A proposal for a tool that helps handling variability and remains compliant with falsifiability

Frédéric Davesne

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Frédéric Davesne
Laboratoire Systèmes Complexes - CNRS FRE2494
University of Evry
40, Rue du Pelvoux
91020 Evry FRANCE
frederic.davesne@iup.univ-evry.fr

Abstract

On one hand, automatics is based on proofs before the experiment in order to validate an a priori (partially) known deterministic interaction between the robot and its environment; in return, the experimenter may expect the reliability of the real system based on his model. On another hand, statistical methods are used when the environment is supposed unknown; in return, the robot may own adaptation capabilities but its behavior is not predictable before the experiment and is not necessary reliable.
We believe that the core issue for the first approach (that we call *deterministic approach*) relies on the fact it cannot handle the variety of all the possible situations in an unconstrain environment (we call this the *variability* issue). Oppositely, we think that the statistical approach is essentially missing a validating stage before the experiment in order to possibly *falsify* a proposed model.
The aim of this article is to suggest a methodology that combines both a validating stage before the experiment and the creation of models that handle unknown environments. To do this, we suggest models that own a validation statement and internal parameters that fix a compromise between falsifiability and robustness. For simple cases, we show that it is possible to fix internal parameters in order to meet the two antagonist constraints. As a consequence, we stress that the precision of the model has a lower bound and we determine a Heisenberg-like uncertainty principle.

1. Context of our study

1.1 The variability challenge

Nowadays, several scientific areas are facing complexity. Nature is complex by itself; artifacts made by humans may be too. However, in some cases, scientific breakthroughs give a framework that breaks the complexity and permit to reduce a physical phenomenon to a model. For example, the trajectory of Earth (which is a complex system by itself) around the Sun may be well approximated by using only a few variables (mass, velocity and position) and by neglecting the other planets of the solar system. Those cases, which I call "favorable cases", gave birth to scientific theories (like Newton's theory of gravitation) which are *falsifiable* in Popper'sense (Popper, 1968). In particular, these theories enable the prediction of what will happen and what will *never happen* in reality.

At the opposite, biological systems - even the simplest - may not be reduced in such a way, mainly because they are bound to *variability*. Variability appears when an entity behaves differently when facing apparently equal situations or when apparently equal entities behaves differently when facing the same situation. In these cases, a system or components of a system cannot be easily isolated to model a phenomenon in an analytical manner. It is interesting to draw a parallel with the study of the behavior of a mobile robot which interacts with a complex and a priori unknown environment. Even if human have built the robot and has programmed it, one must admit that it is not possible to know precisely before the experiment what it will do and will never do: the robot behavior is not really predictable (see (Nehmzow and Walkery, 2003)). Hence it is not possible to calculate the reliability of the robot behavior before the experiment.

The biological and artifact cases share the fact that something in the phenomenon remains unknown by the scientist but cannot be neglected; this carries poor or context dependent results. *Ad hoc* algorithms or learning capabilities may be implemented for artifacts to cope with discovery of a priori unknown characteristics of the environment. However, taking a very simple example from the reinforcement learning domain (which owns theoretical results), we have shown (see (Davesne and Barret, 2003)) that good results mainly depend on the ability of the experimenter to create a proper context to make the learning algorithm work in reality, whereas results may not be predictable before the experiment. It turns to be that, on one side, specific parameters or strategies authorize the fulfillment of a task but, on an other side, they disable the possibility to come up with the variability of the encountered situations: this may lead to context dependency.

1.2 General postulate and method - previous work

We postulate that a behavior or a capability of an entity is the result of an adaptation process in which the interaction between the entity and its environment obeys an action/reaction law and the entity fulfills an internal constraint at any time. The entity may be modeled as a set of real parameters X_i and the internal constraint may be written as follows:

$$F(X_1, X_2, .., X_n) = 0 \tag{1}$$

During the interaction, discovery of new situations tends to break the internal constraint (action of the environment on the entity). The adaptation process may be seen as the reaction of the entity in order to fulfill the constraint. It implies an internal change of the entity (a lot of X_i values may vary simultaneously). If we suppose that the behavior of the entity only depends on the knowledge of the X_i, the reaction process implies a modification of the behavior of the entity. Hence *the modification of the internal parameters is not task-driven but is due to the fulfillment of an internal consistency law.* The underlying idea suggests that *a task cannot be considered independently of the real robot/real environment interaction.*

If the interaction law is supposed to be known, it is possible to determine *before any experiment* all the reachable internal modifications of the entity. The theoretical framework consists on two separate steps:

- proof that it is possible to fulfill the law for every reachable situation.
- determination of the set of environments for which the internal modifications lead to favorable [1] behaviors of the entity.

A model is considered to be suitable if the first item is fulfilled and the resulting set of environments is compatible with the reality.

In (Davesne, 2004), we show that a very simple interaction law gives rise to reinforcement learning capabilities for a navigation task of an artificial rat. We are currently working on the implementation of a physics-like relation between effectors and sensors of a robot to improve the reliability of the environment recognition process (see (Hazan et al., 2005)).

[1] In the experimenter's point of view.

1.3 Purpose of this paper

In the former paragraph, we have briefly described a general method that both includes a falsifiability property for proposed models and permits to get ride of the necessity to model the environment. The toy example exhibited in (Davesne, 2004) shows that this method may be carried out successfully. However, the results are far from being satisfactory for one reason: the set of appropriate environments is too tight to be met by real environments. This is due to the *purely deterministic tools used to design the constraints on our proposed model* in (Davesne, 2004).

The core issue we are facing may be expressed as follows:

- on one side, our method discards the use of statistical tools to create a model because the proof steps have to be performed *before any experiment in the real environment* (data is supposed to be unavailable when the model is designed).
- on the other side, we need a more flexible mathematical tool than the formulation of equation 1 to express the constraints.

In this paper, we develop a proposal to extend the general formulation of equation 1. Our tool is not supposed to be used exclusively with our methodology but keeps *the notion of equality/difference* which is mandatory in the context of a falsifiable method.

Section 2. explains the underlying idea of our proposal. A simple model is analyzed and the flexibility of our approach comparing to the deterministic and correlation methods. Numerical resolution is then proposed. Section 3. shows how our proposal might permit to handle signal processing. Theoretical results are given.

2. Proposal for an alternative methodology

2.1 Introduction

In this section, we will give a tool example that will illustrate our methodology in the rest of this article. Our model considers a linear relation between two real variables X_1 and X_2. We have chosen this example because our method may be compared easily with:

- *the correlation method* used in statistics that permits to determine the best line followed by a set of gathered couples $\left(x_1^i, x_2^i\right)$.

- *the deterministic method* that fixes the parameters of the straight line before the experiment (see equation 2).

$$X_1 + a.X_2 + b = 0 \tag{2}$$

Where a and b are fixed real values.

We will show in 2.3 that our methodology may compares favorably with both the deterministic and the correlation methods for this simple example.

2.2 Underlying idea

Let consider the following relation:

$$X_1 + a.X_2 + b \in [-L, L] \tag{3}$$

Where X_1, X_2 and L are real values. The solutions of this relation are included into a cylinder (figure 1 (a)). When L tends to 0, the set of solutions may be depicted as a straight line (see equation 2 and the straight line of the figure 1 (a)).

Our way of modeling transposes the relation of equation 2. Let's consider the probability distributions P_{X_1} and P_{X_2} made by gathering all the couples (X_1, X_2) that are solutions of the relation 3. In this case, P_{X_1} and P_{X_2} are uniform distributions. Let be an integer n and an other integer $i \in \{1, .., n\}$. Let x_1^i and x_2^i be two values obtained by the ith random draw over P_{X_1} and P_{X_2}. Finally, consider the ith event E_i defined by the sentence "the couple $\left(x_1^i, x_2^i\right)$ fulfills the relation 4":

$$x_1^i + a.x_2^i + b \in [-L, L] \tag{4}$$

We finally consider a validation statement VS_1, including a integer $k \in \{1, .., n\}$: "at least k of the n events E_i have appeared".

In summary, our transposed model, so called $M_{a,b}$, is composed of:

1. 2 integers n and k
2. a set of n relations 4, including a parameter L
3. a validation statement VS_1

A solution of our model $M_{a,b}$ is a set of n couples $\left(x_1^i, x_2^i\right)$ that fulfills the validation statement VS_1 *included in the model*. Figure 1 (b) gives an example of data that validates the model and figure 1 (c) gives another example of data that falsifies it.

It is not difficult to verify that:

- any set of n couples $\left(x_1^i, x_2^i\right)$ that follows the classical equation 2 is a solution of our model for any n, k and L. Oppositely, a couple $\left(x_1^i, x_2^i\right)$ that is an element of a solution of our model may not follow the equation 2. In particular, a model may be validated even if $k < n$: this means that "outliers" may be authorized.

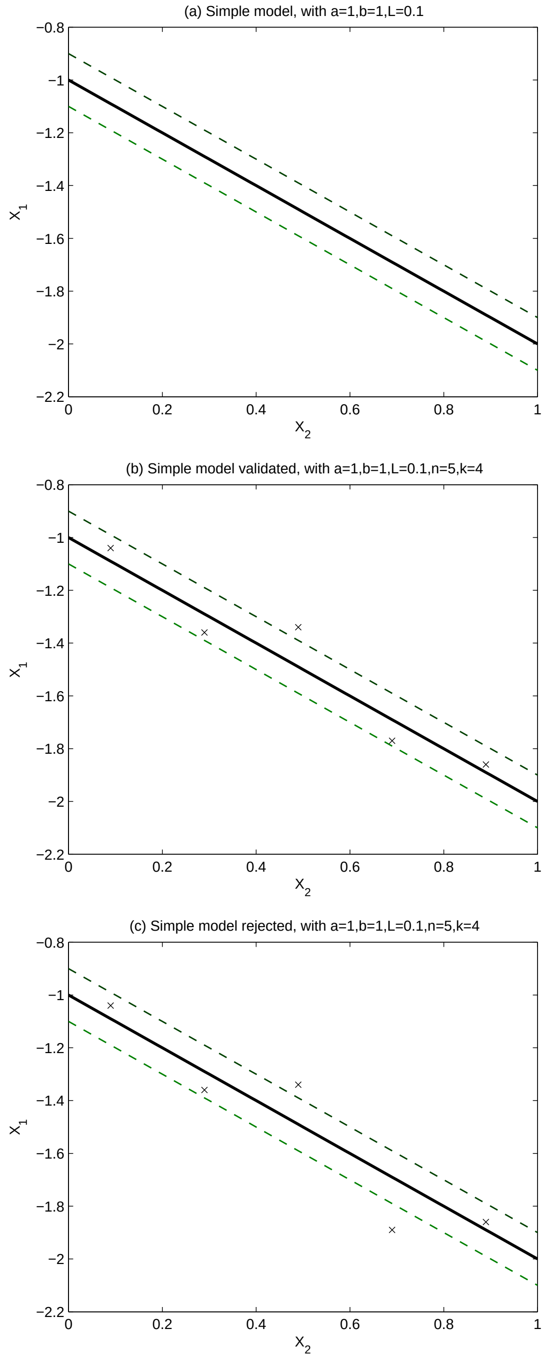

Figure 1: A simple model (a), a validation case (b) and rejection case (c) - gathered data are depicted by crosses.

- in our case, if n grows sufficiently and/or L tends to 0 and k tends to n, the probability $P^{n,k}$ that k events E_i over the n possible events has appeared diminishes and may be as small as we desire [2]. At the limit, and if $k = n$, the couples $\left(x_1^i, x_2^i\right)$ that are part of a solution of our model follow the classical equation 2.

So, our model is more flexible that the one of the standard equation because a larger set of couples $\left(x_1^i, x_2^i\right)$ may be part of a validated solution. The standard equation may be seen as a limit of our model when $k = n$, n tends to infinity and L tends to zero.

2.3 Flexibility and falsifiability

We claim that, in the latter example, our model is falsifiable because it is possible to fix n, k and L in order to diminish $P^{n,k}$ as near as we want of 0. This way of defining falsifiability seems to be appropriate because the standard equation 2, which owns falsifiability properties, is the limit of our model when $P^{n,k}$ tends to 0 and $k = n$.

We will define in 3.4 a quantification of the falsifiability of a model depending on $P^{n,k}$ and show that it is not necessary that $k = n$ or L tends to 0 to make $P^{n,k}$ tends to 0. *So, in our way of modeling, a highly falsifiable model (associated with $P^{n,k}$ near 0) is compatible with unpreciseness and the existence of outliers.* This is a first step to exhibit falsifiable models of a phenomenon that owns variability properties. At this stage, we may say that our methodology compares favorably to the deterministic method because it is more flexible and keeps falsifiability.

But the most outstanding advantage offered by our way of modeling consists on the possibility to both enrich the set of relations within the model as well as keep falsifiability properties. Until now, a and b were considered to be fixed. What if we consider a new model M which is a set of models $M_{a,b}$, each depicted by the following items, when a and b vary ?

1. a varying couple (a, b)
2. n, k
3. a set of n relations 4, including a parameter L
4. a validation statement VS_2

VS_2 that goes with this new model M is specified as follows: "There exists at least one model $M_{a,b}$ that is validated under the validation statement VS_1".

This proposed model is clearly different from the one that *a priori* fixes a and b. It is also reacher: the set of possible solutions consists of all the vectors of couples $\left(x_1^i, x_2^i\right)_{i\in\{1,..,n\}}$ that validate VS_2. It means that they validate at least one $M_{a,b}$. But they may validate simultaneously many $M_{a,b}$ (figure 2 (a)). And what about falsifiability ? Is it possible to diminish $P^{n,k}$ associated with M as near of 0 as we want ? Figure 2 (b) shows that:

- $P^{n,n}$ for model M is greater than $P^{n,n}$ for model $M_{a,b}$.
- But when n increases, $P^{n,n}$ for model M diminishes regularly with the same slope than $M_{a,b}$. So $P^{n,n}$ seems to converge to 0 for $M_{a,b}$.

Hence, M seems [3] to be compatible with our definition of falsifiability.

Another possibility offered by our method is depicted in figure 2 (c): two disconnected sub-models $M_{a,b}$ may be validated simultaneously. It is possible, in this case, because $k \le n/2$.

3. Example of a model analysis that utilizes our methodology

3.1 Introduction

In this section, we study a little further the model M. How is it possible to numerically determine the set of solutions of the validation statement VS_2 ? What is the influence of the parameters n, k and L on the falsifiability of the model and the robustness to noise ? Paragraphs 3.2, 3.3 and 3.4 try to answer those questions.

3.2 Computation of the validation statement output

We are considering model M, which possesses the VS_2 validation statement. The aim of this paragraph is to show how to compute the couples (a, b) that fulfill VS_2. Let be D a set of n couples $\left(x_1^i, x_2^i\right)$ obtained by gathering data during the experiment. We face here an inverse problem in which all the couples (a, b) that fulfill VS_2 must be found. A couple (a, b) is a solution if at least k of the n inequalities following relation 4 are validated given D.

A good way for resolving this problem numerically is to use interval analysis (see (Moore, 1979) for a general presentation). More precisely, we use the SIVIA[4]

[2] In our case, $P^{n,k}$ may be calculated accurately and its limit may be deduced mathematically. We do not include the proof in this article, but French readers may refer to (Davesne, 2002), part II.

[3] We say "seems" because we have not found how to compute $P^{n,k}$ mathematically. $P^{n,k}$ is determined by selecting X_2 with random draws and watch if M is validated or falsified (see 3.2 for the computation of the validation procedure).

[4] SIVIA stands for Set Inverter Via Interval Analysis.

method (see (Jaulin and Walter, 1993)). The advantages of such a method mainly lies on the fact that:

- the set of solutions is approximated by two sets that give with guaranty inner and outer bounds of this set.
- the set of solutions may be parted into non connected sub-sets (see figure 2 (c)).

The general idea for the use of SIVIA algorithm is as follows. Consider two intervals namely I_a and I_b. The relation 4 may be rewritten with the interval formulation:

$$x_1^i + I_a.x_2^i + I_b \subset [-L, L] \tag{5}$$

In the interval analysis formulation, the event E_i which is linked with relation 5

- may appear for all couples $(a, b) \in I_a \times I_b$. In this case, this event is associated with the value $S_i = 1$.
- may never appear for all couples $(a, b) \in I_a \times I_b$. In this case, this event is associated with the value $S_i = 0$.
- may appear for some couples $(a, b) \in I_a \times I_b$ and not appear for other couples. In this case, this event is associated with the interval $S_i = [0, 1]$.

The validation statement VS_2 is reached for all couples $(a, b) \in I_a \times I_b$ if and only if $\sum_{i=1}^{n} S_i \subset [k, n]$. Whereas the validation statement VS_2 is never reached for all couples $(a, b) \in I_a \times I_b$ if and only if $\sum_{i=1}^{n} S_i \cap [k, n] = \emptyset$. Else VS_2 is reached for some couples $(a, b) \in I_a \times I_b$ and not reached for other couples. In this situation, the box $I_a \times I_b$ is divided in two and the algorithm is launched recursively for the two sub-boxes until a stopping criterium is met.

Figure 3 shows the result of the validation process for a perfectly linear input with parameters $n = 50, k = 50, L = 0.1$. The limits of the boxes show the recursive division process of the outer box. At the end of the validation process, it is possible to bound the volume V of the set of solutions (interval $[V_{min}, V_{max}]$).

3.3 *Some examples about the influence of the parameters k and L on the compromise robustness/falsifiability of the model M*

In section 2., we have shown briefly that our methodology permits both flexibility and falsifiability. Falsifiability means that if X_1 and X_2 are chosen by random draws, the probability that VS_2 is validated must be small. At the opposite, we need that if noisy data is gathered there exists some sub-models $M_{a,b}$ of M that are validated.

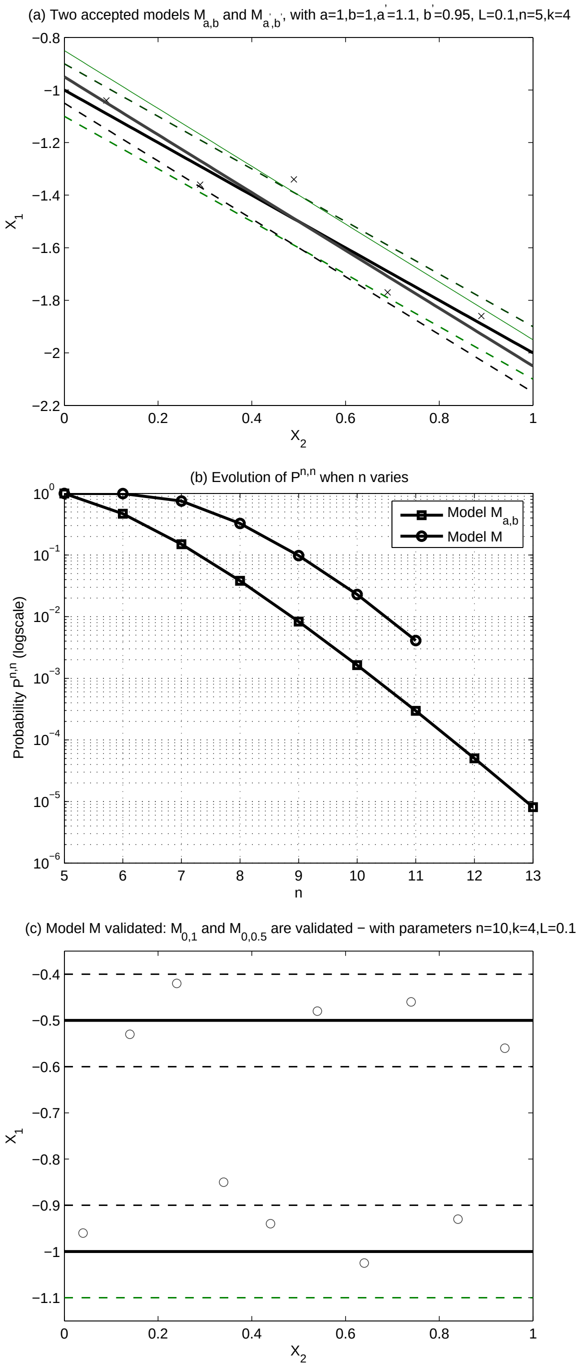

Figure 2: Some properties of model M: many sub-models may be validated (a), M seems to be falsifiable (b), Two disconnected sub-models may be simultaneously validated (c).

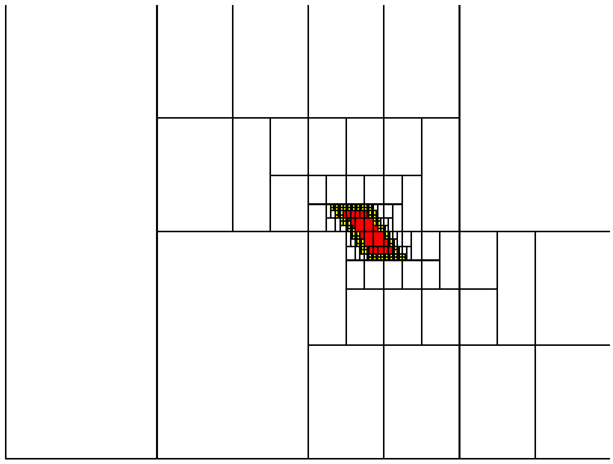

Figure 3: SIVIA algorithm used for the validation process of model M for a perfectly linear input data. X-coordinate: a values, Y-coordinate: b values.

Noise	N_1	N_2	N_3	Fals.
Model 1				
Valid.	100%	0%	0%	0%
Mean $[V_{min}, V_{max}]$	$[7.6e^{-4}, 1.1e^{-3}]$	[0,0]	[0,0]	[0,0]
Model 2				
Valid.	100%	99.9%	7.9%	0%
Mean $[V_{min}, V_{max}]$	$[9.9e^{-4}, 1.1e^{-3}]$	$[7.9e^{-4}, 1.5e^{-3}]$	$[3.2e^{-6}, 2.6e^{-5}]$	[0,0]
Model 3				
Valid.	100%	100%	99.9%	0%
Mean $[V_{min}, V_{max}]$	$[1.5e^{-2}, 1.7e^{-2}]$	$[1.2e^{-2}, 1.4e^{-2}]$	$[7.6e^{-3}, 9.3e^{-3}]$	[0,0]
Model 4				
Valid.	100%	100%	100%	21%
Mean $[V_{min}, V_{max}]$	$[1.1e^{-1}, 1.3e^{-1}]$	$[1.1e^{-1}, 1.3e^{-1}]$	$[1.1e^{-1}, 1.2e^{-1}]$	$[9.6e^{-5}, 2.6e^{-4}]$

Table 1: Validation results. They are obtained after 1000 data gathering iterations.

Four models M are proposed, with different values of k and L. The value of n is fixed to 50. Parameters for model 1 are $k = 50$, $L = 0.03$. Parameters for model 2 are $k = 50$, $L = 0.1$. Parameters for model 3 are $k = 40$, $L = 0.1$. Parameters for model 4 are $k = 25$, $L = 0.1$. These models are exposed to three different kind of data sets which come from a random process:

$$X_1 = 0.2X_2 - 0.5 + N_i \tag{6}$$

Where N_i is a random distribution. $N_i = (1-p_i)G+p_iU$ with G a Gaussian distribution $G\,(0, \sigma_i)$, U a uniform distribution $U\,(0, 1)$ and p_i a real parameter in $[0, 1]$. We have chosen: $p_1 = p_2 = 0$, $p_3 = 0.1$, $\sigma_1 = 1e^{-4}$, $\sigma_2 = 0.02$ and $\sigma_3 = 0.02$.

In order to test the falsifiability property of the four models, we consider a forth data set consisting of random draws over the distribution of X_1.

Results are compiled in table 1. In terms of precision, we may classify the four models, from the more precise to the less: $M_1 > M_2 > M_3 > M_4$. This is deduced from the volume of solutions $[V_{min}, V_{max}]$ obtained with the noiseless signal.

Precision goes with falsifiability: again, we may classify the models from the more falsifiable to the less falsifiable: $M_1 > M_2 > M_3 > M_4$. This result is a consequence of the percent of validation within the 1000 data gathering steps for the different kinds of noises. In particular, the "Fals." column shows us that M_4 is poorly falsifiable (21 % of pure random data sets validate the model) whereas the others seem more falsifiable (no pure random data set validate the three models).

But precision goes against robustness to noise: we may compare the models from the more robust to the less robust: $M_4 > M_3 > M_2 > M_1$.

To summarize this experiment, we may suggest that it is possible to find a compromise between precision and reliability of the signal detection, without discarding falsifiability properties of the model: the first three models have good falsifiability properties, but M_1 is less robust than M_3 whereas M_1 is more precise than M_3. The key issue lies on the determination of appropriate parameters values.

3.4 *Existence of "good" models*

After looking at the former paragraph results, we postulate that it is possible to fix the internal parameters of the model M in order to fulfill two opposite constraints:

- be the more falsifiable as possible
- be the more robust as possible

In terms of probability, we postulate that the following equation may be met:

$$\begin{aligned} \forall \epsilon > 0, \exists n \in N^*, \exists k \in \{0, .., n\}, \exists L \in \Re^{+*}, \\ (P(Fals|pure\ random) > 1 - \epsilon) \wedge \\ (P(Fals|real\ data) < \epsilon) \end{aligned} \tag{7}$$

Where $P(Fals|pure\ random)$ is the probability that the model M is falsified with a data set of random draws over the histogram of all the possible X_1 and $P(Fals|real\ data)$ is the probability that the model M is falsified with a data set coming from gathered data. This is clearly a postulate about the existence of good

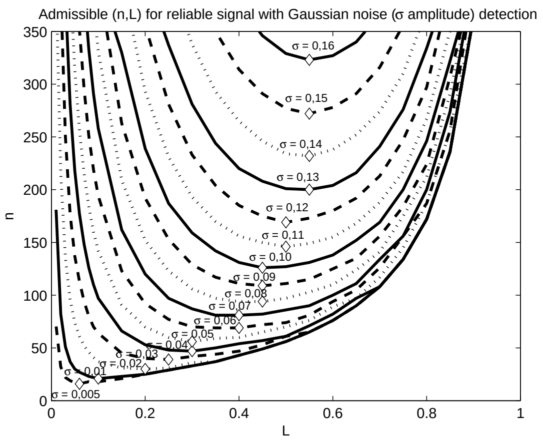

Figure 4: Curves featuring the compromise between ϵ-falsifiability and ϵ-robustness when a signal with Gaussian noise is delivered.

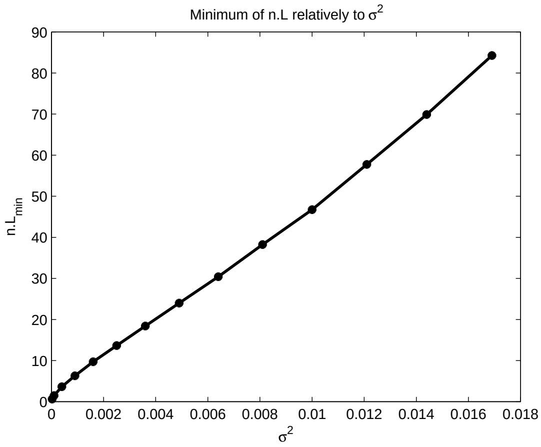

Figure 5: The relation between σ^2 and $n.L_{min}$ is linear.

models. This implies two opposite constraints. The first may be called ϵ-*falsifiability* and the second ϵ-*robustness.*

We have proven that this postulate is true in the restricted case of a model $M_{a,b}$ (the couple (a,b) is fixed) if and only if X_1 is a random process where the density of probability $P(X_1|X_2 = x_2)$ is different from the histogram of all the possible X_1 obtained when X_2 varies: this is a very light condition. Furthermore, given any ϵ, it is possible to calculate the parameters L, n and k that satisfy the two opposite constraints of ϵ-falsifiability and ϵ-robustness.

3.5 Heisenberg inequality

Figure 4 gives the relation between L and n when a signal with Gaussian noise is delivered. In this case, ϵ is fixed to 10^{-15}, so that the unfulfillment of the two constraints is highly unlikely in practice.

Notice that each curve owns a minimum for n. This means that the validation process needs a minimum amount of data to work properly, being both ϵ-falsifiable and ϵ-robust, whatever the value of L is. Besides, if we consider the product $L \times n$, we notice that it reaches a minimum for each different σ. It is an Heisenberg inequality: one cannot be infinitely precise over both X_1 (precision associated with L) and X_2 (precision associated with n) variables. Furthermore, if we find the linear relation between σ^2 and the minimum of the product $n.L_{min}$ for this value of σ (figure 5). We deduce the following inequality:

$$\exists \alpha > 0 \, \forall n \in N, \forall L \in \Re^{+*} \; M_{a,b} \; is \; valid \; \Rightarrow \; n.L \geq \alpha\sigma^2$$

4. Conclusion and further work

In automatics, proof before the experiment is used to validate the model of the interaction between the robot and its environment. Our long-term research focuses on extending this kind of falsifiable method to interactions with *a priori* unknown environments. In order to do this, we have paid attention to the following kind of equations

$$F(X_1, X_2, .., X_n) = 0$$

which represents an invariant (over time) which is commonly used in the validation phase of the building of a model.

Unfortunately, this formulation is too strong if the environment offers a variety of situations which are not predictable.

This paper focuses on a methodology that transposes this equation into a more flexible one, without using statistics because we do not have data before the experiment.

We define a probabilistic approach (and not a statistical one !) to falsifiability (so called ϵ-falsifiability) and robustness (so called ϵ-robustness) which includes models that possess their own validation statement and some internal parameters that fix a compromise between the flexibility (robustness) and falsifiability of the model. For a specific model, we show that it is always possible to fix internal parameters in order to meet simultaneously the two antagonist constraints. We also show that if this double constraint is met, there exists a lower bound to the precision of the model, that is similar to Heisenberg 's uncertainty principle.

The existence proof is suspected to extend to a wider variety of models, but it has not been proved yet.

How a model should be built to meet falsifiability and robustness constraints together, whereas the environment is *a priori* unknown ? A solution we study is to part this problem into two smaller ones:

1. From a model that is known to be (highly) falsifiable, build a class of transformations that is shown to preserve the (high) falsifiability property. This may be done before the experiment.
2. Use the class of transformations in order to obtain the validation of the model at each time during the experiment. This is a learning process.

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