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# The Redistributive Aspects of ELIE: a simulation approach

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**Abstract**

Ce papier analyse les problèmes liés à l'implémentation du schéma Equal Labour Income Equalisation (ELIE) proposé par Kolm (2005). Il étudie successivement l'influence de l'incertitude dans la connaisance des revenus indivduels, l'impact des échelles d'équivalence et finalement les conséquences de l'accumulation du capital. Si l'incertitude ne modifie pas fondamentalement les propriétés d'équité de ELIE, les échelles d'équivalences peuvent avoir des cosnéquences non triviales en fonction de la relation entre fertilité et revenu. Enfin, l'accumulation du capital introduit de fortes inégalités dans la taxation. Le papier repose sur des simulations calibrées de la distribution des revenus en France et sur l'utilisation des divers indices de taxation.

**Abstract**

This paper analyses the problems linked to the implementation of the Equal Labour Income Equalisation (ELIE) scheme proposed by Kolm (2005). It successively studies the influence of uncertainty in the knowledge of individual incomes, the impact of equivalence scales and finally the consequences of capital accumulation. If uncertainty does not modify fundamentally the equity properties of ELIE, equivalence scales can have non trivial consequences depending on the relation between income and fertility. Finally, capital accumulation introduces strong inequalities in taxation. The paper relies on simulations of the income distribution, calibrated on French data and on the use of taxation indices.

JEL Classification : C15, D63, D78, H21.

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## 1 Introduction

Macrojustice, as discussed in Kolm (2005) concerns the rules for distributing the benefits of the main social resources. Because initial dotations are unequally distributed including human capacities, macrojustice implies some kind of redistribution which depends on the socially accepted degree of solidarity or altruism. Kolm (2005) considers that the main resources are labour capacities so that redistribution should imply an equal labour income equalisation or ELIE. "*Practically, each individual receives or yields in proportion to the difference between her wage rate and the average, the proportion being the equalisation labour. Hence, a difference in productivity is compensated by a proportional difference in income transfer.*" As capital was initially produced by labour, the real means of production at the steady state are labour and natural resources. The contribution of labour to total resources is accounted for 97%. Consequently, Kolm (2005) assumes that the question of taxation can be centred on labour only: *Labour is the main source of income in society.*

This being said, let us turn to usual public finance theory in order to situate the possible impact of ELIE. Public finance has different aims and functions that can be summarised as follows.

- A redistributive function which is based essentially on progressive income tax and various policies of assistance such as the RMI (minimum income for insertion) in France or allowances for old people. Obviously an implementation of ELIE should replace these existing taxes and transfers.

- An incentive function. For instance, a carbon tax aiming at reducing $CO_2$ emissions, or family allowances that aim at promoting a greater number of children per family. ELIE cannot be a full substitute for these taxes and transfers.

- The production of public goods. This aspect of public finance aims at organising and improving economic growth and capital accumulation. Here again, we are outside the role and attributions of ELIE.

These functions cannot of course be totaly independent one from another. But we would like to be able to separate as much as possible these different effects. We would not like for instance that a redistributive policy might generate disincentives or that an incentive policy could have regressive distributional features. This is the reason why ELIE is based on capacities and not on effective labour supply. As its basis for taxation is inelastic, there should be no inefficiency disincentives for labour supply with ELIE.

Kolm (2005) assumes that the distribution of talents is perfectly known. We shall detail a number of reasons why this might not be the case. It is thus reasonable to investigate the sensitivity of the ELIE scheme to an uncertainty in the knowledge of the distribution of talents. Are the basic properties greatly modified or is uncertainty only a minor problem? This question is essential for judging the redistributive performances of ELIE. In other words, from a purely redistributive point of view, is it worth changing the existing system for ELIE?

Modern societies and particular France are concerned with the behaviour of households as regards to fertility decisions. Family allowances have been implemented as an incentive to fertility. They have been quite successful in France for maintaining a decent rate of birth, compared to other European countries. But family allowances, on the other side, might have some unwanted redistributive effects. So what is the exact trade-off between family incentives and redistribution? Because of their incentive motives, they cannot be a substitute to ELIE, but have to be a complement. How can the two systems cohabitate? How will family allowances modify the redistributive properties of ELIE? This is also an important question.

Kolm (2005) does not consider capital income by arguing that physical capital is itself produced by labour, so that for macrojustice (not for microjustice) capital can be neglected. This can be true in a steady state, but it ignores the question of accumulation and economic growth and so does not give any hint on how to investigate the trade-off between growth and redistribution. On one hand, individual saving rates rise with the level of income. Any redistributive scheme, by transferring resources from rich to poor tends to lower the aggregate rate of saving and by consequence the rate of capital accumulation. On the other hand, capital income modifies the total income distribution and has a strong influence on inequality. In most countries and certainly in France, wealth inequality is much stronger than income inequality.

To answer the above questions, we shall have to formalise the ELIE scheme in various contexts. The basic ELIE formula is simple and linear. But this attractive simplicity hides in fact many non trivial properties. For instance, the degree of redistribution depends on the characteristics and shape of the initial gross income distribution. When mixed with other public finance mechanisms, the initial model is also much complexified. For these two reasons (shape of the distribution, and more complex models), we have to resort to simulation techniques. We have clearly two ways of proceeding. We could start from a microeconomic sample of French wages and simulate from this empirical distribution. The alternative solution is to simulate a given density and calibrate its parameters so as to match some inequality or redistributive indices computed from the empirical distribution of gross or net incomes in France. de la Croix and Lubrano (2008) adopt the same type of calibrated simulation as well as many authors do in the literature. This approach is much simpler to implement because it is easier to get the figures for some indexes than the figures of individual incomes collected from household surveys. Moreover, calibrated simulation is more adapted when one want to study variants of the original model, because in this case, the income distribution can be simulated in different way across the variants of the initial model.

A fiscal system has been characterised in the econometric literature by a number of indices that we recall in the appendix. They are, apart from the well-known Gini index, the progressivity index of Kakwani (1977), the vertical equity index of Reynolds and Smolensky (1977) and the horizontal inequity index of Atkinson (1980) and Plotnick (1981). We detail these indices in an appendix, because it is rather difficult to find a comprehensive presentation of the related literature which might not be well known to all the readers of this book.

The paper is organised as follows. In section 2, we give a mathematical presentation of the ELIE scheme. In section 3, we simulate this model and verify its redistributive properties. We show that a small value of the fundamental redistributive parameter is needed in order to obtain the same degree of redistribution that exists in France. With section 4, we enter the core of the debate, examining the consequences of uncertainty in the knowledge of the distribution of talents. In section 5, we introduce family allowances while section 6 details a growth model à la Solow Solow (1956) to investigate the relation between capital accumulation and income distribution. Section 7 concludes.

## 2 The initial formulation of the model

The economy we consider is composed of $n$ individuals with a labour supply $\ell_i$ and a potential wage rate of $w_i$ corresponding to individual capacities or productivities. Taxation bears on $w_i$, independently of $\ell_i$, the effective quantity of supplied labour. That, with the assumption that *there is a total freedom for the amount of labour supplied*, means that there is no interaction between labour supply and taxation. Consequently, the question of optimal taxation can be treated in a first best framework provided that *wage rates or productivities are perfectly known*. Kolm imagines a self-financing distributive system where taxes and subsidies, both noted $t_i$, balance with $\sum t_i = 0$. As labour is the main resource, taxes and subsidies are measured in terms of a quantity of labour. An equal quantity of labour $k$ is taken from each individual and measured at his productivity so the tax is equal to $kw_i$. Meantime, an equal amount $k\tilde{w}$, to be determined, is redistributed to each individual so that the net transfer for individual $i$ is equal to

$$t_i = k(\tilde{w} - w_i). \tag{1}$$

The value of $k$ has to be chosen by society while $\tilde{w}$ is determined by the assumption that *transfers are totaly financed by taxes.* As a matter of fact, the system must verify

$$\sum t_i = 0 \Rightarrow k(n\tilde{w} - \sum w_i) = 0. \tag{2}$$

This equation has only two solutions. Either $k = 0$ or $\tilde{w} = \sum w_i/n = \bar{w}$. Thus $\tilde{w}$ is equal to the mean wage $\bar{w}$. It plays a central role in the system and we shall call it here the *pivot*.

**Remark : 1** *$\bar{w}$ determines the position of richer and poorer individuals. In the EU and in the UK as well, any individual with an income below a fraction of the mean income in society is considered as poor. In France, the same definition uses instead the median. When the income distribution is not symmetric, this makes a difference. So the choice of $\bar{w}$ as a pivot is not neutral.*

When accounting for taxes and transfers, disposable income or net total income is given by

$$y_i = w_i\ell_i + k(\bar{w} - w_i) = k\bar{w} + w_i(\ell_i - k). \tag{3}$$

In this framework, taxation is based on capacities and not on actual income. So that an individual determines his labour supply independently of $k$. But he is implicitly constrained to work at least for $k$ days. Everybody receives the same basic income $k\bar{w}$ plus a fraction $\ell_i - k$ of his labour productivity. This is the **E**qual **L**abour **I**ncome **E**qualisation or ELIE. ELIE is thus a kind of universal basic income.[1] Everybody receives $k\bar{w}$. At the difference of a usual basic income, the financing mechanism is already contained in the definition of ELIE as everybody also pays $kw_i$ for financing the system.

## 3 Characterising the initial model

We start from an hypothetical population of $n = 10\ 000$ individuals. For the time being, we restrict our attention to individuals, leaving the question of household composition to section

[1] The interpretation of ELIE as a basic income mechanism is uncontroversial when everybody is working. It can lead to controversies when there are inactive individuals.

5. We suppose also that wages are for the while the sole income. These two hypothesis characterise the initial simple model of Kolm. We are here interested in exploring the consequences of wage heterogeneity on redistribution, or in other words to show that ELIE properties heavily depends on the shape of the gross income distribution. We shall thus simulate a calibrated distribution of gross income and analyse the resulting distribution of net income (after taxes and transfers) when $k$ varies. In the original model of Kolm, labour supply is exogenous. So for the ease of simulation, we shall simply suppose that $\ell_i = 1$. Consequently, the net total disposable income (3) is transformed into

$$y_i = w_i + k(\bar{w} - w_i) \tag{4}$$

This assumption is a non trivial one. In the original model, $w_i$ is the hourly wage rate which has to be multiplied by $\ell_i$ or order to get the gross income $\ell_i w_i$. Assuming $\ell_i$ exogenous and equal to 1 means that $w_i$ now represents gross labour income. In a way the first best framework is lost, in another way is it kept because $\ell_i$ is supposed to be exogenous. The model also supposes that $w_i$ is perfectly known. We shall relax this last assumption in section 3.

We suppose that gross wages are distributed according to a Gamma distribution:

$$f(w|\nu, s) = \frac{1}{\Gamma(\nu)} s^\nu w^{\nu-1} e^{-w\,s}.$$

We recall that the mean of this distribution is equal to $\nu/s$ and its variance to $\nu/s^2$. We then impose the restriction $s = \nu$ in order to get a normalised mean of 1. [2]. We have then to calibrate this distribution by adjusting its remaining parameter $\nu$. For a normalised mean, the empirical distribution of gross wages can be characterised in a simple way by its Gini coefficient. This coefficient was equal to 0.327 in France in 1998. [3] To obtain the same Gini coefficient for our simulated sample, we had to choose $\nu = 2.75$. Note however that these figures are in a way not exactly comparable because here we have individuals and earnings for the while when real data are concerned with households and total incomes. Note also that with the Gamma distribution, very high incomes are not so well represented. In our sample, the maximum income is 6.27 times the mean. We could have chosen another usual distribution such as the logNormal or the Pareto without changing much the results. We choose the Gamma for the sake of algebraic simplicity. All moments exist, contrary to the Pareto.[4]

Let us now apply the ELIE scheme to this sample, taking three different values for $k$: 0.2, 0.3, 0.4, as suggested in Kolm (2005) so as to obtain the distribution of net incomes (after taxes and transfers) as given by (4). A coefficient $k = 0.2$ corresponds to a tax rate of one

[2]The annual mean disposable net income in France was 28 935 euros in 2004 and the median 24 599 euros. As we have chosen a normalized mean, the horizontal axis can be read as x times the mean. It is then easy to interpret the right tail of the distribution. Actual income figures can be obtained just by multiplying the figures of the horizontal axis by the empirical mean given in this footnote.

[3]Some of these data can be found for instance on the Web site of the World Institute for Development and Economic research, UNU-WIDER World Income Inequality Database, Version 2.0a, June 2005. http://www.wider.unu.edu/wiid/wiid.htm .

[4]A real alternative would have been to choose a richer distribution with much more parameters in order to get a better tail behaviour. We can quote the Generalised Beta II distribution which has four parameters. However, the calibration of the parameters is far from trivial. We should have used at least the deciles of the income distribution for this purpose and not just a simple Gini coefficient. The gain for the subsequent analysis is not clear.

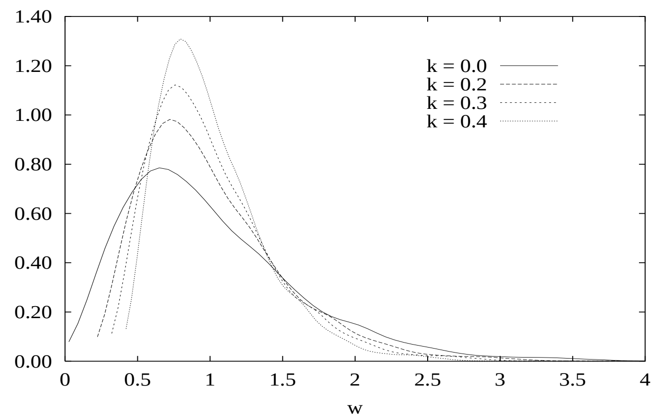


Figure 1: Income distribution before and after ELIE transfers
*Multiply by 28 935 euros (the annual mean disposable net income in France in 2004) to have an idea of the range of net incomes in France.*

working day in a five working day week. And $k = 0.4$ corresponds to two working days. Figure 1 displays a non-parametric estimate of the different income distributions obtained when varying $k$. We immediately notice three facts: *firstly*, ELIE introduces a concentration of net incomes around the mean; *secondly* it raises the minimum disposable income as $k$ grows; *thirdly*, if poverty is defined an income lower than half the mean, a $k$ greater than 0.4 is needed to totaly eliminate it (See Leroux and Leroux (2009) as an illustration).

The decrease of the Gini coefficients given in Table 1 shows that ELIE strongly reduces inequality. But the chosen values for $k$ entail Gini coefficients which are much lower than that computed on French data for net incomes and which is equal to 0.27 in 2000 and 2001. We should have chosen $k = 0.17$ to recover such a value for the Gini coefficient, which would corresponds to slightly less of one working day. This is a relatively low value when compared to the importance of social contributions and payroll taxes in France. However, we are studying here only redistribution and redistribution, when properly measured, is rather weak in France. For instance, income taxes are only a small part of GDP in France. In this simple model, a rather low value for $k$ is finally realistic. When examining various variants of the basic model, we shall see that the French Gini coefficient for net incomes is reproduced for increasing values of $k$ as we depart from the original model.

It is rather easy to verify that ELIE complies with the Pigou-Dalton requirement, which means that it does not change the mean of the distribution after taxes and transfers and that it entails Lorenz curves that get closer to the main diagonal without crossing. A a consequence, all concentration curves, computed on the sole basis of positive taxes are identical.[5]

Let us now detail various indices which characterise taxation, leaving aside transfers. Taxes, noted $tt_i$, concern only individuals who are above the mean:

$$tt_i = \begin{cases} 0 & \text{if } w_i \leq \bar{w} \\ k(w_i - \bar{w}) & \text{otherwise.} \end{cases}$$

[5]For instance, as $w_i \sim G(\nu, \nu)$, then by properties of the Gamma distribution, $y_i = (1-k)w_i + k\bar{w}$ is also Gamma with $y_i \sim G(\nu, \nu/(1-k)) + k$. Its mean is equal to 1 and its variance to $(1-k)^2/\nu$. So ELIE does not change the mean but reduces the variance.

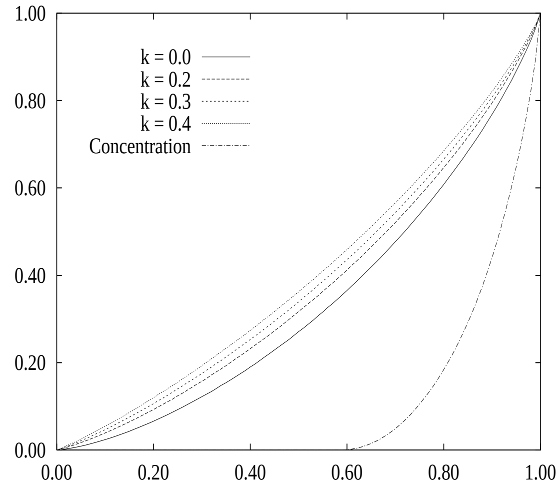


Figure 2: Lorenz and concentration curves associated with ELIE

Those below the mean receive only transfers. The marginal tax rate is constant and equal to $k$ for those paying taxes. However, despite this linear structure, ELIE is a progressive taxation scheme. The mean taxation rate, obtained by dividing taxes by gross wages, is

$$tx_i = tt_i/w_i = k(1 - \frac{\bar{w}}{w_i}). \tag{5}$$

This rate grows with $w_i$, but less than proportionally as illustrated on Figure 3. The shape

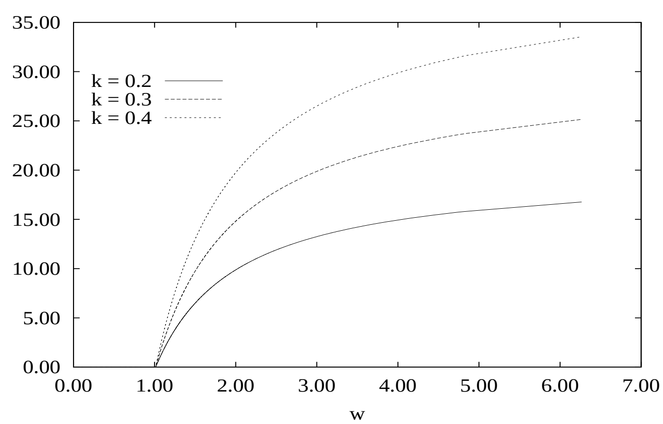


Figure 3: Taxation rates implied by ELIE (in percentage)

of this graph, which illustrates the bearing of taxation on our particular population, depends heavily on the shape of the distribution of income. On the contrary, the marginal rate of taxation is constant and equal to $k$ for individuals above the mean, and of course equal to zero for those below the mean.

The usual index of progressivity of Kakwani (1977) is strong with 0.45 compared to its value computed on actual French data which is only 0.31. It is not a function of $k$ because of the linear structure of ELIE (constant marginal rate). It is only a function of the shape of the distribution and of the chosen *pivot* for ELIE.[6] A smaller index could be obtained with a lower $\nu$ entailing stronger asymmetry and stronger initial inequality. The average fiscal

Table 1: The original ELIE model

| $k$ | Mean wages | Gini | Fiscal pressure | Partial fisc. pres. | Kakwani | Reynolds | Atkinson | Redis-tribution |
|---|---|---|---|---|---|---|---|---|
| 0 | 1.013 | 0.327 | 0.00 | 0.00 | 0.000 | 0.000 | 0.00 | 0.000 |
| 0.2 | 1.013 | 0.262 | 4.70 | 7.23 | 0.454 | 0.065 | 0.00 | 0.065 |
| 0.3 | 1.013 | 0.229 | 7.05 | 10.84 | 0.454 | 0.098 | 0.00 | 0.098 |
| 0.4 | 1.013 | 0.196 | 9.39 | 14.45 | 0.454 | 0.131 | 0.00 | 0.131 |

pressure, computed as the ratio between the sum of total taxes and the sum of gross incomes, is very low, because only incomes which are above the mean are taxed. But even when the fiscal pressure is computed for only those actual paying taxes, it is not very high. The amount of inequality removed as indicated by the Reynolds index as well as the redistribution index grows with $k$.

Horizontal equity, as defined in the appendix is perfect, taxes and transfers imply no rank permutation. The Atkinson-Plotnick horizontal inequity index is always zero. This is due to the fact that there is no effect due to family composition or to the combination of different taxes.

We can conclude that the pure ELIE transfer scheme is a rather powerful mechanism of redistribution. Its implementation in France would require a rather low value for $k$ to reproduce the same Gini coefficient for net income. However, ELIE is much more progressive. That would mean putting the burden of redistribution more on richer people than what it is now.

## 4 Uncertainty in the knowledge of wages

Wages and income are not in general perfectly known by the tax authority. The basic wage rate can be public, but there are bonuses and extra hours for instance that can be more difficult to know for the tax authority. These can be important for some categories. We have then tax evasion and tax avoidance. The former is illegal but some professions are famous for practicing it. The later is perfectly legal and results from an optimisation behaviour as described for instance in Stiglitz (1985). Of course in this game of tax evasion and tax avoidance, richer individuals have more opportunities than poorer ones.

We are now going to modify the initial model so as to take into account this aspect which is important for practical implementation. We have to know how much ELIE looses of its nice redistributive properties when it becomes difficult to have a precise knowledge of the individual talents. We now modify the taxation scheme so that $w_i$ would be lowered by a

[6] The pivot is chosen here equal to the mean because the system is balanced in this case. Choosing for instance the median would have led to an unbalanced system. The pivot can be chosen lower than the mean if extra resources are to be collected for paying for instance civil servants and collective equipments. But in this case the original ELIE is distorted.

random effect $e_i$:

$$t_i = k(\bar{w} - w_i - e_i). \tag{6}$$

We want to implement the idea that uncertainty in the knowledge of wages increases with its level and that the importance of dissimulation is random. For that purpose, $e_i$ is a strictly positive process and once again we chose the Gamma distribution for convenience. Its mean will depend on the level of $w_i$ and on its rank in the distribution. The maximum rate of tax avoidance (expressed as a fraction of gross income) is supposed to be 30%. More precisely, if the $w_i$ are sorted in increasing order, we shall have

$$e_i \sim 0.3 \frac{i}{n} w_i \mathrm{G}(\nu, \nu), \tag{7}$$

where $\mathrm{G}(\nu, \nu)$ means the Gamma distribution. This formula implies that $e_i$ is positive with a mean equal to $0.3 * w_i * i/n$ and a variance equal to $(0.3 * w_i * i/n)^2/\nu$. The richer people will on average manage to hide 30% of their income while the poorer people will manage to hide a negligible part of it. For the system to be balanced, the pivot has to be taken equal to the mean of the declared wages $w_i - e_i$ and no longer equal the mean of the exact wages $w_i$. Introducing a random dissimulation entails little changes when compared to the initial Table 1. Redistribution is of course weaker as well as the fiscal pressure on richer people. A weak horizontal inequity appears due to the random character of the modeled tax avoidance. In order to get a Gini of 0.27, we must chose a $k = 0.24$, a value slightly higher than in the original case where a value of $k = 0.17$ was sufficient.

Table 2: ELIE and uncertainty in the knowledge of wages

| $k$ | Mean wages | Gini | Fiscal pressure | Kakwani | Reynolds | Atkinson | Redistribution |
|---|---|---|---|---|---|---|---|
| 0.0 | 1.013 | 0.327 | 0.00 | 0.000 | 0.000 | 0.0000 | 0.000 |
| 0.2 | 1.013 | 0.282 | 3.16 | 0.437 | 0.044 | 0.0000 | 0.044 |
| 0.3 | 1.013 | 0.261 | 4.73 | 0.437 | 0.066 | 0.0002 | 0.066 |
| 0.4 | 1.013 | 0.239 | 6.31 | 0.437 | 0.089 | 0.0003 | 0.088 |

Despite the fact that we have introduced a rather strong possibility of tax avoidance and evasion, the final influence is weak. We just have to increase $k$ in order to get the same amount of inequality. Redistribution is simply slightly less important. Taxing capacities instead of incomes seems a crucial hypothesis because incomes are rather easy to measure whereas capacities are not. The results of this section show that finally this hypothesis is not so crucial because when relaxing it, the main properties of the model are not so much altered. We must however keep in mind that that we have supposed that labour supply is exogenous and thus does not depend on the value of $k$.

## 5 Family allowances and family composition

ELIE is a fair and powerful redistributive scheme with nice labour incentives properties. However, incentives are not limited to the labour market, as underlined in the introduction. In this section, we want to investigate how the pure redistributive ELIE mechanism can interact with the incentive mechanism of family allowances.

Family allowances exists in many European countries and are very important in France. They have been successfully designed to increase natality. With family allowances, we have a mechanism which mixes redistribution and incentives. Clearly, ELIE cannot be a substitute for family allowances, simply because it is not a targeted mechanism. If we want to experiment the feasibility of implementing ELIE in France, we have to study the properties of a mix system combining ELIE with a complementary mechanism of redistribution based on family composition and not solely on the position of gross income versus mean income.[7]

## 5.1 ELIE and household composition

The basic unit $i$ is no longer an individual but a household. We suppose that a household is composed of two adults earning the same amount each and a random number of children noted $ch_i$ that can be zero.[8] The relationship between income and fertility is rather controversial. Becker (1981) argues for a positive relationship on the ground that children are consumer goods competing with alternative goods. But he also put forward a trade-off between quality of education and quantity, so that the relation can be negative (see also De la Croix and Doepke (2003) for an alternative point of view). How could we decide between these alternative views? We had access to survey data collected from interviews in Marseilles in 2006 at the occasion of an exercise in experimental economics concerning willingness to pay for a better air quality. The sample is made of 549 individuals, coming from distinct households for which we have the number of children, the total net income of the household and the marital status of the responder. We can thus run a regression which links the number of children to family income and marital status. $Cel$ is a dummy variable indicating if the responder is single or not. Expressed in logarithms, this regression has a low $R^2 = 0.205$, but clearly indicates a slightly positive relation between fertility and family income as we have:

$$\ln(1 + ch_i) = \underset{[0.16]}{0.034} + \underset{[2.43]}{0.061} \ln(1 + w_i) - \underset{[-9.01]}{0.40}\, cel.$$

Following this regression[9], we shall suppose that the number of children follows a Poisson distribution with parameter $\theta$ (which represents both the mean and the variance) and we note $ch_i \sim P(\theta)$. The average number of children in France is two per family. We suppose that part of the fertility decision is independent of income, say for the first child, but that the decision for an extra child depends on income. We have thus chosen $\theta = 1+w_i/\bar{w}$. The realised number of children in a family for the 10 000 households of the sample lies in the interval [0,9]. In Figure 4, we compare the distribution of the number of children in a household when fertility depends on gross income to a distribution that would suppose that fertility is totaly independent of income, while keeping the same mean. Clearly, the assumption that fertility depends on income has a strong influence on the decision to have a second child, but lowers the probability to have more than 3 children for the given gross income distribution.

As we now define the income distribution of a household and no longer the income distribution of an individual, comparisons between households become more difficult. Clearly a

[7] In France, family allowances are independent of income, except for the *complement familial* which is proportional to wages.

[8] We exclude for simplicity mono-parental households. This is certainly a limitation of our analysis. We are concerned with the whole French income distribution, of which households with a single adult are a significant fraction. We also do not introduce the possibility of income disparities between the two parents and their consequences on taxation.

[9] A non-parametric regression, not reported here, indicates also a positive and quasi linear relation.

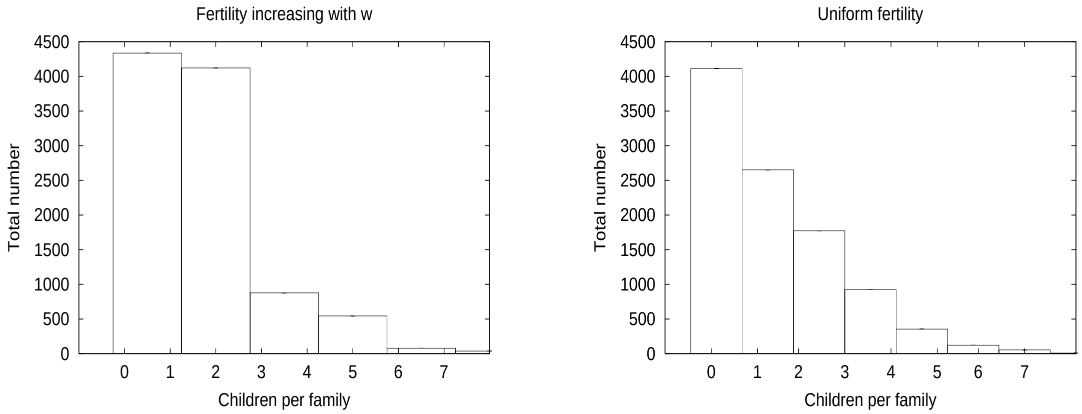


Figure 4: Distribution of the number of children for the two scenarios

couple with many children has not the same needs as a couple without any children. We have to introduce an equivalence scale in order make household incomes comparable. We have chosen a rather mild equivalence scale which gives a weight of 1 to the first adult, 0.7 to the second adult and 0.3 to each child. This is a mix between the Oxford equivalence scale and the OECD equivalence scale. Let us call $sn_i$ the number thus obtained for household $i$. If we divide $w_i$ by this number, we have an income expressed in terms of family needs. The basic ELIE formula is changed into:

$$\frac{y_i}{sn_i} = 2\,\frac{w_i}{sn_i} + 2\,k\left(\bar{w}_s - \frac{w_i}{sn_i}\right) \tag{8}$$

where $\bar{w}_s = n^{-1}\sum_i w_i/sn_i$.

## 5.2 The influence of equivalence scales

Might ELIE gain some incentive properties for fertility decisions just by using an equivalence scale without introducing a specific family allowance mechanism? The answer might depend on the assumption made for fertility which determines in which type of family children are located. It is thus wise to consider two variants: one where fertility is independent of income and one where it is income dependent. Let us first suppose that the number of children per household is drawn from a Poisson distribution with $\theta = 2$. We have simulated an income distribution for 10 000 households composed of two adults earning each the same $w_i$ and having a random number of children, independent of their income. Results are displayed in In Table 3. In column $U$, we have divided the total gross household income by 2 (the number of adults in the household) while in column $S$, total income is divided by an equivalence scale depending on the number of children. We then apply the usual ELIE scheme on the resulting scaled income, using (8). Considering an equivalence scale slightly increases inequality, fiscal pressure and decreases redistribution. The order of magnitude of these changes is small, but significant. It would be stronger with a different equivalence scale. Column $U$ is identical to the initial case of section 3. With an equivalence scale, the French Gini coefficient is recovered for $k = 0.19$ instead of $k = 0.17$ when there is no equivalence scale.

Let us now consider the income distribution conditional on the number of children. What is the more favourable system for large families? In Table 4, we give the mean income of the

Table 3: Equivalence scales and ELIE
with a random fertility

| | $k = 0.0$ | | $k = 0.2$ | | $k = 0.3$ | | $k = 0.4$ | |
|---|---|---|---|---|---|---|---|---|
| | U | S | U | S | U | S | U | S |
| Gini | 0.327 | 0.342 | 0.262 | 0.273 | 0.229 | 0.239 | 0.196 | 0.205 |
| Fisc. pres. | 0.000 | 0.000 | 4.697 | 4.911 | 7.045 | 7.366 | 9.393 | 9.821 |
| Kakwani | 0.000 | 0.000 | 0.454 | 0.448 | 0.454 | 0.448 | 0.454 | 0.448 |
| Reynolds | 0.000 | 0.000 | 0.065 | 0.068 | 0.098 | 0.103 | 0.131 | 0.137 |

In columns U, household income is divided by 2, while in columns S it is divided by an equivalence scale.

10 000 households having zero, one, two, three and more children. We give the total income after ELIE taxes and transfers, the exact income that the household will have in its pocket, once transfers are operated. This figure is obtained by re-multiplying the net income obtained in (8) by $sn_i$. In column $k = 0$ of Table 4, there is of course no difference between $U$ and $S$. For $k > 0$, the $U$ columns are simply an estimate of the conditional mean of the income distribution. The computed marginal mean income is equal to 2.025. Let us concentrate on the columns marked $S$. When $k$ is positive, ELIE seems to redistribute money as a function of the number of children. Households with less than two children receive less that the mean income. Household with more than two children receive more and transfers increase with the number of children.

Table 4: Total net income per family
with a uniform fertility

| | $k = 0.0$ | | $k = 0.2$ | | $k = 0.3$ | | $k = 0.4$ | |
|---|---|---|---|---|---|---|---|---|
| Children | U | S | U | S | U | S | U | S |
| 0 | 2.062 | 2.062 | 2.054 | 1.959 | 2.051 | 1.907 | 2.047 | 1.856 |
| 1 | 2.025 | 2.025 | 2.025 | 1.984 | 2.025 | 1.963 | 2.025 | 1.943 |
| 2 | 2.025 | 2.025 | 2.025 | 2.039 | 2.025 | 2.046 | 2.025 | 2.053 |
| 3 | 2.018 | 2.018 | 2.020 | 2.088 | 2.020 | 2.123 | 2.021 | 2.158 |
| $> 3$ | 2.003 | 2.003 | 2.008 | 2.160 | 2.010 | 2.239 | 2.012 | 2.317 |

As a conclusion, an equivalence scale seems to introduce a trade-off between fertility incentives and fairness.

### 5.3 Equivalence scales and income dependent fertility

If we believe in the incentive effect of family allowances, fertility should be assumed to depend on income. This assumption is going to change dramatically the previous results. We have redone the same simulation exercise as in subsection 5.2, but this time with $ch_i \sim P(\theta)$ where $\theta = 1 + w_i/\bar{w}$ instead of $\theta = 2$. Table 5 shows that the use of an equivalence scale now decreases inequality as shown by the Gini indices computed on net income after ELIE transfers, instead of increasing it as in the case of random fertility. The fiscal pressure is lowered while progressivity is increased. Thus the dependence between fertility and income

Table 5: Equivalence scales and ELIE
with income dependent fertility

| | $k = 0.0$ | | $k = 0.2$ | | $k = 0.3$ | | $k = 0.4$ | |
|---|---|---|---|---|---|---|---|---|
| | U | S | U | S | U | S | U | S |
| Gini | 0.327 | 0.307 | 0.262 | 0.245 | 0.229 | 0.215 | 0.196 | 0.184 |
| Fisc. pres. | 0.000 | 0.000 | 4.697 | 4.393 | 7.045 | 6.589 | 9.393 | 8.753 |
| Kakwani | 0.000 | 0.000 | 0.454 | 0.465 | 0.454 | 0.465 | 0.454 | 0.465 |
| Reynolds | 0.000 | 0.000 | 0.065 | 0.061 | 0.098 | 0.092 | 0.131 | 0.123 |

has a drastic influence on the redistribution properties of ELIE when an equivalence scale is used. This influence is now positive in term of fairness. A value of $k = 0.13$ is now sufficient to get the French Gini coefficient on net income instead of $k = 0.17$ in the original model without equivalence scales.

Table 6: Equivalence scales and ELIE
with income dependent fertility (continued)

| | $k = 0.0$ | | $k = 0.2$ | | $k = 0.3$ | | $k = 0.4$ | |
|---|---|---|---|---|---|---|---|---|
| Children | U | S | U | S | U | S | U | S |
| 0 | 1.502 | 1.502 | 1.607 | 1.500 | 1.659 | 1.499 | 1.711 | 1.498 |
| 1 | 1.712 | 1.712 | 1.774 | 1.720 | 1.806 | 1.724 | 1.837 | 1.729 |
| 2 | 1.997 | 1.997 | 2.002 | 2.001 | 2.005 | 2.003 | 2.008 | 2.005 |
| 3 | 2.271 | 2.271 | 2.222 | 2.273 | 2.197 | 2.273 | 2.173 | 2.275 |
| $> 3$ | 2.892 | 2.892 | 2.719 | 2.860 | 2.632 | 2.843 | 2.546 | 2.827 |

Let us now examine the money which actually enter the household pocket. Table 6 shows first that before any redistribution ($k = 0$), the fertility model implies that households with children have more money than households without children, while the computed average income remains the same at 2.025. This is just a consequence of our assumption made on fertility. When $k > 0$, the mean income per household type is not significantly changed. Families with more than two children get more money with the equivalence scales and family with less than two children get less money as in the previous configuration. So the incentive mechanism introduced by the equivalence scales is unchanged, while the fairness properties of the model were increased.

### 5.4 ELIE and family allowances

ELIE is a mechanism designed for macro justice and not for incentives, because it is not a targeted mechanism. Its sole aim is to reduce the distance between average income and actual income, independently of any other factors. We have here introduced a new dimension, which is family composition and its corollary equivalence scales. Depending on the modeling of fertility, the use of an equivalence scale can confer to ELIE some incentive properties, but at the expense of a lower degree of fairness measured here as progressivity and inequality. When fertility increases with income, the incentive effect disappear and fairness is restored.

Family allowances is a sensitive topic in France and also in many other European countries. A government who would like to implement the redistributive ELIE mechanism cannot politically suppress family allowances, even if we can truly suspect this mechanism to distort the nice properties of ELIE. Let us try to conceive a complementary redistribution scheme based solely on family composition and independent of income. We need extra resources in order to finance explicit family allowances, which means distributing an amount of money only on the basis of family composition, independently of income. The redistributive ELIE system balances because the pivot is equal to the mean $\bar{w}_s$. If we choose a pivot lower than the mean, say $\bar{w}_q < \bar{w}_s$, we get the needed extra amount to redistribute:

$$Tr = 2\sum_{i=1}^{n} k(\bar{w}_q - \frac{w_i}{sn_i}) > 0. \tag{9}$$

So that net income is now defined as

$$\frac{y_i}{sn_i} = 2\,\frac{w_i}{sn_i} + 2\,k\left(\bar{w}_q - \frac{w_i}{sn_i}\right) + Tr\,ch_i/n_{ch}, \tag{10}$$

where $n_{ch}$ is the total number of children in the complete sample. How can we calibrate $\bar{w}_q$? In the ELIE scheme, the Kakwani progressivity index is independent of $k$, but varies with the size of the pivot. In the previous sections, the Kakwani index was much higher than its French value of 0.31. We can decide to chose $\bar{w}_q$ so as to match the value of 0.31 for the Kakwani index. Compared to Table 5, inequality has slightly increased, but mainly

Table 7: Family allowances and ELIE
with income dependent fertility

| | $k = 0.0$ | $k = 0.2$ | $k = 0.3$ | $k = 0.4$ |
|---|---|---|---|---|
| Gini | 0.306 | 0.249 | 0.222 | 0.197 |
| FiscPres | 0.000 | 8.154 | 12.231 | 16.308 |
| Kakwani | - | 0.310 | 0.310 | 0.310 |
| Reynolds | 0.000 | 0.059 | 0.089 | 0.119 |

progressivity was calibrated to be much lower. This is translated in a huge increase in the fiscal pressure because now much more households are taxed. The Reynolds index is smaller indicating that taxes are less successful in reducing inequality.

Table 8 show that large families get much more money. But we have seen that this extra amount of money is not obtained in a fair way. If ELIE had to be applied, explicit family allowances should be avoided. Equivalence scales are enough.

## 6 Introducing capital stock

Kolm (2005) claims that *since capital is by definition produced, the other primary resources finally account for about 97.5% for labour and 2.5% for the non-human natural resources.* So that *the problem of overall distribution in macrojustice is the allocation of the rights in the value of productive capacities*, i.e. labour capacities.[10] The idea is attractive from a normative

[10] See also section 3.3 of Kolm (2009), this book.

Table 8: Equivalence scales and ELIE
with income dependent fertility (continued)

| | $k = 0.0$ | $k = 0.2$ | $k = 0.3$ | $k = 0.4$ |
|---|---|---|---|---|
| Children | | | | |
| 0 | 1.488 | 1.387 | 1.337 | 1.286 |
| 1 | 1.731 | 1.676 | 1.649 | 1.621 |
| 2 | 1.955 | 1.967 | 1.974 | 1.980 |
| 3 | 2.344 | 2.409 | 2.441 | 2.474 |
| $> 3$ | 2.857 | 3.090 | 3.206 | 3.322 |

point of view and in a steady state, but it is not adapted if the objective is to confront the ELIE scheme to an empirical reality. Kolm's point of view implicitly assumes that capital and credit markets are efficient so that there is no problem of capital reallocation in the long term.

### 6.1 Stylised facts

We observe a fundamental inequality in the sharing of the fruits of capital and of labour which sets forward the question of the efficiency of any redistributive policy. At the macro level, the share of capital in total value added can remain relatively constant despite of large wage rises as in the US, or experience a marked increase as in Europe and especially in France as shown in Figure 5 and as noted in Piketty (2004). The modification of the capital share

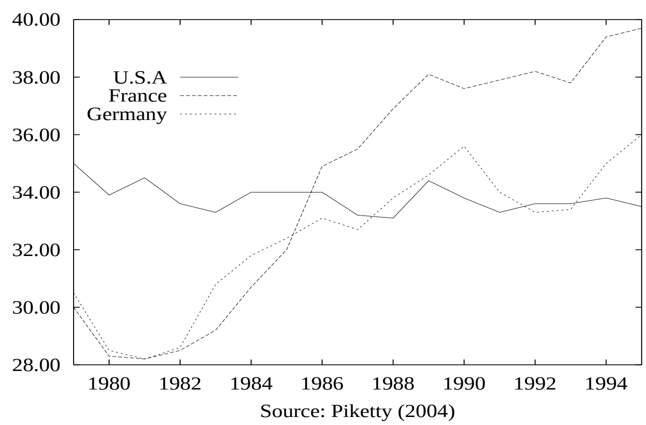


Figure 5: Capital share in some OECD countries

in France implied a reallocation of more than 10% of total GNP on a rather short period of 10 years. This is more than what any fiscal redistributive policy could have operated. At the micro level, we can notice that capital allocation remains extraordinary stable in the long term between individuals. This is due to several reasons. There is *first* the question of initial conditions in the dynamic process of wealth transmission: death duty does not manage to reallocate efficiently ownership because rates of taxation are too low for that purpose. *Secondly*, the rate of saving is different between poor and rich individuals. A high rate of saving is necessary for accumulation. Moreover, as documented in Direr and Weitzenblum (2007), mostly rich individuals have access to really profitable investments. *Thirdly*, credit market is rationed, banks avoid financing risky projects initiated by lower income holders as exemplified for instance in Piketty (1994). *Finally*, wages and capital gains are not taxed in the same way. In France, for the same level of income, wage earners are more taxed than holders of capital gains. All these features put together contribute to inequality in the income and wealth distributions. As a matter of fact, in every country, there is a stronger inequality in wealth distribution, than in income distribution. For instance, the Gini index for net income distribution was reported to be 0.27 in France. It rises up to 0.65 for wealth inequality in 2004 (see for instance Cordier, Houdré, and Rougerie (2006)).

Regulating the distribution of income between capital and labour can become thus one of the major goals of income distribution policies. This justifies introducing capital accumulation in the ELIE framework and see what is the performance of ELIE for regulating the inequality in income distribution generated by capital accumulation.

## 6.2 General features

The model we shall use for capital accumulation[11] is inspired from Solow (1956) where the rate of saving determines the rate of accumulation. In the original growth model of Solow, population grows at a fixed rate while there is no depreciation for capital. Here we want to keep the labour force constant with the same wage distribution for ease of comparison with our previous results. We shall thus suppose that the rate of growth of the labour supply is zero while capital depreciates at a fixed rate. The fundamental properties of the model should not be affected. We consider that there is one active generation with an economic activity lasting for 30 periods. In this framework, we can leave aside the question of inheritance taxes (death duty). We just have to precise initial conditions, saving behaviour and transfers.

The first key assumption concerns the rate of saving. As we have an heterogenous population (contrary to the model of Solow), we shall suppose that the rate of saving varies across the population. Our key assumption is that rich people save more than less rich people. More precisely, the rate of saving will be a positive function of income. This assumption summarises the stylised facts detailed above and is central to obtaining a distribution of wealth more concentrated than the distribution of income. The second key assumption follows Direr and Weitzenblum (2007) where only rich people have access to really profitable investments. Capital profitability will also be dependent on income.

With this model, we want to compare two types of taxation-redistribution in the presence of capital accumulation. In a first scenario, we introduce ELIE only for labour income. There is a flat tax on capital income, very much like what happens in France. The fruit of these taxes is redistributed uniformly among the individuals as would be a basic income. This scenario aims at reproducing more or less the French situation or a minimal adaptation of ELIE to France. The second scenario is a kind of full application of ELIE when there is capital accumulation. Capital income is added to labour income and serves a basic to the ELIE formula. Capital and income are equally treated.

[11] A complete model would have to consider overlapping generations such as in Direr and Weitzenblum (2007). We would like, at least in a first attempt, to leave this question aside.

### 6.3 A dynamic model of capital accumulation

Initial conditions are crucial for any non-stationary processes, because they have a direct influence on their trajectories. We have calibrated the wage distribution on the French Gini coefficient and normalised it with a mean equal to one. We have to do the same for the initial wealth distribution. We suppose that the initial capital stock is a function of labour income and that it is not equally distributed in such a way so as to reproduce the French Gini coefficient computed for wealth. Our proposed formula is

$$s_{0,i} = \alpha_0 \frac{w_i}{\mu_r} \mathbb{1}(w_i > \bar{w}) + (1 - \alpha_0) \frac{w_i}{\mu_r} \mathbb{1}(w_i < \bar{w}), \tag{11}$$

where $\mu_r$ is the mean rate of return on capital and $\mathbb{1}$ is the indicator function. With $\alpha_0 = 0.75$, we get a Gini coefficient of 0.651 for the initial capital stock. These initial conditions are interpreted as a bequeath of the previous generation. With an initial mean rate of return of 3%, the income generated by the initial capital stock will represent roughly 39% of total income at $t = 0$.

The rate of saving determines the rate of accumulation. Capital stock available at time $t$, $s_{i,t}$ comes from capital at time $t-1$, $s_{i,t-1}$, diminished of a fixed depreciation rate $\delta$, but augmented of savings. It is commonly admitted that the saving rate increases with income, but less than proportionally (see e.g. Dynan, Skinner, and Zeldes (2004)). We propose modeling the saving rate as

$$\gamma_i = \mu_\gamma \sqrt{w_i}, \tag{12}$$

where $\mu_\gamma$ is a scaling parameter. The mean saving rate is equal to 15% in 2005 in France. This rate is reproduced for $\mu_\gamma = 0.156$. For individuals with a labour income lower than $\bar{w}$, this rate is on average equal to 12%; for individuals with a labour income greater than $\bar{w}$, this rate is equal to 19% on average, given our wage distribution.

Following Direr and Weitzenblum (2007), profitability of capital must be a function of income to illustrate the fact that only rich individuals have access to really profitable investments. We shall suppose the rate of return on capital, $r_{i,t}$, is a random variable with a mean which is a linear function of the square root of $w_i$ and a fixed variance:

$$r_{i,t} \sim G(\mu_r \sqrt{w_i}, \sigma_r^2) \tag{13}$$

where $G(.,.)$ is a Gaussian random variable. At each period, we will have a new draw for the returns. We have previously chosen $\mu_r = 0.03$ and now we chose $\sigma_r = 0.01$. This formulation does not exclude temporary negative returns. Capital income at time $t$, $sr_{i,t}$, is given by:

$$sr_{i,t} = s_{i,t} r_{i,t}, \tag{14}$$

while capital stock is defined by

$$s_{i,t} = (1 - \delta) s_{i,t-1} + \gamma_i y_{i,t}. \tag{15}$$

The growth rate of the capital stock depends on the balance between the proportion of depreciation $\delta$ and the share of net income devoted to savings. As savings depends on net income, the growth rate of capital is a function of $k$, the redistribution parameter. There is a clear trade-off between growth and redistribution. Net income is given by the combination of labour income, capital income and the implementation of a given redistribution scheme.

Table 9: The trade off between growth and inequality

| $k$ | Mean net total inc. | Mean capital share | Mean capital stock | Gini net income | Gini capital income | Gini capital stock |
|---|---|---|---|---|---|---|
| | | | Initial situation | | | |
| 0.000 | 1.659 | 0.389 | 16.540 | 0.417 | 0.706 | 0.651 |
| | | | ELIE on labour income only | | | |
| 0.00 | 1.670 | 0.393 | 17.004 | 0.410 | 0.680 | 0.601 |
| 0.20 | 1.660 | 0.390 | 16.842 | 0.367 | 0.673 | 0.590 |
| 0.30 | 1.655 | 0.388 | 16.761 | 0.346 | 0.670 | 0.584 |
| 0.40 | 1.650 | 0.386 | 16.680 | 0.324 | 0.666 | 0.578 |
| | | | ELIE on total income | | | |
| 0.000 | 1.684 | 0.399 | 17.240 | 0.470 | 0.692 | 0.619 |
| 0.200 | 1.660 | 0.390 | 16.842 | 0.367 | 0.673 | 0.590 |
| 0.300 | 1.648 | 0.385 | 16.650 | 0.318 | 0.664 | 0.575 |
| 0.400 | 1.636 | 0.381 | 16.462 | 0.269 | 0.654 | 0.561 |

In our *first scenario*, ELIE is applied only to labour income, while capital income is taxed at a fixed rate $\tau$ of 20% (a realistic value in the French context). The product of capital taxation is used to finance a basic income. We have:

$$y_{i,t} = w_i - k(w_i - \bar{w}) + (1-\tau)sr_{i,t} + al_t/n, \tag{16}$$

where $al_t$ is the fruit of taxation on capital income and is given by

$$al_t = \tau \sum_i sr_{i,t}. \tag{17}$$

We can now calibrate $\delta$. As there is a trade-off between capital growth and redistribution, we chose $\delta$ so as to maintain over the 30 periods a constant ratio between capital income and total income for a given value of $k$. For $k = 0.3$, the corresponding value of $\delta$ is found to be 0.0175.

In our *second scenario*, capital and labour incomes are treated in the same way (which they are not in reality). We add capital and labour income and then apply the ELIE formula to total income;

$$y_{i,t} = w_i + sr_{i,t} - k(\bar{w}_{cap} - w_i - sr_{i,t}), \tag{18}$$

with $\bar{w}_{cap} = n^{-1}\sum(w_i + sr_{i,t})$. Simulating these two variants for our 10 000 individuals will give us clues on the properties of ELIE in the presence of capital accumulation.

### 6.4 A trade-off between growth and inequality

We first analyse the trade-off between growth and inequality in our simple calibrated growth model. Table 10 clearly illustrates the trade-off resulting from ELIE. Increasing $k$ deceases the net total income but also inequality both for net income, capital income and capital stock. There is still more inequality on capital income than on labour income. The two systems of taxation-redistribution, that is to say, on one side ELIE applied only to labour income but coupled with a flat tax on capital income, and a basic income and on the other side the full ELIE applied to the sum of labour and capital income, are strictly equivalent for $k = 0.2$. But for greater values of $k$, the full ELIE is much more powerful to reduce inequality. For $k = 0.2$, the share of capital income increases slightly in both cases, compared to the initial situation. For $k = 0.3$, it diminished with the full ELIE while it remains slightly constant in the first scenario. The redistributive power of ELIE is made even more apparent if we examine Figure 6 which displays for both scenarios the income distribution. With the full ELIE, the right

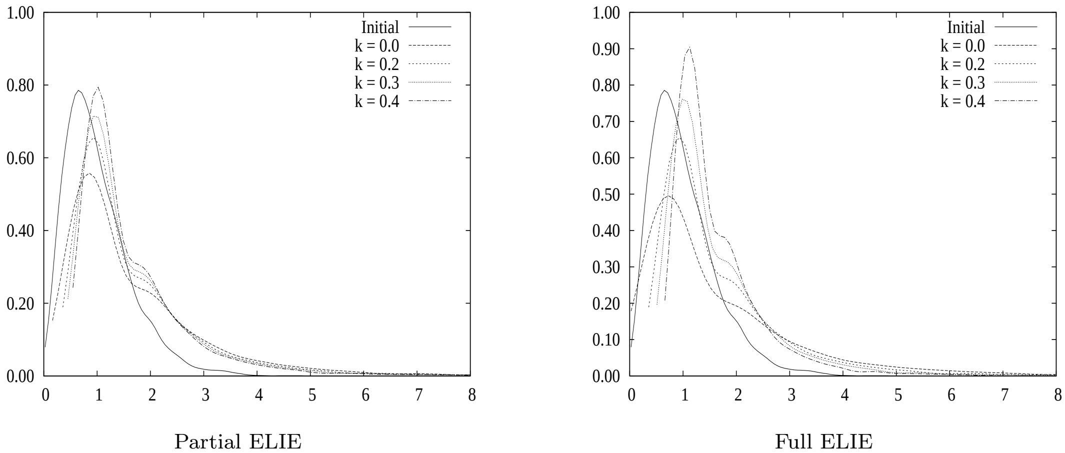


Figure 6: Income distribution with capital accumulation

tail of the income distribution is much flatter. The group of very rich individuals is strongly and negatively affected by ELIE taxes. With a progressive taxation, the extreme right tail disappears. Moreover, the importance of this group collapses for earners beyond 5 times the mean, while individual around twice the mean beneficiate largely from the full ELIE scheme.

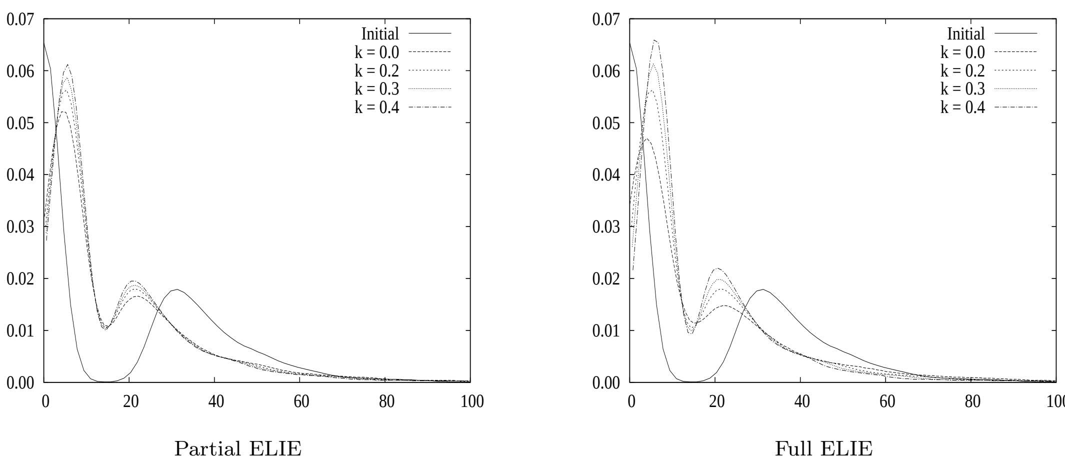


Figure 7: Capital distribution

The change of the initial income distribution has its roots in the modification of capital distribution as illustrated in Figure 7. Initial conditions are largely modified after thirty

Table 10: The trade off between growth and inequality

| $k$ | Mean net total inc. | Gini total inc. | Fiscal pressure | Kakwani | Reynolds |
|---|---|---|---|---|---|
| ELIE on labour income only | | | | | |
| 0.00 | 1.670 | 0.410 | 7.868 | 0.214 | 0.053 |
| 0.20 | 1.660 | 0.367 | 10.663 | 0.238 | 0.092 |
| 0.30 | 1.655 | 0.346 | 12.072 | 0.247 | 0.111 |
| 0.40 | 1.650 | 0.324 | 13.490 | 0.254 | 0.131 |
| ELIE on total income | | | | | |
| 0.000 | 1.684 | 0.470 | 0.000 | - | 0.000 |
| 0.200 | 1.660 | 0.367 | 6.945 | 0.350 | 0.092 |
| 0.300 | 1.648 | 0.318 | 10.298 | 0.352 | 0.136 |
| 0.400 | 1.636 | 0.269 | 13.574 | 0.355 | 0.179 |



periods, because everybody can save and accumulate in our model, even if all are not equal in their opportunities and saving rates. But the capital distribution is still bimodal. The full ELIE has a larger impact on holders of capital stock greater than 60 times the mean wage. The full ELIE is able to greatly diminish their importance. This corroborates the remarks made in Piketty (2001) on the disappearance of large fortunes at the beginning of the XX$^{\text{th}}$ century with the introduction of a progressive taxation on income. However, the redistributive scheme of ELIE should be combined with a careful study for tuning death duties. Initial conditions are of a considerable importance, even after 30 periods, to determine the shape of the capital distribution.

### 6.5 Analysing taxation

The previous picture can be completed by analysing some taxation indices. For $k = 0.20$, we have seen that the macroeconomic situation is the same for both scenarios. However, we see now that the full ELIE system implies a lower fiscal pressure and a stronger progressivity of taxation in order to achieve at the same redistribution. Fiscal pressure starts to become greater with the full ELIE only for $k = 0.40$. Finally, we note that with capital accumulation, our calibrated model needs a $k = 0.40$ together with the full ELIE in order to reach the French value of 0.270 for the Gini coefficient on net income. We can thus conclude that introducing capital accumulation leads to a very strong modification of the income distribution and of the ELIE principle of equity based on the sole labour factor. Clearly, ELIE has to be completed by a taxation redistribution of bequeath.

## 7 Conclusion

The ELIE scheme is a principle of taxation and transfers that has many attractive properties in terms of macrojustice and equity. In this paper, we have tried to address the question of its implementation. We examined that nice principle of taxation-redistribution and tried to see how its properties would be transformed when it has to be applied in a specific context with its own characteristics where everything cannot be erased. Taxation systems cannot be changed completely in one year. Reformation is always discussed at the parliament which means that some categories will pay or receive more or less than what the original idea would imply. In France family allowances are historically very important and certainly been successful in maintaining a decent rate of natality, even if we have seen that their impact in term of equity is dubious. Since capital is much more mobile than labour, the impact of capital taxation is always discussed in terms of incentives and international fiscal competition more than in terms of equity.

We have tried to propose options which make the basic ELIE model more realistic for the French case. We have found that uncertainty on wages does not modify fundamentally the properties of the model and this is a nice result. However it might lower greatly its efficiency. Family allowances represents a greater challenge. The incentive properties of family allowances can be recovered simply by introducing equivalence scales. Their impact is very sensitive to the relation between income and fertility. If fertility is increasing with income, an equivalence scale can increase the redistributive efficiency of the system in the sense that the same degree of inequality can be obtained with a lower $k$. But we can have exactly the reverse property with a different model for fertility. The incentive effect can be greatly increased if we introduce a specific family allowance mechanism, but at cost of a huge increase in inequality. Finally, capital accumulation has the strongest distortion impact and changes many of the ELIE properties. In the presence of capital accumulation, ELIE should be completed by a mechanism of taxation-redistribution on bequeath which is outside the scope of this paper.

Our model was calibrated in such a way so as to reproduce most of the inequality and taxation indexes obtained on French data. Various values for $k$ were considered. They are directly linked to the desired degree of aversion for inequality in society. In the pure ELIE model, $k$ had to be rather low to fit French data. But in this basic model, the income distribution does not reproduce correctly the importance of high incomes. A model with capital accumulation is necessary to obtain an distribution representing correctly high incomes. In this case $k$ has to be rather high in order to reproduce the value of the various taxation indexes. The determination of $k$ thus heavily depends on the model considered. The more we depart from the original model, the greater $k$ has to be. Further investigation is needed so as to estimate the desired degree of aversion for inequality in society. Many papers have been written on this topic. Building on the seminal work of Kolm (1969) and Atkinson (1970), we can quote Gevers, Glejser, and Rouyer (1979) followed by Van Praag, Goedhart, and Kapteyn (1980) or Amiel, Creedy, and Hurn (1999), all using experiments and questionnaires. This is planned for future work.

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## APPENDIX : About the instruments

Most of the indices used in the literature devoted to taxation and transfer analysis are based on the geometry of the Gini index. Let us start from a series of gross income $X$ with density $f(x)$ and distribution $F(q)$. The Lorenz curve is defined as

$$p = F(q) \Rightarrow L_X(p) = \frac{1}{\bar{x}} \int_0^q x f(x)\, dx.$$

Let us now suppose that the observations have been ordered in ascending order. The Lorenz curve can be estimated using

$$L_X(p = i/n) = \frac{1}{n\,\bar{x}} \sum_{j=1}^{i} x_j.$$

The Lorenz curve indicates the proportion $L_X(p)$ of total income that is hold by the proportion $p$ of the total population. Perfect equity corresponds to the main diagonal $L_X(p) = p$. The Gini index **G** is defined as the surface between that diagonal and the Lorenz curve:

$$\mathbf{G} = 2 \int_0^1 [p - L_X(p)]\, dp = 1 - 2 \int_0^1 L_X(p))\, dp$$

which can be approximated by, using ascending order

$$\mathbf{G} = 1 + \frac{1}{N} + \frac{1}{\bar{x}N^2} \sum_{i=1}^{N} (N - i + 1) x_i.$$

Let us now define a tax function $t(x)$ and the amount of taxes $t_i = t(x_i)$. The mean tax rate is obtained as the ratio of total taxes by total gross income:

$$g = \frac{1}{\bar{x}} \int_0^\infty t(x) f(x)\, dx = \frac{\bar{t}}{\bar{x}}.$$

It will be used extensively in the next definitions. The concentration curve $C_T(p)$ indicates the proportion of total taxes that the population holding the proportion $p$ of the total income pays:

$$p = F(q) \Rightarrow C_T(p) = \frac{1}{g\,\bar{x}} \int_0^q t(x) f(x)\,dx.$$

Let us now order the observations $t_i$ according to the ascending order of $x_i$. The concentration curve can be estimated by

$$C_T(p = i/n) = \frac{1}{n\,g\,\bar{x}} \sum_{j=0}^{i} t_j = \frac{1}{n\,\bar{t}} \sum_{j=0}^{i} t_j.$$

A concentration index **CI** is defined in the same way as the Gini index as a measure of the surface between the concentration curve and the diagonal

$$\mathbf{CI} = 1 - 2\int_0^1 C_T(p)\,dp.$$

Let us go from gross income $x_i$ to net income $y_i$ by subtracting taxes $t_i$. The concentration curve of net incomes $C_Y(p)$ gives the proportion of total net income that is held by the the population holding the proportion $p$ of the total gross income:

$$p = F(q) \Rightarrow C_Y(p) = \frac{1}{(1-g)\bar{x}} \int_0^q [x - t(x)] f(x)\,dx,$$

which can be estimated by

$$C_Y(p = i/n) = \frac{1}{n\,(1-g)\,\bar{x}} \sum_{j=0}^{i} (x_j - t_j) = \frac{1}{n\,\bar{y}} \sum_{j=0}^{i} y_j.$$

There exists a deterministic relationship between the three curves:

$$L_X(p) = gC_T(p) + (1-g)C_Y(p).$$

The various indexes that characterise a taxation and redistribution scheme are defined in the same way as the Gini index. They correspond to surfaces between two Lorenz and/or concentration curves.

**The progressivity index** of Kakwani (1977) measures the surface between the Lorenz curve associated to gross incomes and the concentration curve of taxes. It characterises progressivity as a departure from proportionality:

$$\mathbf{K} = 2\int_1^0 [L_X(p) - C_T(p)]dp = \mathbf{CI}_T - \mathbf{G}_X.$$

This index is positive if a tax is progressive, equal to zero if the tax is proportional and negative if the tax is regressive. The negative bound is $-(1 + \mathbf{G}_X)$ and the positive bound $(1 - \mathbf{G}_X)$ where $\mathbf{G}_X$ is the Gini index for gross income $x_i$.

**The index of vertical equity** of Reynolds and Smolensky (1977) measures the reduction in inequality brought in by taxes as the surface between the concentration curve of net income and the Lorenz curve of gross income

$$\mathbf{RS} = 2\int_1^0 [C_Y(p) - L_X(p)]dp = \mathbf{G}_X - \mathbf{CI}_Y.$$

It equivalently measures the reduction operated by taxation on the Gini index of gross income. Because of the deterministic relationship existing between Lorenz and concentrations curves, these two indices are related by

$$\mathbf{RS} = \frac{g}{1-g}\mathbf{K}.$$

**The index of horizontal inequity** of Atkinson (1980) and Plotnick (1981) is concerned by the unequal treatment of equals or re-ranking operated by taxation. It measures the surface between the concentration curve and the Lorenz curve of net income:

$$\mathbf{AP} = 2\int_1^0 [C_Y(p) - L_Y(p)]dp = G_Y - CI_Y.$$

If taxes do not induce a rank permutation, then the concentration and Lorenz curves of net income are identical and this index is zero. Or the Gini and concentration index of net income are identical.

**The redistributive index** measures the surface between the Lorenz curve of net and gross incomes:

$$\mathbf{IR} = 2\int_1^0 [L_Y(p) - L_X(p)]dp = \mathbf{G}_X - \mathbf{G}_Y.$$

This index is at value between 0 and 1. It increases with the degree of redistribution. It is simple to discover that this index is equal to the difference between the index of vertical equity and the index of horizontal inequity: $\mathbf{IR} = \mathbf{RS} - \mathbf{AP}$.

For more details on the topic, see Lambert (2001), Duclos and Tabi (1999) or Essama-Nssah (2000).