Self Field Effect Compensation in an HTS Tube

Bruno Douine, Kévin Berger, Jean Lévêque, Denis Netter, Olivia Arbey, and Nancy Barthelet

***Abstract*—It is well known that the critical current density J_c of a superconducting material depends on the magnetic flux density B. There exists an electric method to measure the $J_c(B)$ deduced from the $U(I)$ measurements. The problem with this method is the self field effect because the magnetic flux density is always the sum of the applied magnetic flux density and the self magnetic flux density. This paper presents a special experimental arrangement, compensating fully or partially the self magnetic flux density in an HTS tube. It allows characterizing the true zero magnetic flux density behaviour of the superconducting material. The experimental results of the compensation are discussed. A theoretical analysis based on Bean's model is presented and gives results close to the experimental ones. The proposed compensation is not perfect but the experiments and the theoretical analysis allow validation of the compensation principle.**

***Index Terms*—Critical current density, self field effect, superconductor.**

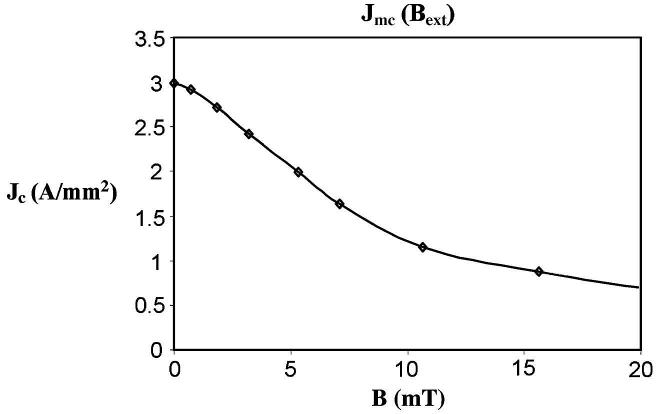

Fig. 1. Measured $J_{\mathrm{mc}}(B_{\mathrm{ext}})$ for a BiSCCO tube at 77 K.

I. Introduction

It is well known that the critical current density J_c in a superconducting material depends on the magnetic flux density B. The $J_c(B)$ characteristic is very important to calculate AC losses in superconducting wires. Indeed, the evaluation of these losses is necessary for the design of the cooling system for any superconducting device. The superconducting power cable geometry is similar to that of a tube [1], [2]. Despite the inhomogeneous nature of power cables, the loss is often compared to the monoblock model using average current densities and magnetic flux density. For this reason it is important to know well the $J_c(B)$ dependence of a tube.

Measurements of $J_c(B)$ can be achieved by electrical transport (direct) [3] or magnetic (indirect) [4] methods. In this paper, the electrical method used to define the $J_c(B)$ of a superconducting tube is studied. Electrical transport measurements are achieved by the standard four-probe technique, which involves injecting a dc current I in the superconducting tube and measuring the voltage drop U [3]. Throughout this paper, the extremity effects are neglected because the voltage taps are sufficiently distant from the tube extremities. The critical current density J_c can be deduced from the $U(I)$ measurements thanks to the following hypotheses:

1) the distribution of the current density is uniform in the superconducting material section S and so $I = J.S$;
2) the electric field has only one component along the tube axis and so $U = E.L$ (L is the distance between the voltage taps).

In this case, one can deduce $E(J)$ from $U(I)$. The value of the critical current density $J_c = I_c/S$ depends on the measurement criterion. This criterion corresponds to the critical electric field E_c that is often chosen as 1 μV/cm. To theoretically calculate, for example, ac losses or magnetization, we can use Bean's critical state model that defines the relation between electric field E and current density J as $J = \pm J_c$ or $J = 0$.

To obtain the experimental curve of $J_c(B)$, the tube was fed with direct current I and submitted to an external magnetic flux density B_{ext} parallel to the axis $[Oz]$ [3]. The sample voltage drop U versus I is measured for different external magnetic flux densities B_{ext}. From the measured critical current $I_{\mathrm{mc}}(B_{\mathrm{ext}})$ corresponding to the voltage drop equal to 1 μV/cm $*$ L, we can deduce $J_{\mathrm{mc}}(B_{\mathrm{ext}}) = I_{\mathrm{mc}}(B_{\mathrm{ext}})/S$ supposing complete current penetration. Fig. 1 shows the curve related to this function for a BiSCCO tube at 77 K.

$J_{\mathrm{mc}}(B_{\mathrm{ext}})$ and $J_c(B)$ are different because:

1) $J_c(B)$ is a microscopic quantity and $J_{\mathrm{mc}}(B_{\mathrm{ext}})$ is a macroscopic quantity;
2) B is not equal to B_{ext} because of the self magnetic flux density B_{SF}

$$\overrightarrow{B} = \overrightarrow{B}_{\mathrm{SF}} + \overrightarrow{B}_{\mathrm{ext}} \qquad \text{and} \qquad B = \sqrt{B_{\mathrm{SF}}^2 + B_{\mathrm{ext}}^2}.$$

When the value of B_{ext} is sufficiently large, B_{SF} becomes negligible and the values of $J_{\mathrm{mc}}(B_{\mathrm{ext}})$ are very close to $J_c(B)$. When the value of B_{ext} is close to B_{SF}, $J_{\mathrm{mc}}(B_{\mathrm{ext}})$ can be very different from $J_c(B)$.

Manuscript received October 5, 2007; revised January 14, 2008, March 5, 2008, and March 8, 2008. First published June 27, 2008; current version published September 4, 2008. This paper was recommended by Associate Editor J. Willis.

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Digital Object Identifier 10.1109/TASC.2008.2000903

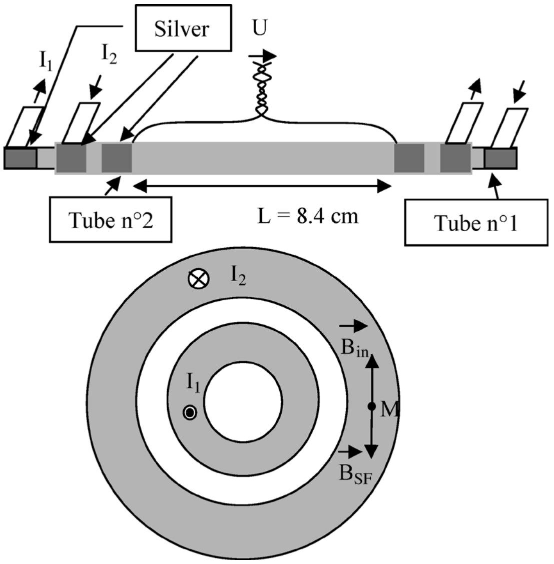

Fig. 2. Self magnetic field compensation setup and principle.

In this paper, a method of self field compensation in the electrical measurement method of $J_{\mathrm{mc}}(B_{\mathrm{ext}})$ for a superconducting tube is presented. This compensation allows one to make a link between $J_{\mathrm{mc}}(B_{\mathrm{ext}})$ and $J_c(B)$. The principle of compensation is valid as long as the material is isotropic.

Some authors have already studied the self field effect in superconducting tapes and its compensation [5] but not in superconducting tubes. First, the principle of our compensation method and measurements are presented. Then, a theoretical analysis is proposed.

II. PRINCIPLE OF SELF MAGNETIC FIELD COMPENSATION AND MEASUREMENTS

A. *Self Magnetic Field Compensation Principle*

Knowing that the self magnetic flux density in the tube is azimuthal, one has to create another azimuthal magnetic flux density $\overrightarrow{B}_{\mathrm{in}}$ opposed to $\overrightarrow{B}_{\mathrm{SF}}$ (Fig. 2). In this paper, we propose to generate $\overrightarrow{B}_{\mathrm{in}}$ using another superconducting tube inside the one studied (Fig. 2). We choose a superconducting tube to minimize the losses that would be generated by a copper tube. The internal tube is noted tube no. 1 and the studied one is noted tube no. 2 (Fig. 2). The currents passing through tube no. 1 and tube no. 2 are noted, respectively, I_1 and I_2. To compensate the self field effect, I_1 and I_2 have to be opposed (Fig. 2). Without an external magnetic field, the magnetic flux density in tube no. 2 is equal to $\overrightarrow{B_2} = \overrightarrow{B}_{\mathrm{SF}} + \overrightarrow{B}_{\mathrm{in}}$. So $B_2 = 0$ if $B_{\mathrm{in}} = B_{\mathrm{SF}}$. It is *a priori* impossible to cancel B_2 everywhere in the tube section but one can reduce the B_2 average

$$B_{2\mathrm{AV}} = \frac{\varphi_2}{S} = \frac{2\pi}{S} \iint_S \overrightarrow{B}_2 . \overrightarrow{\mathrm{ds}}.$$

The $B_{2\mathrm{AV}}$ reduction reduces the self field effect and increases the critical current for several magnetic fields as can be seen in the next sections. One question remains: what is the value of I_1 that minimizes the self field effect in tube no. 2?

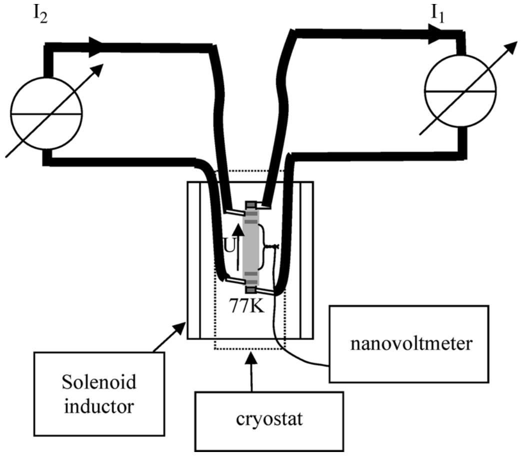

Fig. 3. Compensation experimental bench.

B. *Experimental Setup*

To perform the self field compensation, a special experimental setup (Fig. 3) was made.

The studied tube no. 2 consists of a hollow cylindrical current lead. It is a compacted composite of $(\mathrm{Bi,Pb})_2\mathrm{Sr}_2\mathrm{Ca}_2(\mathrm{Cu}_3\mathrm{O}_{10+x})$ with a critical temperature of 108 K (CAN Superconductors). The section S of the superconductor is about 30 mm^2 ($+/-10\%$). The internal and external diameters of tube no. 2 are 8 and 10.1 mm. The minimal critical current I_{c2} of tube no. 2, guaranteed by the manufacturer, is 80 A (at 77 K with the 1-μV criterion). So the minimal critical current density J_{c2} is about 2.7 A/mm^2. The critical current I_{c1} of tube no. 1 is higher than I_{c2} because we want to vary I_2 between 0 and above I_{c2}.

There are two independent current supplies, one for I_1 and another for I_2. A nanovoltmeter is used to measure the voltage U between the two taps of tube no. 2 (the external one) (Fig. 3).

C. *Measurement Results and Discussions*

To answer to the question at the end of Section II-A, six measurements are presented:

1) $U(I_2)$ without external magnetic flux density ($B_{\mathrm{ext}} = 0$) and without compensation ($I_1 = 0$) (Fig. 4);
2) $U(I_1)$ for several I_2 with $B_{\mathrm{ext}} = 0$ to determine the current $I_{1\,\mathrm{min}}$ that minimizes U (Fig. 5);
3) $U(I_2)$ with $B_{\mathrm{ext}} = 0$ and with compensation ($I_1 = I_{1\,\mathrm{min}}$) (Fig. 4);
4) $U(I_2)$ with $B_{\mathrm{ext}} > 0$ and without compensation ($I_1 = 0$) (Fig. 6);
5) $U(I_1)$ for several I_2 with $B_{\mathrm{ext}} > 0$ to determine the current $I_{1\,\mathrm{min}}$ that minimizes U (Figs. 7 and 8);
6) $U(I_2)$ with $B_{\mathrm{ext}} > 0$ and compensation ($I_1 = I_{1\,\mathrm{min}}(B_{\mathrm{ext}})$) (Fig. 9).

From the first curve $U(I_2)$ (Fig. 4), the critical current without external magnetic field I_{c20} of tube no. 2 is deduced: $I_{c20} = 84.5$ A. In a second step, several high values of I_2 (80, 82, 85, and 87 A) are chosen around I_{c20} to have sufficient voltages for measurement. Then, for each I_2 value, a $U(I_1)$ curve is determined (Fig. 5). One observes that the minimum of $U(I_1)$ is when I_1 is about $I_2/2$. The determination of the exact

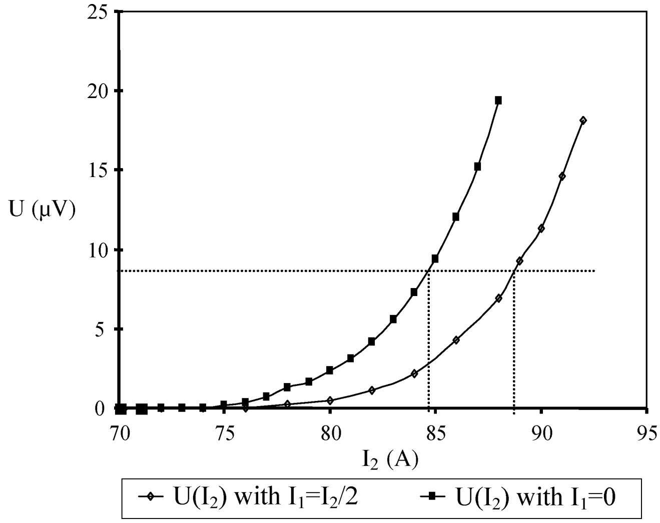

Fig. 4. Measured $U(I_2)$ curve of tube no. 2 with and without compensation.

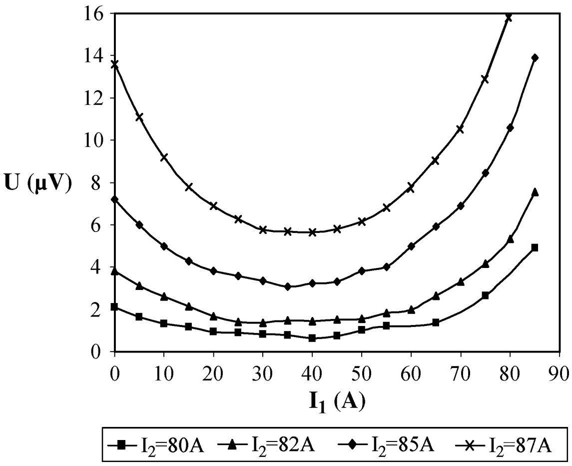

Fig. 5. Measured $U(I_1)$ curves for several I_2 values.

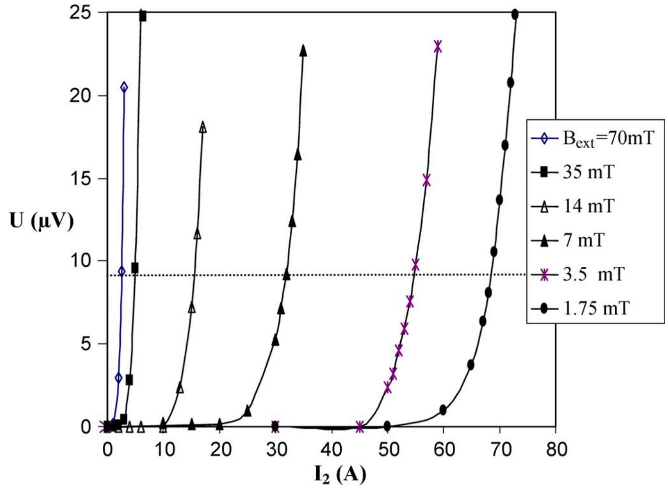

Fig. 6. Measure of sample voltage drop versus direct current for different external magnetic flux densities.

value of $I_{1\,\mathrm{min}}$ is difficult because the curve $U(I_1)$ is flat around $I_{1\,\mathrm{min}}$ so we decided to perform the self field compensation with $I_{1\,\mathrm{min}} = I_2/2$. From the curve $U(I_2)$ with $I_1 = I_{1\,\mathrm{min}}$

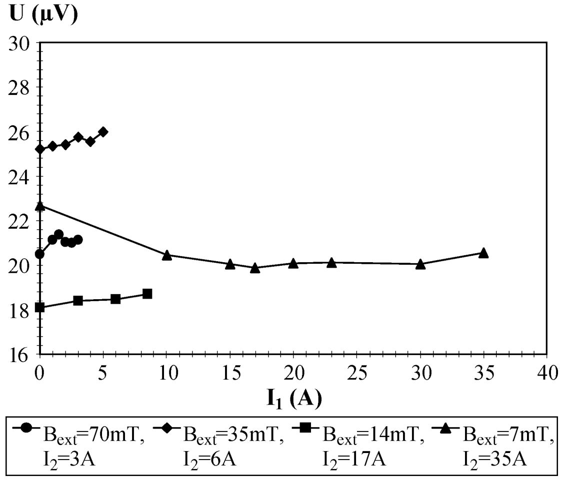

Fig. 7. Measured $U(I_1)$ curves for high different values of B_{ext} and one value of I_2 each.

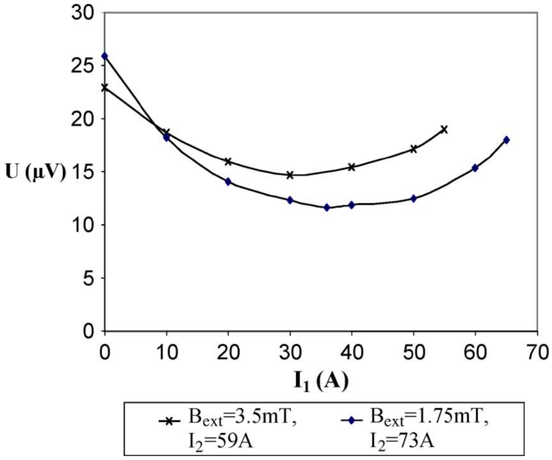

Fig. 8. Measured $U(I_1)$ curves for low different values of B_{ext} and one value of I_2 each.

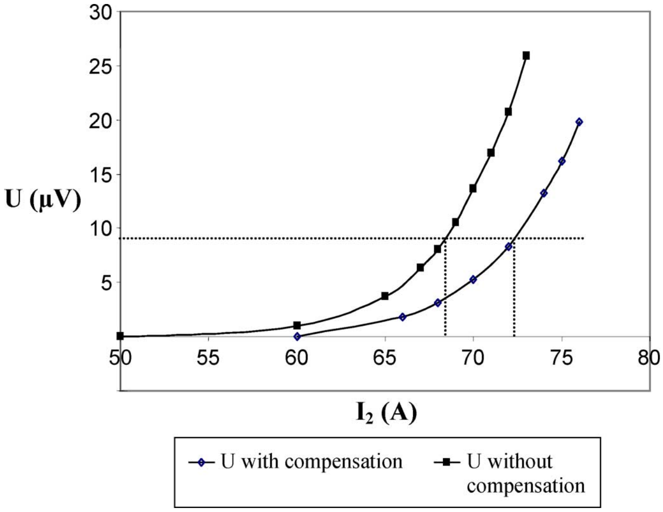

Fig. 9. Measured $U(I_2)$ curves of tube no. 2 with and without compensation for $B_{\mathrm{ext}} = 1.75$ mT.

(Fig. 4), the compensated critical current without external magnetic field $I_{\mathrm{cc}20}$ of tube no. 2 is deduced: $I_{\mathrm{cc}20} = 88.5$ A.

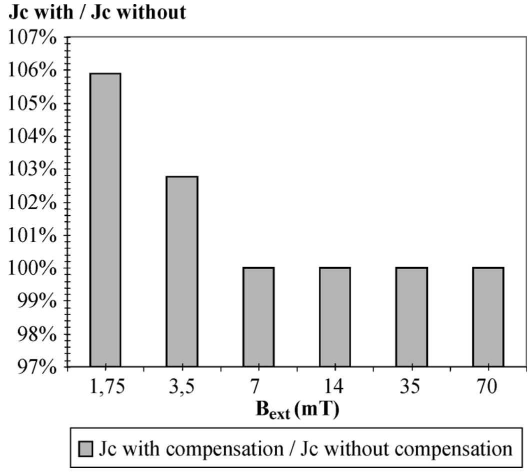

Fig. 10. Measured J_{mc} with and without self field compensation versus B_{ext}.

Thanks to this partial self field effect compensation, the critical current was increased by almost 5%. In a fourth step, the sample voltage drop $U(I_2)$ is measured without compensation ($I_1 = 0$) for different external magnetic flux densities B_{ext} (Fig. 6) from a higher value than self field (70 mT) to a lower value (1.75 mT; the maximal self magnetic flux density of tube no. 2 is about 4 mT for $I_2 = 100$ A). The curve $J_{\mathrm{mc}}(B_{\mathrm{ext}})$ without compensation is deduced (Fig. 1). For each value of B_{ext}, we choose only one value of I_2 for one $U(I_1)$ curve to have significant voltage (15 μV $< U <$ 25 μV). There are two kinds of $U(I_1)$ curves. If B_{ext} is significantly higher than the self magnetic flux density (7 to 70 mT, Fig. 7), the curve is flat and the compensation is not valid. For example, for $B_{\mathrm{ext}} = 7$ mT and $I_2 = 35$ A, the self magnetic flux density is about 1.4 mT and $B = \sqrt{7^2 + 1.4^2} = 7.1 \approx B_{\mathrm{ext}}$. If B_{ext} is close to the self magnetic flux density (3.5 and 1.75 mT, Fig. 8), one still observes that the minimum of $U(I_1)$ appears when I_1 is about $I_2/2$. For example, for $B_{\mathrm{ext}} = 3.5$ mT and $I_2 = 59$ A, the self magnetic flux density is about 2.4 mT and $B = \sqrt{3.5^2 + 2.4^2} = 4.3 \gg B_{\mathrm{ext}}$. In the sixth step, the curve $U(I_2)$ (Fig. 9) with $I_{1\ \mathrm{min}}$ and for $B_{\mathrm{ext}} = 1.75$ mT is presented. The compensated critical current $I_{\mathrm{cc}2}$ with the external magnetic field of tube no. 2 is deduced: $I_{\mathrm{cc}2}(B_{\mathrm{ext}} = 1.75\ \mathrm{mT}) = 72$ A. For $B_{\mathrm{ext}} = 3.5$ mT, $I_{\mathrm{cc}2}(B_{\mathrm{ext}} = 3.5\ \mathrm{mT}) = 56$ A. Finally, $J_{\mathrm{mc}}(B_{\mathrm{ext}})$ curves with and without compensation are plotted (Fig. 10). In the low field regime, the superposition of external magnetic field with the self magnetic field has an influence on J_{mc}, and the compensation permit to increase J_{mc}. Improvements made to J_{mc} are weak but this experiment validates the compensation principle. In the high magnetic field regime, the self magnetic field has no influence on J_{mc}, and one can say that $J_c(B) = J_{\mathrm{mc}}(B_{\mathrm{ext}})$.

III. THEORETICAL ANALYSIS

In this section, a theoretical analysis of the self field compensation is presented. We want to find the value of $I_{1\ \mathrm{min}}$ that maximizes the critical current I_{c2} and so the critical current density J_{c2} for $B_{\mathrm{ext}} = 0$. We choose $B_{\mathrm{ext}} = 0$ because it is the best compensation case and it is the simplest calculation case. Indeed in this case, the current density is along the tube axis. On the other hand, if $B_{\mathrm{ext}} \neq 0$, the current density adopts a helical path around the tube.

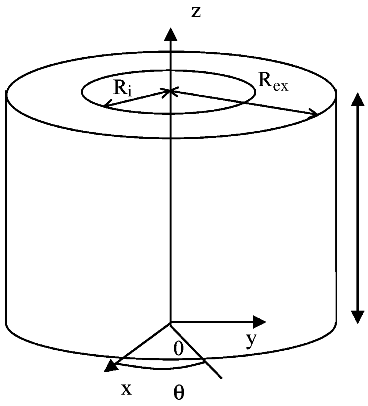

Fig. 11. Reference frame and dimensions of the superconducting tube.

The case of a superconducting tube fed by a dc current I_2 and exposed to an internal magnetic flux density B_{in} is studied. The dimensions of the tube (Fig. 11) are internal radius R_{in}, external radius R_{ex}, section S, and length L. The z-axis is along the axis of the tube. The relation between the magnetic flux density and the current density is governed by the Maxwell equation

$$\overrightarrow{\mathrm{rot}}\vec{B} = \mu_0.\vec{J}. \tag{1}$$

By symmetry, the current density is oriented along the $[Oz]$ axis. In addition $B_{\mathrm{in}}(r)$, $B_{\mathrm{SF}}(r)$, and the current density in tube no. 2 $J_2(r)$ have only one component

$$\overrightarrow{B}_{\mathrm{SF}}(r) = B_{\mathrm{SF}}(r).\vec{u}_\theta, \overrightarrow{B_{\mathrm{in}}}(r) = -B_{\mathrm{in}}(r).\vec{u}_\theta, \overrightarrow{J_2}(r) = J_2(r).\vec{u}_z.$$

The magnetic flux density in the superconducting material is the sum of the different magnetic flux densities

$$\overrightarrow{B}(r,t) = \left(B_{\mathrm{SF}}(r) - B_{\mathrm{in}}\right).\vec{u}_\theta.$$

The analytical calculation of the magnetic flux density and current density distributions in the superconducting material is difficult [3] because (1) with $J_2(r) = J_{c2}(B)$ has to be solved. One can simplify calculation of these distributions with three hypotheses.

1) To calculate $B_{\mathrm{SF}}(r)$, J_{c2} is taken as a constant (Bean's model) and so

$$\frac{\mathrm{dB_{SF}}(r)}{\mathrm{dr}} = J_{c2}. \tag{2}$$

2) To calculate $J_2(r,t) = J_{c2}(B)$, a linear relation is chosen $J_{c2}(B) = J_{c0}(1 - (|B(r,t)|/B_{j0}))$. J_{c0} and B_{j0} are constants and depend on the superconducting material.
3) The critical current $I_{c2} = \int_S J_{c2}.ds$ and the full penetration current I_{p2} [3] are equal.

With these hypotheses one can calculate I_{c2} as follows:

$$I_{c2} = \iint\limits_S \vec{J}.d\vec{s} = \iint\limits_S J_c ds = \iint\limits_S J_{c0}\left(1 - \frac{|B(r,t)|}{B_{j0}}\right) ds$$
$$= J_{c0}.S - \frac{J_{c0}}{B_{j0}}\left|\iint\limits_S B(r,t) ds\right| = J_{c0}.S - \frac{J_{c0}S}{B_{j0}}|B_{\mathrm{AV}}|$$

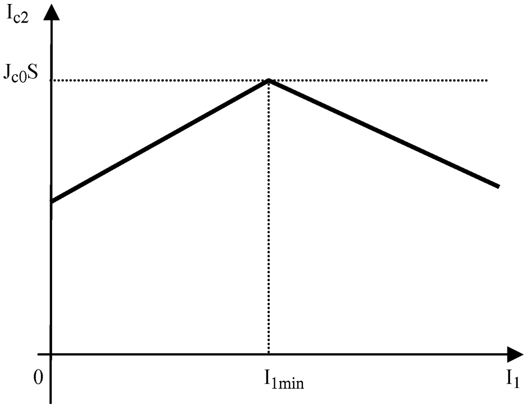

Fig. 12. Critical current I_{c2} of the studied superconducting tube versus I_1.

with $B_{\rm AV} = (\varphi_2/S) = (2\pi/S)\int_{R_{\rm in}}^{R_{\rm ex}} B(r).r.dr$ and $B(r) = B_{\rm SF}(r) - B_{\rm in}(r)$.

So maximizing I_{c2} is equivalent to minimizing $|B_{\rm AV}|$.

Equation (2) and $B_{\rm SF}(R_{\rm in}) = 0$ allows one to calculate $B_{\rm SF}(r)$

$$B_{\rm SF}(r) = \mu_0.J_{c2}.(r - R_{\rm in}).$$

The internal tube that creates $B_{\rm in}$ is viewed as an infinite long wire and so

$$\begin{aligned}
B_{\rm in}(r) &= \frac{\mu_0 I_1}{2\pi r}\\
B_{\rm AV} &= \frac{2\pi}{S}\int\limits_{R_{\rm in}}^{R_{\rm ex}} \left[\mu_0 J_{c2}(r - R_{\rm in}) - \frac{\mu_0 I_1}{2\pi r}\right].r.\mathrm{dr}\\
&= \frac{2\pi}{S}\left[\mu_0 J_{c2}\left(\frac{R_{\rm ex}^3}{3} + \frac{R_{\rm in}^3}{6} - \frac{R_{\rm in}R_{\rm ex}^2}{2}\right)\right.\\
&\qquad\left. - \frac{\mu_0 I_1}{2\pi}(R_{\rm ex} - R_{\rm in})\right].
\end{aligned}$$

The relation $I_{c2}(I_1)$ is represented in Fig. 12 and we can easily obtain $I_{1\,\rm min}$ from $B_{\rm AV} = 0$

$$I_{1\,\rm min} = \frac{2\pi J_{c2}}{R_{\rm ex} - R_{\rm in}}\left(\frac{R_{\rm ex}^3}{3} + \frac{R_{\rm in}^3}{6} - \frac{R_{\rm in}R_{\rm ex}^2}{2}\right).$$

We can apply this formula to our superconducting tube. The critical current without external magnetic field is $I_{c2} = 84.5$ A and $J_{c2} = 84.5/30.10^{-6} = 2.8.10^6$ A/mm^2. One thus finds $I_{1\,\rm min} = 45.2$ A. This value is close to $I_{c2}/2$ as it was shown in the experimental part. Despite the strong hypothesizes, the $I_{1\,\rm min}$ calculation validates the experimental results.

IV. CONCLUSION

This paper presented the self field effect compensation in a superconducting tube for the measurements of the $J_c(B)$ curve. A special experimental arrangement allows results in an increase in the measured critical current when the external magnetic field is close to the self field. It consists of placing another superconducting tube inside the studied one. This second tube creates a magnetic field opposed to the self field to reduce the self field effect. Several presented experiments show that the compensation is real and allow determining the $J_c(B)$ curve with compensation. This curve is slightly higher than the curve without compensation. A theoretical analysis based on the Bean model and some strong hypotheses are presented. For compensating the self field effects, experimental and theoretical results show that the internal tube current must be equal to one half of the studied tube current. The proposed compensation is not perfect, but the experiments and the theoretical analysis allow validating the compensation principle. Finally, this work opens the door to a further work: what new arrangement would allow for a better compensation of the self field inside the superconducting tube? This is an inverse problem that consists of finding the external magnetic field configuration which creates an azimuthal magnetic field perfectly opposed to the self field of the studied tube.

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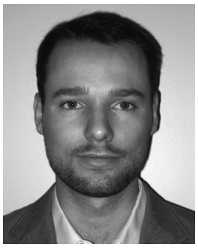

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