{i,i+N}$, depending on which service mode is obtained. It has mean value and variance value of, respectively:

$$\widehat{c}_{i,i+N} \equiv \mathrm{E}[c_{i,i+N}] = \sum_{m=0}^{N} \pi_{i,i+N}^{m}.c_{i,i+N}^{m} \tag{3a}$$

$$v_{i,i+N} \equiv \mathrm{var}[c_{i,i+N}] = \sum_{m=0}^{N} \pi_{i,i+N}^{m}.(c_{i,i+N}^{m} - \widehat{c}_{i,i+N})^2 \tag{3b}$$

2.5 Discussion

The two priority rules induce the probability to sit either from the previous segment or at boarding, so by successive segments along the transit leg (transit section) a probability results for each "service mode" made up of a sequence of *m* segments at standing followed by $n-m$ segments at seating in a leg of *n* segments. At the leg level, the service mode is obtained in a somewhat random way (due to first rule), and yields a random leg cost, with distribution characterized by both the segment-state costs and the service mode probabilities.

To the leg user, the leg generalized cost is derived from the cost of the random service mode. The random leg cost has a given distribution hence given mean value and variance. Every trip-maker includes the leg cost in the route cost from origin to destination: this enables him to choose his preferred route from among the available routes on the basis of their respective costs.

3. MATHEMATICAL FORMULATION AND ALGORITHMS

Let us now state *in an abridged way* the mathematical analysis and algorithms for the model of seat congestion. First, a line loading algorithm is provided to load an access-egress trip matrix onto the line, in a way consistent with the priority rules and yielding all sitting probabilities in an efficient way (subsection 3.1). Then, a line costing algorithm is provided to compute the mean leg costs along the line (subsection 3.2). After stating how to represent the network and which variables to consider (subsection 3.3), the problem of traffic equilibrium on the transit network is defined (subsection 3.4). A solution algorithm is provided, which involves iterations made up of node-to-destination costing, flow loading and flow update (subsection 3.5).

3.1 Line flow loading algorithm

The line loading problem is to assign a line access-egress trip matrix to the seating and standing states along the line segments, and hereby to establish the sitting probabilities at station i for standing rider either through or boarding. Apart from line seat capacity κ, the inputs consist in the line access-egress trip matrix $[q_{ij}^{\ell}]_{i<j\in\ell}$. Let us consider as state variables at station i the residual capacity κ_i and two flow vectors of riders indexed by egress station j: $\underline{x}_j$ for seated riders and $\overline{x}_j$ for standing riders.

The line loading algorithm deals with station i from beginning to end of line along the following steps:

- The seated riders that exit at i release their seats, yielding $\kappa_i^o := \kappa_{i-1}^- + \underline{x}_i$.
- The on-board standing riders that do not exit at i, in number of $y_i^o := \sum_{j>i} \overline{x}_j$, try to get a seat, with success probability of $p_i^o := \min\{1, \kappa_i^o / y_i^o\}$. This decreases the residual capacity to $\kappa_i^+ := \kappa_i^o - p_i^o . y_i^o$, whereas the egress flows are changed into $\underline{x}_j := \underline{x}_j + p_i^o . \overline{x}_j$ and $\overline{x}_j := (1 - p_i^o) . \overline{x}_j$ by station $j > i$.
- The boarding riders in number of $y_i^+ := \sum_{j>i} q_{ij}^{\ell}$ try to get a seat, with success probability of $p_i^+ := \min\{1, \kappa_i^+ / y_i^+\}$. This decreases the residual capacity to $\kappa_{i+1}^- = \kappa_i^+ - p_i^+ y_i^+$, whereas the egress flows are changed into $\underline{x}_j := \underline{x}_j + p_i^+ . q_{ij}^{\ell}$ and $\overline{x}_j := \overline{x}_j + (1 - p_i^+) . q_{ij}^{\ell}$ by station $j > i$.

This involves a number of elementary operations in $O(\overline{n}_{\ell}^2)$, with $\overline{n}_{\ell}$ the number of stations i along line ℓ.

3.2 Line leg costing algorithm

The flow share and cost of each service mode, as stated in (1) and (2), may be readily evaluated on the basis of the sitting probabilities. A recursive algorithm in $O(\overline{n}_{\ell}^3)$ was given in (*1*), together with streamlined algorithms in $O(\overline{n}_{\ell}^2)$ targeted at only the mean and variance of leg cost. Let us recall here the algorithm to evaluate the mean leg cost, which deals with a given station j of egress by taking the access stations $i < j$ in decreasing order from $j-1$ down to 1. To obtain the mean leg cost $\widehat{c}_{ij}$ from access station i to egress station j, let us associate two costs to that leg, namely:

- the seating cost $c_{ij}^0 = \sum_{k=i}^{j-1} \underline{c}_{a\approx(k,k+1)}$,
- an auxiliary mean cost γ_{ij} which is a mean cost from i to j conditional on standing on segment $(i, i+1)$.

These costs satisfy the following recursive equations, due to the law of total probability:

$$c^0_{i,j} = \underline{c}_{a\approx(i,i+1)} + c^0_{i+1,j} \quad (4a)$$

$$\gamma_{i,j} = \overline{c}_{a\approx(i,i+1)} + p^o_{i+1}.c^0_{i+1,j} + (1-p^o_{i+1}).\gamma_{i+1,j} \quad (4b)$$

$$\hat{c}_{i,j} = p^+_i.c^0_{i,j} + (1-p^+_i).\gamma_{i,j} \quad (4c)$$

3.3 Network representation and assignment variables

The obvious way to address seat congestion is to associate one network arc to each leg of a transit line, tailed at the access station and headed to the egress station. For an oriented transit line ℓ with $\overline{n}_\ell$ stations i numbered in forward order from 1 to $\overline{n}_\ell$, there are $\overline{a}_\ell = \overline{n}_\ell - 1$ line segments $(i,i+1)$. The set of network nodes required to model the line is $N_\ell = N^+_\ell \cup N^-_\ell$ with $N^+_\ell = \{\mathrm{n}^+_{\ell,i} : i = 1..\overline{n}_\ell \text{ -}1\}$ the subset of nodes for access at station i and $N^-_\ell = \{\mathrm{n}^-_{\ell,i} : i = 2..\overline{n}_\ell\}$ for egress nodes. The line segments give rise to set $A_\ell = \{a \approx (\mathrm{n}^+_{\ell,i}, \mathrm{n}^-_{\ell,j}) : 1 \le i < j \le \overline{n}_\ell\}$ of line leg arcs, in number of $|A_\ell| = \frac{1}{2}(\overline{n}_\ell - 1)(\overline{n}_\ell - 2)$.

A leg arc has a travel cost which depends on the line access-egress trip matrix, which is associated to the set of leg arcs for that line.

In large networks this representation of the legs yields a very large assignment network; however no other network representation would save on the computer cost, since it is required to identify the line trip matrix which is analogous to the line subvector of a network flow.

The network $G = [N, A]$ is comprised of a set N of nodes n and a set A of arcs a with tail node n^+_a and head node n^-_a in N. Any node or arc belongs to at most one transit line $\ell \in L$ set of lines. Arcs that do not belong to a transit line are called "non-transit" arcs and correspond to other transportation modes such as by foot. Among non-transit arcs, those headed to an access node $\mathrm{n}^+_{\ell,i}$ along a transit line ℓ are called access arcs to that line.

Arc a has average traversal cost $\hat{c}_a$, and a service frequency f_a which is set to either line frequency f_ℓ when *a* is an access arc to line ℓ, or infinity otherwise.

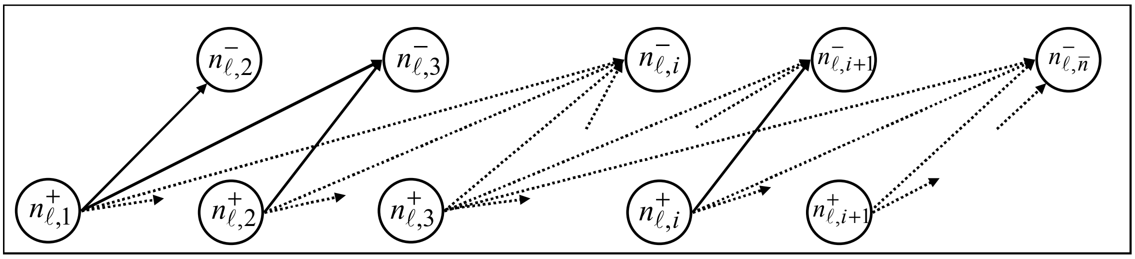

FIGURE 1 Representation of transit line within assignment network.

The set of destinations is denoted by S – for sink nodes. To a destination s is associated a set W^s of origin-destination (OD) pairs $w=(o,s)$ with OD flow of q_{os}. To the set $W=\bigcup_{s\in S} W^s$ is associated the vector of OD flows $\mathbf{q}=[q_w]_{w\in W}$.

At the arc level, let x_{as} denote a flow of users on arc a destined to s.

A vector of arc-destination flows $\mathbf{x}_{AS}=[x_{as}:a\in A,s\in S]$ is called a **flow state**. Its restriction to the legs of a transit line ℓ is denoted as $\mathbf{x}_{\ell S}=[x_{as}:a\in A_\ell,s\in S]$.

3.4 Transit network traffic equilibrium problem

A flow state $\mathbf{x}_{AS}$ yields an access-egress trip matrix $\mathbf{q}^\ell$ by transit line ℓ on the basis of $\mathbf{q}^\ell=[q_a^\ell:a\in A_\ell]$ in which $q_a^\ell=\sum_{s\in S} x_{as}$. Application of the line loading algorithm to $\mathbf{q}^\ell$ yields uniquely defined sitting probabilities $\mathbf{p}_\ell^o=[p_{\ell,i}^o]_{i\in\ell}$ at through nodes and $\mathbf{p}_\ell^+=[p_{\ell,i}^+]_{i\in\ell}$ at access nodes. Then, the line costing algorithm induces uniquely defined average line leg costs: in fact this algorithm is a cost-flow relationship at the line level, on the basis of seated and standing segment costs (which can be functions of the flow state).

Several options are available to capture the seat congestion effects in a route choice model of transit assignment: static or dynamic, hyperpath-based or route-based to address the common line problem: i.e. how to split traffic at a station node where several transit lines are attractive for the rider to get closer to his destination.

Hereafter we focus on a static, hyperpath-based model: a rider travels along a sub-network, called a hyperpath, which is a bundle of OD paths such that route choice takes place at specific nodes, depending on which alternative line is service first.

As shown in (*5*), hyperpath model includes two specific network algorithms: the first one to search for OD hyperpaths on the basis of segment costs and line frequencies; the other one to load the OD volume onto the associated hyperpath. In our adaptation of hyperpath transit assignpment, the two network algorithms are left unchanged: the significant modification pertains to the cost-flow relationship at the line level.

Notice that at an access node, the sitting probabilities do not make routing probabilities similar to those associated to route choice at station nodes: this is because the seated and standing states are not modelled by distinct arcs for a given line segment.

Overall, based on a flow state the route costs by OD pair can be derived; associated to these route costs are optimal routing structures (hyperpaths or transit routes), along which the assignment of the OD trip volumes would yield another flow state.

A *transit network traffic equilibrium* is reached at a given flow state such that it is optimal with respect to its own route costs.

The network assignment problem is to find a transit network traffic equilibrium: it has been cast into a mathematical program of nonlinear complementarity, and equivalently into a variational inequality problem, in the framework of hyperpath assignment (*2*). A convergence criterion is available, based on the duality gap of the variational inequality at a given flow state.

This formulation is endowed with a theorem of existence for a traffic equilibrium, provided that the arc cost functions are continuous. Multiple equilibria may arise with no structural feature of uniqueness, as was demonstrated in a simple case.

3.5 Network assignment algorithms

Traffic assignment to a transit network with seat congestion can be performed by means of the following equilibration algorithm, which makes use of two network flow states $\mathbf{x}_{AS}$ for a current state and $\mathbf{y}_{AS}$ for an auxiliary state, two related overall arc flow vectors $\mathbf{x}_A$ and $\mathbf{y}_A$, one set of node potentials $\mathbf{u}=[u_n]_{n\in N}$, an iteration counter k, variables β, Γ, U, W and Z. Input variables consist in G, L, arc costs $\underline{\mathbf{c}}=[\underline{c}_a]_{a\in A}$ and $\bar{\mathbf{c}}=[\bar{c}_a]_{a\in A}$, $\mathbf{f}=[f_\ell]_{\ell\in L}$, $\boldsymbol{\kappa}=[\kappa_\ell]_{\ell\in L}$, the OD trip matrix $\mathbf{q}=[q_{os}]_{o\in O_s, s\in S}$, a tolerance ε on the convergence level, and a sequence of decreasing positive numbers $(\zeta_k)_{k\geq 0}$ with $\zeta_0=1$.

The equilibration algorithm is made up of five steps:

Initialization. Set $\mathbf{x}_{AS}:=0$ and $\mathbf{x}_A:=0$. Let $k:=0$, $\beta:=0$ and $\Gamma:=0$.

Cost-Flow Relationship. Evaluate the arc costs $c_a=\mathrm{C}_a(\mathbf{x}_{AS})$ for all $a\in A$ as in subsection 4.3.

Network Costing and Flow Loading. Let $U:=0$ and $\mathbf{y}_A:=0$. For every destination node $s\in S$:

- Find the optimal hyperpath destined to s under the current arc costs, yielding node potentials u_n.
- Load the OD flows $[q_{ns}:n\in N]$ on the currently optimal hyperpaths to s, yielding arc flows y_{as}.
- Let $U:=U+\sum_{n\in N} q_{ns}u_{ns}$. Let $y_a:=y_a+y_{as}$ for all $a\in A$.

Flow Update. Let $W:=U-\sum_{a\in A} c_a y_a$. Let $\Gamma:=\Gamma+\zeta_k$ and $\beta:=\beta+\zeta_k W$. Let $Z:=\frac{\beta}{\Gamma}-U+\sum_{a\in A} c_a x_a$. Then let $\mathbf{x}_{AS}:=\mathbf{x}_{AS}+\zeta_k(\mathbf{y}_{AS}-\mathbf{x}_{AS})$ and $\mathbf{x}_A:=\mathbf{x}_A+\zeta_k(\mathbf{y}_A-\mathbf{x}_A)$.

Convergence Test. If $Z\leq\varepsilon$ then terminate, else let $k:=k+1$ and go to step *Cost Flow Relationship*.

This is a mere method of successive averages, in which the auxiliary flow state $\mathbf{y}_{AS}$ is a user-optimized assignment of all the OD flows on the basis of the costs induced by the current flow state $\mathbf{x}_{AS}$. The convergence criterion Z is the duality gap in the variational inequality program associated to the transport model (*2*).

4. APPLICATION TO THE PARIS TRANSIT NETWORK

After introducing the application data in the Paris case (subsection 4.1), we shall report on the computation of a traffic equilibrium (subsection 4.2) and on the assignment results, with comparison to a reference, uncapacitated model (subsection 4.3).

4.1 Application case and data

The Greater Paris Area (the "Île-de-France" region) is populated by 11.6 million inhabitants as of 2007, over a land area of 12,011 km².

Network overview. The Public Transport (PT) system is comprised of rail and heavy metro ("RER"), subway, tramway and bus lines. The rail lines include 8 "Transilien" lines of suburban rail serviced by 96 bidirectional vehicle routes, 5 RER lines serviced by 79 vehicle routes, 16 metro lines and 4 tramway lines, with 806 stations along a total length of 1,642 km. There are 1,371 bus lines with 30,068 bus stops along track mileage of 18,700 km.

Amongst the lines of regional train and heavy metro, the busiest line is RER line A, operated at morning peak using 30 trains per hour and direction on its trunk section, thus providing local capacity of about 200 thousand passengers per hour. Table 1 gives some indication about the heavy modes by rail. Whereas these modes cover the region in a mostly radial pattern centred on the Paris city, the subway lines span the city and its close suburbs, hence the densest area, in a dense manner: there are about 300 stations spaced by half a kilometre on the average. Subway line 13 is the busiest one, serviced at morning peak by up to 35 trains per hour and direction: at several stations there are queues of riders waiting for the next trains, making it the most acute instance of transit congestion in the metropolitan area; whereas along RER line A congestion mostly occurs in the form of densely crowded trains. Table 2 provides indication about subway lines and capacity.

The network database pertains to year 2004 and contains 1,314 service routes with 27,815 service nodes. It is comprised of 250,432 links, among which 27,853 are service links, 138,490 transfer links and 84,089 connectors.

Transit trips. On the average weekday as of 2002, out of 10 million (M) trip-makers (including inhabitants but not visitors), about 3 M use a transit mode, yielding about 7 M transit trips and 11 M legs. The trip and leg distance are much varied by transit mode and with respect to origin-destination pair. The modal share of transit is increased from 29% within Paris city, up to 62% between Paris city and the distant suburbs, passing by 57% between Paris city and its close suburbs. Season tickets are held by about 2.5 M inhabitants who make 80% of the transit trips. The structure of tariffs with respect to location has a monocentric pattern.

The study area is divided into 1,921 traffic assignment zones. The OD matrix at the morning peak contains about 1 M trips per hour.

TABLE 1 Heavy Rail Lines with Vehicle Capacity, as of 2004

Heavy rail lines	# line endpoints	# service routes	Frequency range by route (train/h)	Maximum frequency* (train/h)
RER A	5	13	1 - 6	30
RER B	4	37	1 - 2	20
RER C	7	11	2 - 4	20
RERD	3	11	4 - 12	28
RER E	3	7	2 - 8	16
⇔ Montparnasse Station	3	16	1 - 4	4
⇔ North Station	9	26	1 - 4	4
⇔ East Station	7	15	1 - 6	6
⇔ St Lazare Station	7	32	1 - 6	6
⇔ Lyons Station	6	7	1 - 2	4

* Maximal number of trains per direction using the line trunk.

TABLE 2 Subway Lines with Rider Capacity at AM Peak Hour, as of 2005

Line	Seats / vehicle	Frequency (train/h) per direction	Total capacity# (riders/h)	Line	Seats / vehicle	Frequency (train/h) per direction	Total capacity# (riders/h)
1	96	34	24,480	**7bis**	64	15	5,265
2	120	29	16,675	**8**	128	27	15,498
3	120	31	17,825	**9**	120	31	17,825
3bis	72	18	6,138	**10**	120	20	11,500
4	144	31	21,700	**11**	96	34	15,776
5	120	30	17,250	**12**	120	28	16,100
6	120	31	17,825	**13**	128	35	20,090
7	128	34	19,516	**14**	144	33	23,826

#l Local capacity of both seats and standing areas (with 4 passengers per m²).

4.2 On the computation of traffic equilibrium

Traffic equilibrium in the transit model with seat congestion was computed by the Method of Successive Averages, as indicated in subsection 3.5. The stepsize was set to $\zeta_k = 1/(k+1)$. An acceptable level of convergence was reached after 30 iterations, based on the duality gap criterion.

The model was run on a UNIX workstation SUN W21002 with two processors AMD Opteron at 2.6 GHz and RAM of 2 Gb, yielding an average run time per iteration of 313 seconds (to be compared to the 139 s required to perform the single-iteration assignment for the unbounded model).

In order to test the robustness of the solution, keeping in mind that multiple equilibria may arise, a second assignment was performed using an alternative strategy of initialization, by setting all of the initial sitting probabilities to zero rather than to one. The aggregate indicators of assignment were preserved up to very minor changes.

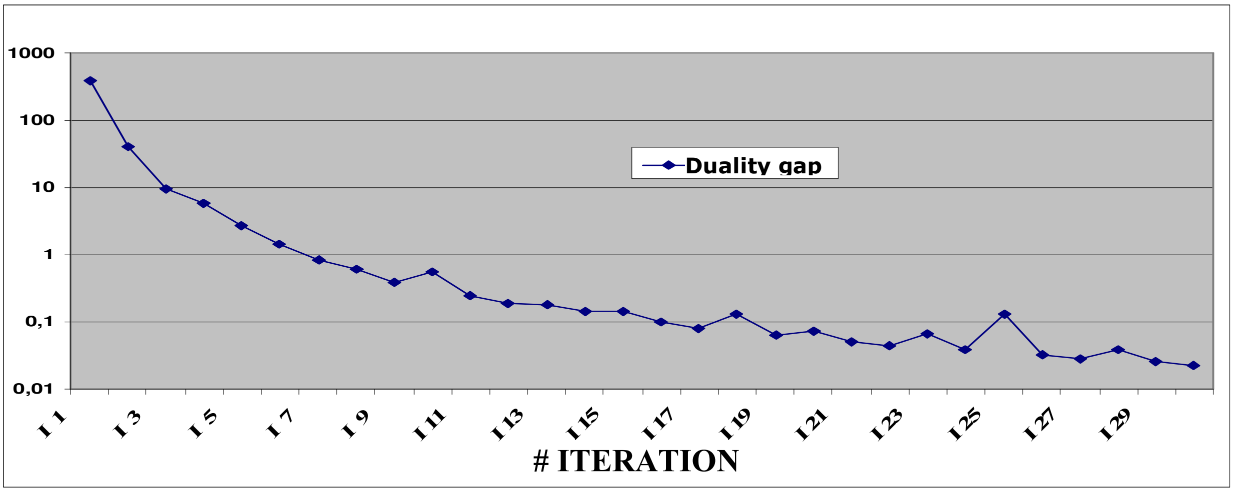

FIGURE 2 Convergence criterion against iteration number

4.3 Assignment results, with comparison to unbounded model

Based on the Stif survey mentioned in the Introduction, the cost of standing in-vehicle was set to 1.8 times that of being seated. The equilibrium assignment, illustrated in Figure 3, yields generalized cost of 9.6 euros per trip on the average (assuming a unit cost of time of 8 €/h and neglecting the tariffs). As shown in Table 3, the share of in-vehicle time is increased from 41% to 46% of the generalized time, yielding a proportionate reduction in the share of both terminal time and transfer time; however the waiting time maintains its share and even increases in absolute value, implying that riders are pushed into changing their routing strategy. This is confirmed by the indicators of route number per OD pair, evaluated in terms of number of transit lines or number of service routes, respectively: riders are likely to diverse to routes of relatively low frequency, which at first look would be less attractive and less congested.

Looking into the detailed results of assignment at the morning peak, the link flows along major transit lines may be changed by some thousand passengers per hour from previous, unbounded model to the model with seat congestion. Such changes are relatively less important for lines of heavy train, where traffic load is heavy and there are few alternative routes, than for subway lines, of which the load can be modified by 20%. Figure 4 depicts the variation of segment load along the stations of subway line 1: the Westbound direction which ends in the major business centre "La Défense" is submitted to large reduction, especially so in the central part for which attractive alternative routes are available.

TABLE 3 Decomposition of Generalized Cost and Number of Routes per OD Pair

Indicator / **Model**	Mean Generalized Cost (€)	In-vehicle time	Waiting time	Transfer time	Terminal time	# Line routes / OD pair
Seat comfort	9.93	46%	10%	11%	33%	5.6
Unbounded	8.89	40%	11%	13%	36%	4.6

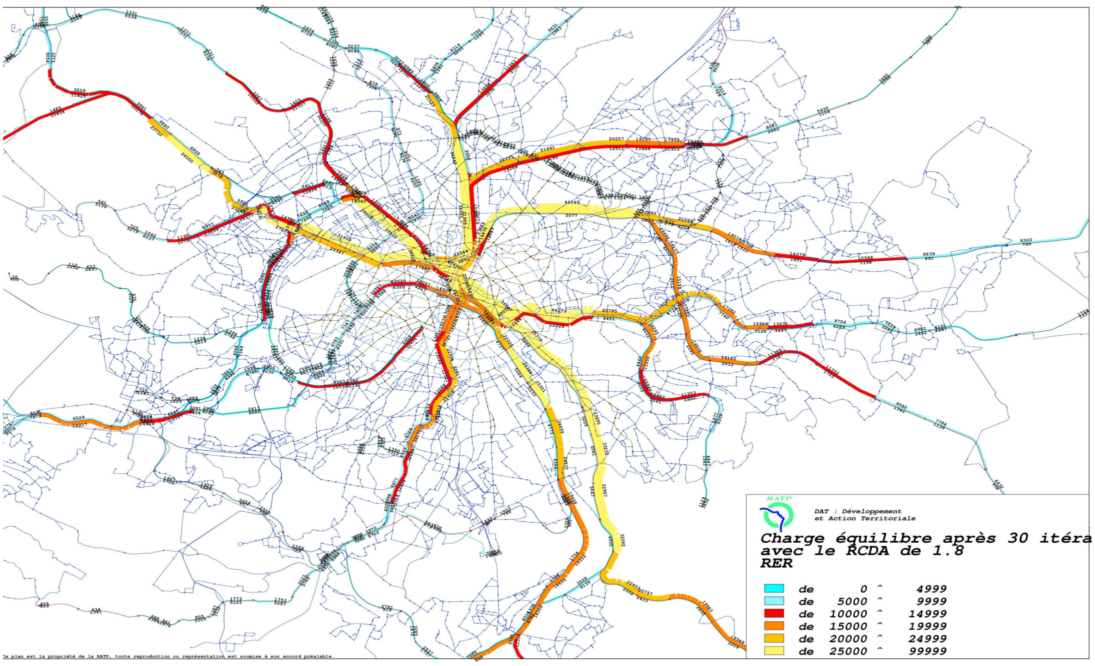

FIGURE 3 Map of passenger load at morning peak on main rail network

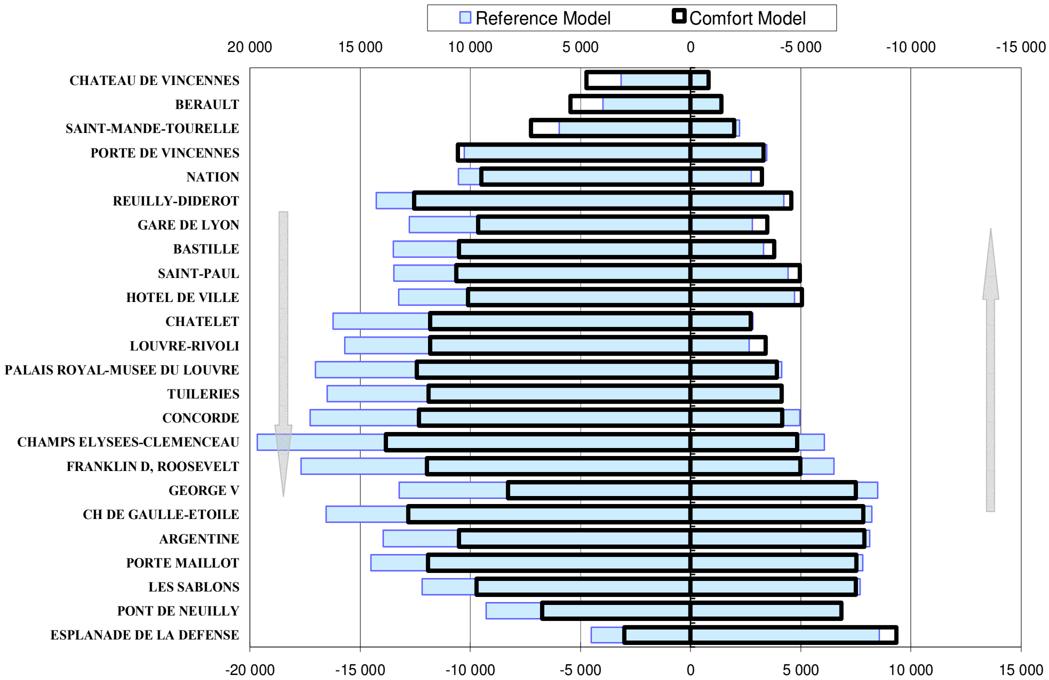

FIGURE 4 Model comparison along subway line 1 at the morning peak (Westbound vs. Eastbound): seated capacity about 5,000 places per hour and per direction

5. CONCLUSION

Seat congestion in urban transit services has been modelled in both physical and behavioural respects. The physical respects pertain to: the evolution of the residual seat capacity at any stage along a service line; the randomness of the service mode that the rider gets on his transit leg from access station to egress station; the priority rules amongst riders trying to get a seat on the basis of their order of arrival in the vehicle. The behavioural respects pertain to the rider's preference to be seating rather than standing, and to the inclusion of the leg cost into the generalized route cost that is considered by the rider to make his route choice. These modelling assumptions have been cast into a mathematical formulation, yielding a problem of traffic equilibrium in transit network. A solution algorithm has been provided.

Application to the Paris metropolitan transit network, indeed a problem of very large size with 30 lines and 800 stations of rail, 1,300 bus lines with 30,000 stops, and 1,900 traffic assignment zones, yielded significant change from the previous, unbounded assignment model of Ratp: mean generalized cost is increased by 15%, subway loads are changed by about 30%, thus driving the riders to make use of transit legs in a more elaborate way.

Such changes are expected to exert significant consequences in the appraisal of transport schemes that reduce passenger congestion, by correcting for the bias of benefit underestimation that such schemes incur when simulated by a model that does not capture seat congestion and passenger comfort.

Related topics for further research include:

- Passenger congestion at stations: between platform and vehicles, in corridors for access and transfer, on platform.
- More realistic representation of tariff prices and structure, in consistency with complex individual routing structures such as hyperpath and transit route. This is all the more required since improved modelling of service quality calls for improved modelling of quality to price trade-offs.
- Estimation of behavioural parameters in route choice and mode choice, on combining realistic representation of transit quality with Revealed Preference data coming e.g. from household travel survey.

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