

Licensing Uncertain Patents: Per-Unit Royalty vs. Up-Front Fee*

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Abstract

We examine the implications of uncertainty over patent validity on patentholders' licensing strategies. Two licensing schemes are investigated: the per-unit royalty rate and the up-front fee. We provide conditions under which uncertain patents are licensed in order to avoid patent litigation. It is shown that while it is possible for the patentholder to reap some "extra profit" by selling an uncertain patent under the per-unit royalty scheme, the opportunity to do so does not exist under the up-front fee scheme. We also establish that the relatively high bargaining power the licensor has even when its patent is weak can be reduced if the patentholder cannot refuse to license an unsuccessful challenger or if collective challenges are allowed for. Furthermore we show that the patentee may prefer to license through the per-unit royalty mechanism rather than the fixed fee mechanism if its patent is weak whereas it would have preferred the latter to the former if the patent were strong. This finding gives a new explanation as to why the per-unit royalty scheme may be preferred by a patentholder to the up-front fee scheme.

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1 Introduction

Licensing intellectual property is a key element in the innovation process and its diffusion. A license is a contract by which the owner of intellectual property authorizes another party to use it, in exchange for payment.¹ The properties and virtues of licensing (Kamien, 1992, Scotchmer, 2004) have mainly been analyzed in a framework in which intellectual property rights guarantee perfect protection and give their owners a right to exclude as strong as physical property rights do. This framework does not correspond to what we observe in practice. In the real world patents do not give the right to exclude but rather a more limited right to "*try to exclude*" by asserting the patent in court (Ayres and Klemperer, 1999, Shapiro, 2003, Lemley and Shapiro, 2005). The exclusive right of a patentholder can be enforced only if the court upholds the patent's validity. For this reason, patents are considered as *probabilistic rights* rather than *iron-clad rights*. This paper is devoted to the analysis of licensing patents that are *uncertain*, i.e. patents that have a positive probability to be invalidated by a court if they are challenged.²

Many reasons explain the inherent uncertainty attached to a patent. First, the standard patentability requirements, namely the subject matter, utility, novelty and non-obviousness (or inventive step in Europe) are difficult to assess by patent office examiners. Legal uncertainty over the patentability standards is especially pervasive in the new patenting subject matters for which the prior art is rather scarce, like software or business methods. Moreover, the claims granted by the patent office are supposed to delineate the patent scope, but their *ex post* validation depends on the judicial doctrine adopted by the court, and it may be difficult for a patentholder and a potential infringer to know exactly what the patent protects. Second, the resources devoted to the patentability standards review by the patent office are in general insufficient to allow an adequate review of each patent application.³ Many innovations are granted

¹According to some surveys (Taylor and Silberstone, 1973, Rostoker, 1984, and Anand and Khanna, 2000), the per-unit royalty rate and the fixed fee mechanism are the most frequent licensing schemes.

²Uncertainty does not necessarily imply asymmetric information or different beliefs about patent's validity among involved parties. Uncertainty may occur even if the parties share the same beliefs on the patent validity. For a different view, see Bebchuk (1984), Reinganum and Wilde (1986), Meurer (1989), Hylton (2002).

³The average time spent by an examiner on each patent is about 15-20 hours in the USPTO (Jaffe & Lerner, 2004) and around 30 hours in the EPO. The gap between the massive growth of patent applications and the insufficient resources at the patent office creates a "*vicious circle*" (Caillaud and Duchêne, 2005). Incentives to file "bad applications" increase the patent office overload, and a larger

patent protection even though they do not meet patentability standards. This results in many "weak patents", i.e. patents that have a high probability to be invalidated by a court if they are challenged. Finally, it has been argued that incentives inside the patent offices make it easier and more desirable for examiners to grant patents rather than reject them (Farrell and Merges, 2004, IDEI report, 2006).

The patent quality problem raises many concerns particularly in the US.⁴ We may ask, first: are bad quality patents harmful or not? Lemley (2001) claims that it is reasonably efficient to maintain a low standard of patent examination, in accordance with the "*rational ignorance principle*". Specifically, he argues that the cost of a thorough examination for each application would be prohibitive while inducing only a small benefit. Firstly, the majority of patents turn out to have insignificant market value implying that the social cost of granting them is small even if they are invalid. Secondly, if a weak but profitable patent is granted, some market players will probably bring the case before a court to settle the validity issue, if the patent is licensed at too high a price.

These arguments have attracted much criticism. First, there are many reasons to think that individual incentives to challenge a weak patent are rather low. A patentee generally cares more about winning than a potential infringer does, since by winning against a single challenger, a patentee establishes the validity of the patent against many other potential infringers. By contrast, when infringers are competitors, a successful challenge obtained by one of them benefits all (Farrell and Merges, 2004, Lemley and Shapiro, 2005).⁵ Consequently, according to the free-riding argument, the individual incentives to challenge a patent validity are weak. Moreover, according to the so-called pass-through argument, licensees are induced to accept a high per-unit royalty rate when they can decide to pass-on the royalty to their customers.⁶ Finally, an unsuccessful attacker may be in jeopardy or even evicted from the market once deprived from the

overload leads to further deterioration of the examination process.

⁴Europe is also concerned by the patent quality problem even though the post-grant opposition at the EPO alleviates it (see Graham *et al.* 2003). The European situation in terms of patent quality is analyzed in Guellec and von Pottelsberghe de la Potterie (2007) and the IDEI report (2006).

⁵Following the *Blonder-Tongue* decision (1971), it became clear that "the attacker is not able to exclude others from appropriating the benefit of its successful patent attack", *Blonder-Tongue Labs., Inc. v. Univ. of Illinois Found*, 402, U.S. 313, 350 (1971).

⁶"When multiple infringers compete in a product market, royalties are often passed-through, at least in part, to consumers downstream. The pass-through will be stronger the more competitive the product market, the more symmetric the royalties, the more elastic the industry supply curve, and the less elastic the industry demand curve" (Farrell and Merges, 2004).

new technology, or required to pay a higher price than the licensees who have accepted the licensing contract.

All these arguments suggest that individual incentives to challenge a patent may be rather low. The probabilistic nature of patent protection and the low individual incentives to challenge a patent may thus strengthen the market power of the licensor. The owner of a probabilistic right and a potential user will come to a licensing agreement as a private settlement to avoid the uncertainty of a court resolution. An agreement benefits the holder of a weak patent while litigation and possible invalidation by a court would deprive the licensor from any licensing revenue. However the licensing contract will be accepted by the licensee only if its expected profit is at least as large as when the patent validity is challenged. Therefore licensing an uncertain patent under the shadow of patent litigation raises an interesting trade-off. We show in this paper that different factors explain the issue of this trade-off: i) the nature of the licensing scheme (per-unit royalty vs. up-front fee); ii) the patent's strength measured by the probability that it will be upheld; iii) the importance of the innovation; iv) the type of commitment when dealing with an unsuccessful challenger; v) the possibility to engage in collective negotiations of the licensing contract; vi) some market structure variables such as the size of the downstream industry and the intensity of market competition.⁷

The literature on licensing and the properties of the different licensing mechanisms has extensively examined the case of perfect patent protection. Based largely on previous works by Arrow (1962), Katz and Shapiro (1985, 1986), Kamien and Tauman (1984, 1986), Kamien *et al.* (1992), the survey by Kamien (1992) summarizes the major results, especially by comparing the patentholder's profits under different licensing schemes. The patentee's profits are highest when licensing is made through an auction, in which the patentee announces the number of licenses on offer and the latter accrue to the highest bidders. The per-unit royalty scheme and the up-front fee mechanism have been set against each other. While the earlier literature claimed that a per unit royalty always generates lower profits than a fixed fee, regardless of the industry size and the magnitude of the innovation (Kamien and Tauman, 1984 and 1986), a more recent work has shown that when the number of firms in the industry is sufficiently high, the innovator's payoff is higher with royalty licensing than with a fixed fee or an auction (Sen, 2005). Moreover, some licensing methods induce full diffusion, while

⁷Since a non-licensee suffers a negative externality when a competitor becomes a licensee, more intense competition in the product market increases the licensor's market power.

others lead to only partial diffusion of the innovation: the number of licensees depends on the licensing method and the magnitude of the cost reduction. In a more recent contribution, Sen and Tauman (2007) generalize these findings by allowing the optimal combination of an auction and a per-unit royalty in situations where the innovator may be either an outsider or an insider in the downstream industry.⁸

Let us now consider how licensing is affected when a patent is a probabilistic right. Rough intuition suggests that licensing an uncertain patent in the shadow of patent litigation leads to a license price which is proportional to the patent strength. This intuition is not always correct for the following reason: when imperfect competition occurs in the downstream industry, the free riding argument mentioned above lowers the individual incentives to challenge the patent's validity and this benefits the patentholder. Farrell and Shapiro (2007) establishes two important properties for a minor cost reducing innovation: (i) For weak patents, the royalty rate is as high as if the patent were certain: it is equal to the magnitude of the cost reduction allowed by the innovation; (ii) Whatever the patent strength, the royalty rate obtained in the shadow of patent litigation exceeds the expected value of the royalty resulting from the patent challenge. These strong properties have been obtained by considering a two-part licensing contract mechanism combining a per-unit royalty and a fixed fee, allowing for instance a high royalty rate to be compensated by a negative transfer (i.e. an up-front fee paid by the licensor to the licensee).⁹ Two restrictive assumptions have been used in Farrell and Shapiro to obtain these results: first, they restrict their analysis to small process innovations, i.e. innovations leading to a small cost reduction; second, they assume that the best patentholder's licensing strategy is to sell a license to all firms in the downstream industry, rather than to restrict the license supply to some firms,

⁸ Another burgeoning literature explores the consequences of informational asymmetries on licensing. Aoki and Hu (1996) examines how the choice between strategic licensing and litigating is affected by the levels of the litigation costs and their allocation between the plaintiff and the defendant. Brocas (2006) identifies two informational asymmetries: the moral hazard due to the inobservability of the innovator's R&D effort, and the adverse selection due to the private value of holding a license. Macho-Stadler *et al.* (1996) introduces know-how transfer and shows that the patentholder prefers contracts based on per-unit royalties rather than fixed fee payments. Other contributions, emphasizing either risk aversion (Bousquet *et al.*, 1998), strategic delegation (Saracho, 2002), strategic complementarity (Muto, 1993, Poddar and Sinha, 2004), or the size of the oligopoly market (Sen, 2005) reach the same conclusion stating the superiority of the royalty licensing scheme.

⁹ Farrell and Shapiro also investigate a two-part tariff in which the fixed fee is constrained to be non negative. However, in this case, their main result holds only under the two additional restrictions that the magnitude of the cost reduction innovation is small and all downstream firms accept the licensing contract at equilibrium.

leaving it to others to refuse and possibly initiate a litigation process.

In this paper we assess the robustness of these results by avoiding these assumptions, and separately investigating two of the most common licensing mechanisms, namely the per-unit royalty rate and the up-front fee. We analyze the properties of these mechanisms, letting the licensor choose the number of licensees whatever the innovation size. For both types of licensing schemes, we develop a three-stage game in which the patentholder, acting as a Stackelberg leader, determines either a royalty rate or a fixed fee at the first stage. At the second stage, each firm independently decides whether to accept the licensing contract. If it does not, it challenges the patent validity. If the patent is found valid, the unsuccessful challenger is bound to use the old technology. If the patent is found invalid, all the firms in the oligopolistic industry have free access to the technology. In the last stage, licensees and non-licensees compete in the product market. Different variants of this basic model are examined in this paper, by introducing the possibility of a collective challenge or by allowing renegotiation between the patentholder and an unsuccessful challenger.

Our paper departs from Farrell and Shapiro (2007) in several ways. First, unlike Farrell and Shapiro who focus on a single licensing scheme combining a per-unit royalty and a fixed fee, we separately investigate these two schemes; second, while they only consider the case where the cost reduction is small, we investigate the consequences of any cost reduction; third, we relax the crucial assumption of their paper stating that the patentholder licenses every firm in the industry, by endogeneizing the number of licensees. We show below that this endogeneization has important consequences, particularly when comparing the properties of the per-unit royalty rate and the up-front fee licensing schemes. We also challenge the assumption that an unsuccessful challenger is offered a license at a price that captures its entire surplus.

We contribute to the literature on licensing uncertain patents on five points. First, we show that while it is generally possible for the patentholder to reap some "extra profit" by selling an uncertain patent under the per-unit royalty regime, the opportunity to do so under the up-front fee regime disappears. This is due to the fact that the patentee's profit under a fixed fee regime is always equal to the expected profit in case of litigation. Second, we show in the case of a linear demand under Cournot competition that the patentee's profits may be higher with a per-unit royalty than with a fixed fee. This result - which confirms Sen (2005) - rests on a completely different argument based on patent uncertainty. Third, for the per-unit royalty regime, we obtain necessary and

sufficient conditions under which the royalty rate resulting from a collective challenge is lower than the expected royalty from an individual challenge. Fourth, we show that there exist situations in which the per-unit royalty for a weak patent is below the expected royalty in case of litigation. The latter result is obtained under general assumptions on the profit functions and is confirmed when post-trial renegotiation is introduced. Finally, we show that the results obtained with perfect patents also hold when patents are uncertain but strong: in this case, litigation never occurs.

The paper is organized as follows. Section 2 examines the per-unit royalty scheme. It starts with the derivation of the maximum value of the per-unit royalty that deters any litigation. This value is compared to two benchmarks: i/ the expected value of the royalty in case of litigation; ii/ the royalty that would prevail under collective challenge of the patent validity. The patentholder's optimal royalty rate and its licensing revenues are then determined. The conditions under which litigation is avoided at the subgame perfect equilibrium are established. Section 3 analyzes the fixed fee licensing scheme. It derives the demand for licenses and the licensing revenues as a function of the up-front fee. These revenues are then compared to the expected revenues in case of litigation. In Section 4, the two licensing mechanisms are compared from the licensor's perspective. Section 5 concludes by summarizing the results, putting them in an economic policy perspective, and suggesting new research directions.

2 Royalty licensing schemes

In this section, we examine licensing schemes involving a pure royalty rate. More precisely, we seek to determine the subgame perfect Nash equilibria of the following three-stage game:

At the first stage the patentholder proposes a licensing contract by which a licensee can use the new technology to reduce its marginal production cost from c to $c - \epsilon$ against the payment of a per-unit royalty rate r .

At the second stage n firms in a downstream industry simultaneously and independently decide whether or not to purchase a license at the price r . If a firm does not accept the license offer, it can challenge the patent's validity before a court. The outcome of such a trial is uncertain: with probability θ the patent is upheld by the court and with probability $1 - \theta$ it is invalidated. The parameter θ measures the patent's strength. If the patent is upheld, then a firm that does not purchase the license uses

the old technology¹⁰ hence producing at marginal cost c whereas those who accepted the license offer use the new technology and pay the royalty rate r to the patentholder, having thus an effective marginal cost equal to $c - \epsilon + r$. If the patent is invalidated, all the firms, including those who accepted the offer can use for free the new technology and their common marginal cost is $c - \epsilon$.

At the third stage the downstream firms compete in an oligopolistic product market. The kind of competition that occurs is not specified. It is simply assumed that there exists a unique Nash equilibrium in the competition game between the members of the oligopoly for any cost structure of the downstream firms.

We sum-up the outcome of the third stage by denoting $\pi(x, y)$ the equilibrium profit function of an active firm producing with marginal cost x while its $(n - 1)$ competitors produce with marginal cost y . The case where $\pi(x, y) = 0$ is not excluded.

We assume the following general properties that are satisfied by a large class of profit functions (See Boone, 2001, and Amir and Wooders, 2000).

A1. The equilibrium profit function $\pi(x, y)$ is continuous in both its arguments over $[c - \epsilon, c] \times [c - \epsilon, c]$ and twice differentiable in both its arguments over the subset of $[c - \epsilon, c] \times [c - \epsilon, c]$ in which $\pi(x, y) > 0$.

A2. The equilibrium profit of a firm is decreasing in its own cost : if $\pi(x, y) > 0$ then $\pi_1(x, y) < 0$ and if $\pi(x, y) = 0$ then $\pi(x', y) = 0$ for any $x' \geq x$.

A3. The equilibrium profit of a firm is increasing in its competitors' costs : if $\pi(x, y) > 0$ then $\pi_2(x, y) > 0$ and if $\pi(x, y) = 0$ then $\pi(x, y') = 0$ for any $y' \leq y$.

A4. In a symmetric oligopoly, an identical drop in all firms' costs raises each firm's profit: $\pi_1(x, x) + \pi_2(x, x) < 0$.

Given A2 and A3, A4 means that own cost effects dominate rival's cost effects.

The subgame perfect Nash equilibria of the game are obtained as usual by backward induction.

¹⁰This assumption may seem quite strong but recall that IP laws do not compel patentholders to license others, particularly those who challenge the validity of a patent or sue the patentholder for infringement of their own patents. To illustrate, when Intergraph (a company producing graphic work stations) sued Intel (micro-processors) for infringement of its Central Processing Unit patent, Intel countered by removing Intergraph from its list of customers and threatening to discontinue the sale of Intel microprocessors to Intergraph (See Encaoua and Hollander, 2002). We relax later this assumption by introducing renegotiation between the unsuccessful challenger and the patentholder.

2.1 Accepting or not the patentholder's offer: second stage

We start by determining the set of royalty rates r such that a Nash equilibrium leads to an outcome in which every downstream firm accepts such a royalty. This occurs if and only if no single firm has an incentive to deviate by refusing to buy a license at this rate and challenging the patent's validity. Since an unsuccessful challenger produces at cost c while its competitors that have accepted the licensing contract produce at cost $c - \epsilon + r$, a per-unit royalty rate r that is accepted by every firm at equilibrium satisfies the condition:

$$\pi(c - \epsilon + r, c - \epsilon + r) \geq \theta\pi(c, c - \epsilon + r) + (1 - \theta)\pi(c - \epsilon, c - \epsilon) \quad (1)$$

The following lemma characterizes the set of royalty rates that satisfy inequality (1).

Lemma 1 *A royalty rate r is accepted by all firms if and only if $r \leq r(\theta)$ where $r(\theta) \in [0, \epsilon]$ is the unique solution in r to the equation $\pi(c - \epsilon + r, c - \epsilon + r) = \theta\pi(c, c - \epsilon + r) + (1 - \theta)\pi(c - \epsilon, c - \epsilon)$*

Proof. See Appendix. ■

When analyzing the maximum value of the per-unit royalty such that all firms accept the contract, two cases emerge.

Case 1: the magnitude ϵ of the cost reduction is such that $\pi(c, c - \epsilon) = 0$.

This case occurs for a sufficiently large innovation (high value of ϵ) or for a sufficiently intense competition (e.g. large number n of downstream firms). In such a case, according to assumptions A1 and A3, there exists a threshold $\hat{r} \in [0, \epsilon]$ such that $\pi(c, c - \epsilon + r) = 0$ if $r \leq \hat{r}$ and $\pi(c, c - \epsilon + r) > 0$ if $r > \hat{r}$. An unsuccessful challenger will get zero profit if the royalty rate is below the threshold ($r \leq \hat{r}$), and a positive profit if the royalty rate is above the threshold ($r > \hat{r}$).

First consider a contract involving a royalty rate $r \leq \hat{r}$. According to condition (1), it will induce a Nash equilibrium where all the firms will accept the licensing contract if and only if:

$$\pi(c - \epsilon + r, c - \epsilon + r) \geq (1 - \theta)\pi(c - \epsilon, c - \epsilon) \quad (2)$$

Denote $r_2(\theta)$ the solution in r to the equation $\pi(c - \epsilon + r, c - \epsilon + r) = (1 - \theta)\pi(c - \epsilon, c - \epsilon)$. It is easy to show that inequality (2) is equivalent to $r \leq r_2(\theta)$.

Second consider a contract involving a royalty rate $r > \hat{r}$. It will be accepted by all firms if and only if inequality (1) is satisfied with $\pi(c, c - \epsilon + r) > 0$. Denote $r_1(\theta)$ the solution to the equation in r $\pi(c - \epsilon + r, c - \epsilon + r) = \theta\pi(c, c - \epsilon + r) + (1 - \theta)\pi(c - \epsilon, c - \epsilon)$ when this solution is greater than \hat{r} . Denote by $\hat{\theta} \in [0, 1]$ the unique solution to the equation $r_2(\theta) = \hat{r}$. The existence of $\hat{\theta} \in [0, 1]$ can be derived from the two following properties: i/ $r_2(0) = 0 \leq \hat{r}$ and $r_2(1) = \epsilon \geq \hat{r}$; ii/ $r_2(\cdot)$ is continuous over $[0, \epsilon]$. Its uniqueness follows from the strict monotonicity of $r_2(\cdot)$. Note that $r_1(\hat{\theta}) = r_2(\hat{\theta}) = \hat{r}$.

Summing-up these possibilities, a royalty rate r will be accepted by all firms if the following condition holds:

$$r \leq \min(\hat{r}, r_2(\theta)) \text{ or } \hat{r} \leq r \leq r_1(\theta)$$

Note that if $\theta \leq \hat{\theta}$, the previous condition is equivalent to $r \leq r_2(\theta)$, and if $\theta > \hat{\theta}$, it is equivalent to $r \leq r_1(\theta)$. This means that the maximum royalty rate inducing a Nash equilibrium where all firms accept the license offer is given by:

$$r(\theta) = \begin{cases} r_2(\theta) & \text{if } \theta \leq \hat{\theta} \\ r_1(\theta) & \text{if } \theta > \hat{\theta} \end{cases}$$

Case 2: the magnitude ϵ of the cost reduction is such that $\pi(c, c - \epsilon) > 0$

In this case, whatever the royalty rate fixed by the patentholder, the profit of a firm challenging the patent's validity remains positive: $\pi(c, c - \epsilon + r) \geq \pi(c, c - \epsilon) > 0$. This implies that the equilibrium value $r(\theta)$ of the per-unit royalty that makes all firms accept the contract is equal to $r_1(\theta)$. In this case $r(\theta) = r_1(\theta)$ for all $\theta \in [0, 1]$.

2.1.1 Royalty rate benchmarks

Having characterized the per-unit royalty level $r(\theta)$, it is interesting to compare it to two benchmarks: i/ the expected value of the maximum royalty rate in case of litigation denoted $r^e(\theta)$; ii/ the royalty rate deterring a collective challenge denoted $r^c(\theta)$.

First benchmark: the expected value of the maximum royalty rate in case of litigation.

This benchmark is easily computed: with probability θ the patent is upheld by the court, hence becoming an ironclad right that can be licensed at a maximum per-unit royalty $r(1) = \epsilon$, and with probability $1 - \theta$ the patent is invalidated and the firms can use it for

free, leaving the patentholder with a royalty $r(0) = 0$. Therefore, the expected value of the maximum royalty rate in case of litigation is equal to $r^e(\theta) = \theta r(1) + (1 - \theta)r(0) = \theta\epsilon$. The expected value of the maximum royalty in case of litigation is thus proportional to the patent's strength θ . This benchmark is interpreted in Farrell and Shapiro (2007) as the *ex ante* value of the per-unit royalty rate that an applicant of a process innovation reducing the cost by ϵ can expect when the patent has a probability θ to be granted by the patent office.

Second benchmark: the royalty rate deterring a collective challenge.

Suppose that at stage 2 the firms cooperatively agree on whether buying the license or challenging all together the patent's validity. In this case, the firms will cooperatively accept a licensing contract involving a royalty rate r if and only if:

$$\pi(c - \epsilon + r, c - \epsilon + r) \geq \theta\pi(c, c) + (1 - \theta)\pi(c - \epsilon, c - \epsilon)$$

The function w defined by $w(r) = \pi(c - \epsilon + r, c - \epsilon + r) - \theta\pi(c, c) - (1 - \theta)\pi(c - \epsilon, c - \epsilon)$ is continuous, strictly decreasing (A3) and satisfies the conditions $w(0) \geq 0$ and $w(\epsilon) \leq 0$. Hence there exists a unique solution $r^c(\theta) \in [0, \epsilon]$ to the equation $w(r) = 0$, and the inequality $w(r) \geq 0$ is equivalent to $r \leq r^c(\theta)$. All firms cooperatively accept to buy a license at a royalty rate r if and only if $r \leq r^c(\theta)$. Some properties of this second benchmark $r^c(\theta)$ are easily obtained.

Proposition 2 *The function $r^c(\theta)$ is increasing. (i) It is concave over $[0, 1]$ if and only if the function $x \rightarrow \pi(x, x)$ is concave over $[c - \epsilon, c]$. In this case $r^c(\theta) \geq r^e(\theta) = \theta\epsilon$ for all $\theta \in [0, 1]$; (ii) It is convex over $[0, 1]$ if and only if the function $x \rightarrow \pi(x, x)$ is convex over $[c - \epsilon, c]$. In this case $r^c(\theta) \leq r^e(\theta) = \theta\epsilon$ for all $\theta \in [0, 1]$.*

Proof. See Appendix. ■

The convexity of the equilibrium profit function $\pi(x, x)$ is satisfied for different demand specifications including for instance a linear demand and a Cournot behavior, while it is difficult, if not impossible, to find a specification of the demand function leading to a concave equilibrium profit function $\pi(x, x)$.¹¹ This suggests that the inequality $r^c(\theta) \leq \theta\epsilon$ is more likely satisfied than the reverse one. Thus the royalty rate deterring

¹¹With a linear demand function $Q = a - p$, a marginal cost x , and an oligopoly of n firms, the Cournot profit equilibrium is $\pi(x, x) = \frac{(a-x)^2}{(n+1)^2}$ which is a convex function of x .

a collective challenge ($r^c(\theta)$) is likely to be lower than the expected royalty rate in case of an individual challenge ($r^e(\theta)$).

2.1.2 Comparison of $r(\theta)$ to $r^e(\theta) = \theta\epsilon$

Analyzing the shape of the function $\theta \rightarrow r(\theta)$ allows to compare the per-unit royalty rate $r(\theta)$ that deters individual challenge to the benchmark $r^e(\theta) = \theta\epsilon$ which represents the expected royalty rate in case of individual litigation.

Recall first that when the innovation ϵ is such that $\pi(c, c - \epsilon) = 0$, we have $r(\theta) = r_2(\theta)$ over the interval $[0, \hat{\theta}]$. It is easy to show that $r_2(\theta)$ is increasing in θ . Indeed, differentiating with respect to θ the equation $\pi(c - \epsilon + r_2(\theta), c - \epsilon + r_2(\theta)) = (1 - \theta)\pi(c - \epsilon, c - \epsilon)$ we get:

$$r_2'(\theta) = \frac{-\pi(c - \epsilon, c - \epsilon)}{(\pi_1 + \pi_2)(c - \epsilon + r_2(\theta), c - \epsilon + r_2(\theta))}$$

which implies that $r_2'(\theta) > 0$ since $\pi_1 + \pi_2 < 0$ (A4). Therefore $r_2(\theta)$ increases with the patent strength θ .

Furthermore, we can derive some properties about the monotonicity of $r_2'(\theta)$ and consequently about the convexity or concavity of $r_2(\theta)$. Note that $(\pi_1 + \pi_2)(c - \epsilon + r_2(\theta), c - \epsilon + r_2(\theta))$ is increasing (resp. decreasing) in θ if $(\pi_1 + \pi_2)(x, x)$ is increasing (resp. decreasing) in x , which is equivalent to $\pi(x, x)$ convex (resp. concave) in x . Hence $r_2(\theta)$ is convex (resp. concave) over the interval $[0, \hat{\theta}]$ if $\pi(x, x)$ is convex (resp. concave) in x .

We can also compare $r_2'(0)$ to ϵ . This comparison matters when comparing $r(\theta) = r_2(\theta)$ to the benchmark $r^e(\theta) = \theta\epsilon$ for small values of θ (weak patent). Indeed, if $r_2'(0) > \epsilon$ (resp. $r_2'(0) < \epsilon$) then for θ sufficiently small we will have $r_2(\theta) > \theta\epsilon$ (resp. $r_2(\theta) < \theta\epsilon$).

Since $r_2(0) = 0$, we have:

$$r_2'(0) = \frac{-\pi(c - \epsilon, c - \epsilon)}{(\pi_1 + \pi_2)(c - \epsilon, c - \epsilon)}$$

Therefore,

$$r_2'(0) > \epsilon \iff \frac{-\pi(c - \epsilon, c - \epsilon)}{\epsilon(\pi_1 + \pi_2)(c - \epsilon, c - \epsilon)} > 1$$

Denoting $\lambda(\epsilon) = \pi(c - \epsilon, c - \epsilon)$, we obtain:

$$r_2'(0) > \epsilon \iff \frac{\lambda(\epsilon)}{\epsilon\lambda'(\epsilon)} > 1 \iff \eta(\epsilon) < 1$$

where $\eta(\epsilon) = \frac{\epsilon\lambda'(\epsilon)}{\lambda(\epsilon)} = \frac{\epsilon n\lambda'(\epsilon)}{n\lambda(\epsilon)}$ is the elasticity of the industry profits with respect to a cost reduction ϵ in the marginal cost of all the industry's firms. These results lead to the following proposition:

Proposition 3 *If $\pi(x, x)$ is concave in x over $[c - \epsilon, c]$ then $r_2(\theta)$ is concave over $[0, \hat{\theta}]$ and $r(\theta) = r_2(\theta) \geq \theta\epsilon$ for any $\theta \in [0, \hat{\theta}]$*

If $\pi(x, x)$ is convex in x over $[c - \epsilon, c]$ then $r(\theta) = r_2(\theta)$ is convex over $[0, \hat{\theta}]$ and the location of $r_2(\theta)$ with respect to $\theta\epsilon$ depends on $\eta(\epsilon)$:

- *if $\eta(\epsilon) < 1$ then $r_2(\theta) \geq \theta\epsilon$ for any $\theta \in [0, \hat{\theta}]$*
- *if $\eta(\epsilon) > 1$ then there exists $\check{\theta}$ such that $r_2(\theta) < \theta\epsilon$ for $0 < \theta < \check{\theta}$ and $r_2(\theta) \geq \theta\epsilon$ for $\theta \geq \check{\theta}$.*

Since the equilibrium profit function $\pi(x, x)$ is more likely to be convex than concave in x , the per-unit royalty rate $r_2(\theta)$ is more likely to have a convex shape for small values of θ . In this case, the maximum royalty rate $r_2(\theta)$ that deters individual challenge may be lower than the expected royalty $r^e(\theta) = \theta\epsilon$ for weak patents (small value of θ) if the industry profits are elastic with respect to ϵ (i.e. $\eta(\epsilon) > 1$). We illustrate this possibility in the following example.

Example: *Consider a Cournot oligopoly with a constant-elasticity demand function: $D(p) = p^{-\frac{1}{\beta}}$ where $\beta < 1$. It is straightforward to show that the equilibrium profit of a firm in a symmetric oligopoly with marginal cost c is given by: $\pi(c, c) = \frac{\beta}{n-\beta} \left(\frac{n-\beta}{n^{1-\beta}}\right)^{\frac{1}{\beta}} c^{1-\frac{1}{\beta}}$ which is convex in c when $\beta < 1$. The elasticity of the industry profits with respect to ϵ is given by $\eta(\epsilon) = \frac{\epsilon}{c-\epsilon} \left(\frac{1}{\beta} - 1\right)$. Therefore, $\eta(\epsilon) < 1 \iff \epsilon < \beta c$. Hence for "major innovations", i.e. innovations such that $\beta c < \epsilon < c$, the royalty rate $r_2(\theta)$ is lower than the benchmark level $r^e(\theta) = \theta\epsilon$ for "weak patents" ($\theta < \check{\theta}$).*

Let us now turn to the properties of $r_1(\theta)$. Recall that $r(\theta) = r_1(\theta)$ over $[\hat{\theta}, 1]$ if $\pi(c, c - \epsilon) = 0$ and $r(\theta) = r_1(\theta)$ over $[0, 1]$ if $\pi(c, c - \epsilon) > 0$. By definition of $r_1(\theta)$, we have:

$$\pi(c - \epsilon + r_1(\theta), c - \epsilon + r_1(\theta)) = \theta\pi(c, c - \epsilon + r_1(\theta)) + (1 - \theta)\pi(c - \epsilon, c - \epsilon)$$

Differentiating this equation with respect to θ we get:

$$r_1'(\theta) = \frac{\pi(c, c - \epsilon + r_1(\theta)) - \pi(c - \epsilon, c - \epsilon)}{(\pi_1 + \pi_2)(c - \epsilon + r_1(\theta), c - \epsilon + r_1(\theta)) - \theta\pi_2(c, c - \epsilon + r_1(\theta))} \quad (3)$$

We have $\pi(c, c - \epsilon + r_1(\theta)) \leq \pi(c - \epsilon + r_1(\theta), c - \epsilon + r_1(\theta)) < \pi(c - \epsilon, c - \epsilon)$. The first inequality follows from $r_1(\theta) \leq \epsilon$ and the second one from $\pi_1(x, x) + \pi_2(x, x) < 0$. Hence the numerator in (3) is negative. The denominator is negative as well since $\pi_1(x, x) + \pi_2(x, x) < 0$ and $\pi_2(x, y) > 0$. Consequently $r_1'(\theta) > 0$, that is $r_1(\theta)$ is increasing in the patent strength θ .

We can derive the position of $r_1(\theta)$ relative to $r^e(\theta) = \theta\epsilon$ for θ sufficiently close to 1, i.e. sufficiently strong patents, from the comparison of $r_1'(1)$ to ϵ . Note that $r_1'(1) = \frac{\pi(c, c) - \pi(c - \epsilon, c - \epsilon)}{\pi_1(c, c)}$. Therefore if the slope $\frac{\pi(c, c) - \pi(c - \epsilon, c - \epsilon)}{\epsilon}$ is strictly greater (resp. smaller) than the negative partial derivative $\pi_1(c, c)$ than $r_1(\theta) < \theta\epsilon$ (resp. $r_1(\theta) > \theta\epsilon$) for sufficiently strong patents.

2.1.3 Comparison of $r(\theta)$ to $r^c(\theta)$

The effect of free-riding is measured by the difference $r(\theta) - r^c(\theta)$. It is easy to see that this difference is positive. This follows from the fact that $\pi(c, c) \geq \pi(c, c - \epsilon + r)$ for any $r \geq 0$. This will in particular be true for $r = r^c(\theta)$. Since $\pi(c - \epsilon + r^c(\theta), c - \epsilon + r^c(\theta)) = \theta\pi(c, c) + (1 - \theta)\pi(c - \epsilon, c - \epsilon)$ we obtain that $\pi(c - \epsilon + r^c(\theta), c - \epsilon + r^c(\theta)) \geq \theta\pi(c, c - \epsilon + r^c(\theta)) + (1 - \theta)\pi(c - \epsilon, c - \epsilon)$. This last inequality implies that a royalty rate $r = r^c(\theta)$ will be non cooperatively accepted by all firms if proposed by the patentholder. Therefore $r^c(\theta) \leq r(\theta)$ for all $\theta \in [0, 1]$. The public good nature of the challenge implies that the maximum royalty rate that the patentholder can obtain in the licensing of an uncertain patent is higher under individual challenge ($r(\theta)$) than under collective challenge ($r^c(\theta)$).

Moreover, if $\pi(x, x)$ is convex in x over $[c - \epsilon, c]$, then $r^c(\theta) \leq \theta\epsilon$ for all $\theta \in [0, 1]$ whereas $r(\theta)$ may be above or below than $\theta\epsilon$. For instance if $\eta(\epsilon) < 1$ then $r(\theta) \geq \theta\epsilon$ for $\theta \in [0, \hat{\theta}]$ (see Lemma 3) while if $\eta(\epsilon) > 1$ then $r(\theta) \leq \theta\epsilon$ for θ sufficiently small ($\theta < \min(\hat{\theta}, \check{\theta})$). We show in subsection 2.4 that with Cournot competition and linear demand, $r(\theta)$ is above $\theta\epsilon$ while $r^c(\theta)$ is below $\theta\epsilon$: $r^c(\theta) < r^e(\theta) = \theta\epsilon < r(\theta)$.

All these results are summarized in Figure 1 which represents four possible shapes of $r(\theta)$ relative to the expected royalty $r^e(\theta)$ in case of litigation (represented by the straight line $\theta \rightarrow \theta\epsilon$).

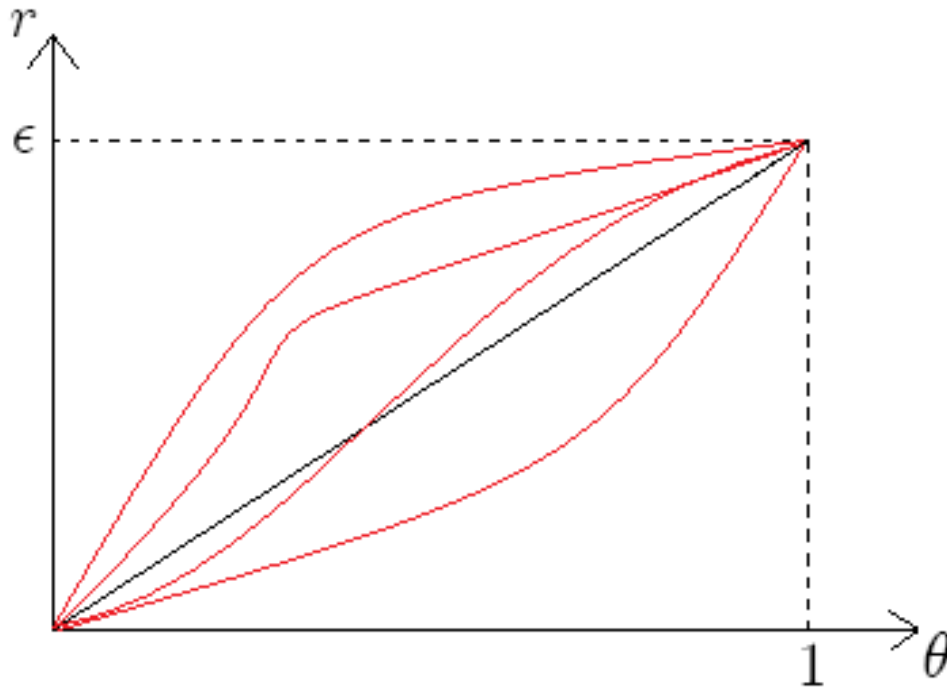


Figure 1: Possible shapes of $r(\theta)$ compared to $r^e(\theta) = \theta\epsilon$

2.1.4 The second stage equilibria

The following proposition gives a complete characterization of the possible second stage equilibria.

Proposition 4 *For a patentholder's offer involving a royalty rate r , the equilibria of the second stage are as follows: i/ if $r \leq r(\theta)$ then the unique equilibrium is given by all firms accepting the license offer; ii/ if $r(\theta) < r \leq \epsilon$ all the equilibria involve a number of $(n - 1)$ license buyers; iii/ if $r > \epsilon$ the unique equilibrium is given by all firms refusing the license offer.*

Proof. See Appendix ■

This proposition states that two possibilities are offered to a holder of an uncertain patent with strength θ when selling licenses through a per-unit royalty rate: either the royalty r is chosen just equal to the maximal value $r(\theta)$ that deters any challenge,

and in this case n licenses are sold, or the chosen royalty rate r is above this value ($r(\theta) < r \leq \epsilon$), and in this case one and only one firm challenges the patent validity ($n - 1$ licenses are sold).

2.2 The patentholder's optimal license offer: first stage

We turn now to the patentholder's optimal decision at the first stage of the game. Denote $q(c - \epsilon + r, k)$ the individual output of a licensee when the per-unit rate r is accepted by k firms, and the $n - k$ remaining firms produce at marginal cost c .

The patentholder's licensing expected revenues $P(r)$ are given by

$$P(r) = \begin{cases} nrq(c - \epsilon + r, n) & \text{if } r \leq r(\theta) \\ \theta(n - 1)rq(c - \epsilon + r, n - 1) & \text{if } r(\theta) < r \leq \epsilon \\ 0 & \text{if } r > \epsilon \end{cases}$$

Note that when $r \in]r(\theta), \epsilon]$, one firm refuses to buy a license and challenges the patent validity and the other $(n - 1)$ firms buy a license (proposition 5). Therefore the patentholder's licensing expected revenues depend on the issue of the trial (the patent is upheld with probability θ).

Let us introduce the following assumptions:

A5. A licensee's output is nonincreasing in the number of licenses: $q(c - \epsilon + r, n - 1) \geq q(c - \epsilon + r, n)$ for all $r \in [0, \epsilon]$.

A6. The aggregate output is nondecreasing in the number of licenses: $Q(c - \epsilon + r, n) \geq Q(c - \epsilon + r, n - 1)$ for all $r \in [0, \epsilon]$.

A7. The function $krq(c - \epsilon + r, k)$ is concave in r for $k \in \{n - 1, n\}$.

Denote $\tilde{r}_k(\epsilon) = \arg \max_{r \geq 0} krq(c - \epsilon + r, k)$ for $k \in \{n - 1, n\}$. As a function of the royalty rate the licensing revenue is a concave function (A7) that reaches its maximum at the value $\tilde{r}_k(\epsilon)$ when k licenses are sold.

In order to determine the maximum of $P(r)$ over $[0, r(\theta)]$ and $[r(\theta), \epsilon]$, we need to compare ϵ and $\tilde{r}_k(\epsilon)$. To do so, we must distinguish between different cases according to the location of ϵ with respect to $\tilde{r}_{n-1}(\epsilon)$ and $\tilde{r}_n(\epsilon)$.

The following lemma is useful for the subsequent analysis:

Lemma 5 *If $\epsilon \leq \tilde{r}_{n-1}(\epsilon)$ then $\epsilon \leq \tilde{r}_n(\epsilon)$.*

Proof. See Appendix ■

A straightforward consequence of the lemma is that if $\epsilon > \tilde{r}_n(\epsilon)$ then $\epsilon > \tilde{r}_{n-1}(\epsilon)$ as well. Therefore, only three cases must be investigated: i/ $\epsilon \leq \tilde{r}_{n-1}(\epsilon)$; ii/ $\tilde{r}_{n-1}(\epsilon) < \epsilon \leq \tilde{r}_n(\epsilon)$; iii/ $\epsilon > \tilde{r}_n(\epsilon)$.

The following propositions determine the patentholder's optimal choice $r^*(\theta)$ in each of these cases and identify the conditions under which litigation is deterred at the subgame perfect equilibrium.

Proposition 6 *If $\epsilon \leq \tilde{r}_{n-1}(\epsilon)$, the function $s(\theta)$ defined as the unique solution in r to the equation $nrq(c - \epsilon + r, n) = \theta(n - 1)\epsilon q(c, n)$ is convex over $[0, 1]$, satisfies $s(0) = 0$, $s(1) < \epsilon$, and the per-unit royalty that maximizes the licensing revenues is given by*

$$r^*(\theta) = \begin{cases} r(\theta) & \text{if } r(\theta) \geq s(\theta) \\ \epsilon & \text{if } r(\theta) < s(\theta) \end{cases}$$

In this case, litigation is deterred at equilibrium if and only if $r(\theta) \geq s(\theta)$

Proof. See Appendix ■

This proposition characterizes the optimal royalty rate for the patentholder when the magnitude ϵ of the cost reduction is such that $\epsilon \leq \tilde{r}_{n-1}(\epsilon)$. First, the function $s(\theta)$ defines the royalty rate level for which the patentholder is indifferent between selling n licenses at the price $r(\theta)$ and selling $(n - 1)$ licenses at the higher price ϵ (in which case litigation occurs and the expected licensing revenues are $\theta(n - 1)\epsilon q(c, n)$). Note that when $\epsilon \leq \tilde{r}_{n-1}(\epsilon)$, if the license is sold to only $(n - 1)$ firms, the optimal royalty rate is ϵ because the licensing revenue is an increasing concave function of r over $[0, \epsilon]$. Second, the comparison between the maximum rate $r(\theta)$ satisfying equation (1) and the royalty rate $s(\theta)$ leads to the following decision: if $r(\theta) \geq s(\theta)$ it is optimal to set $r^*(\theta) = r(\theta)$ and this choice deters litigation; if $r(\theta) < s(\theta)$ it is optimal to set a higher price $r^*(\theta) = \epsilon$ and let one firm challenge the patent validity. Note that if $r(\theta)$ is convex and the curves $r(\theta)$ and $s(\theta)$ meet in only one point over $]0, 1[$, then the curve $r(\theta)$ necessarily intersects the curve $s(\theta)$ from below since $r(0) = s(0) = 0$ and $s(1) < r(1) = \epsilon$. This implies that for low values of θ , we have $r(\theta) < s(\theta)$ and the optimal per-unit royalty rate is then independent of θ and is the same as if the patent were certain. The same result appears in Farrell and Shapiro (2007) but the justification is different here. While Farrell and Shapiro consider only the case where the cost reduction magnitude ϵ is small enough and assume that all firms buy a license

at equilibrium, we obtain the same result by allowing the number of licensees to depend on the per-unit royalty. It is precisely when the royalty at which all firms accept to buy a license is too low (i.e. $r(\theta) < s(\theta)$) that the holder of a weak patent prefers to sell it at the higher price ϵ , triggering thus a patent litigation.

We turn now to the second case where $\tilde{r}_{n-1}(\epsilon) < \epsilon \leq \tilde{r}_n(\epsilon)$.

Proposition 7 *If $\tilde{r}_{n-1}(\epsilon) < \epsilon \leq \tilde{r}_n(\epsilon)$, then defining $v(\theta)$ as the unique solution in r to the equation $nrq(c - \epsilon + r, n) = (n - 1)\theta\tilde{r}_{n-1}(\epsilon)q(c - \epsilon + \tilde{r}_{n-1}(\epsilon), n - 1)$, and $\tilde{\theta}_{n-1}$ as the solution to the equation $r(\theta) = \tilde{r}_{n-1}(\epsilon)$, the function $v(\theta)$ is convex over $[0, 1]$, $v(0) = 0$, $v(1) < \epsilon$, and we have*

$$r^*(\theta) = \begin{cases} \tilde{r}_{n-1}(\epsilon) & \text{if } \theta < \tilde{\theta}_{n-1} \text{ and } r(\theta) < v(\theta) \\ r(\theta) & \text{otherwise} \end{cases}$$

In this case, litigation is deterred at equilibrium if and only if at least one of the two following conditions hold: $\theta \geq \tilde{\theta}_{n-1}$ or $r(\theta) \geq v(\theta)$

Proof. See Appendix. ■

To interpret this proposition, one must first note that if the patentholder finds it optimal to trigger a litigation by selling at a royalty $r > r(\theta)$, the optimal royalty rate is given by $\tilde{r}_{n-1}(\epsilon)$ since $\tilde{r}_{n-1}(\epsilon) < \epsilon$. The expected licensing revenues are therefore equal to $(n - 1)\theta\tilde{r}_{n-1}(\epsilon)q(c - \epsilon + \tilde{r}_{n-1}(\epsilon), n - 1)$. The function $v(\theta)$ defines the royalty rate level for which the patentholder is indifferent between selling n licenses at the price $r(\theta)$ and selling $(n - 1)$ licenses at the price $\tilde{r}_{n-1}(\epsilon)$. Second, it is optimal to sell only $(n - 1)$ licenses at the per-unit royalty $\tilde{r}_{n-1}(\epsilon)$ as long as $v(\theta) > r(\theta)$ and $\theta < \tilde{\theta}_{n-1}$ where $\tilde{\theta}_{n-1}$ is the solution to the equation $r(\theta) = \tilde{r}_{n-1}(\epsilon)$. This means that the holder of a weak patent ($\theta < \tilde{\theta}_{n-1}$) prefers to trigger a patent litigation by selling licenses at a per-unit royalty rate $\tilde{r}_{n-1}(\epsilon)$ when the royalty that all the firms accept is too low ($r(\theta) < v(\theta)$). Again, this extends the result obtained by Farrell and Shapiro (2007) in the sense that the optimal per-unit royalty rate $r^*(\theta)$ for a weak patent ($\theta < \tilde{\theta}_{n-1}$) is independent of the patent strength θ and is the same as if the patent were certain (i.e. $r^*(\theta) = \tilde{r}_{n-1}(\epsilon)$). In our model, it is because the per-unit royalty accepted by all the firms for a weak patent is too low that the patentholder prefers to sell at the royalty rate that maximizes its profit as if the patent were certain, triggering thus a patent litigation.

The last case occurs when $\epsilon > \tilde{r}_n(\epsilon)$

Proposition 8 *If $\epsilon > \tilde{r}_n(\epsilon)$ then, defining $\tilde{\theta}_n$ as the unique solution to the equation $r(\theta) = \tilde{r}_n(\epsilon)$, we have*

$$r^*(\theta) = \begin{cases} \tilde{r}_{n-1}(\epsilon) & \text{if } \theta \leq \min(\tilde{\theta}_{n-1}, \tilde{\theta}_n) \text{ and } r(\theta) < v(\theta) \\ \tilde{r}_n(\epsilon) & \text{if } \theta \geq \tilde{\theta}_{n-1} \\ r(\theta) & \text{otherwise} \end{cases}$$

In this case, litigation is deterred at equilibrium if and only if at least one of the two following conditions hold: $\theta > \min(\tilde{\theta}_{n-1}, \tilde{\theta}_n)$ or $r(\theta) \geq v(\theta)$.

Proof. See Appendix. ■

The interpretation is the same as in the two previous propositions except that for $\epsilon > \tilde{r}_n(\epsilon)$ (implying that $\epsilon > \tilde{r}_{n-1}(\epsilon)$), when it is optimal for the patentholder to trigger a litigation by selling at a royalty $r > r(\theta)$, the optimal royalty rate is given either by $\tilde{r}_{n-1}(\epsilon)$ or $\tilde{r}_n(\epsilon)$. Again the optimal per-unit rate of a weak patent is the same as if the patent were certain, but in this case a patent litigation does not necessarily occur when $r^*(\theta) = \tilde{r}_n(\epsilon)$.

The following corollary gives a sufficient condition for litigation deterrence.

Corollary 9 *If $r(\theta) > \theta\epsilon$ then the patentholder finds it optimal to deter litigation and $P(r^*(\theta)) > \theta P(r^*(1))$.*

Proof. See Appendix. ■

This corollary states that if the maximum royalty rate $r(\theta)$ acceptable by all firms is above the expected value of the royalty in case of litigation $r^e(\theta) = \theta\epsilon$, then the patentholder will prefer to deter litigation. It gives thus a sufficient condition under which the patentholder takes advantage of both the uncertainty of its patent and the externalities between the downstream competitors. The consequence is clear. Insofar as the maximum royalty rate that deters litigation is higher than the expected royalty in case of litigation, the patentholder gets a higher profit than the *ex ante* expected profit it could get if the patent were granted by the patent office with the same probability that the court upholds the patent validity: the per-unit royalty licensing scheme gives to the patentholder a profit $P(r^*(\theta))$ that is higher than the expected profit $\theta P(r^*(1))$ resulting from the uncertainty on the patent validity. The corollary fully justifies the use of the benchmark $r^e(\theta) = \theta\epsilon$.

2.3 Introduction of renegotiation

So far we have assumed that in case of litigation, an unsuccessful challenger produces with marginal cost c because the patentholder refuses to sell him a license. Whether such a commitment to refuse a license to an unsuccessful challenger is credible or not must be discussed. From the challenger's perspective this commitment is equivalent to an offer of a new licensing contract involving a royalty rate $\bar{r} = \epsilon$. However, from the patentholder's perspective, this equivalence does not hold. Moreover a situation where an unsuccessful challenger is offered a new licensing contract involving a royalty rate $\bar{r} < \epsilon$ may be preferred to a license refusal. Such an issue is important since a potential challenger will take the decision whether to accept the license or contest the patent's validity, anticipating what will happen if the patent is validated.

Formally if we allow for renegotiation when $(n - 1)$ firms accept a licensing contract based on a royalty rate r and the remaining firm challenges the patent unsuccessfully, then the patentholder will offer to the challenger a contract involving a royalty rate $\bar{r} \in [0, \epsilon]$ that maximizes its licensing revenues

$$P(r, \bar{r}) = (n - 1) r q^L(c - \epsilon + r, c - \epsilon + \bar{r}) + \bar{r} q^{NL}(c - \epsilon + r, c - \epsilon + \bar{r})$$

where $q^L(c - \epsilon + r, c - \epsilon + \bar{r})$ denotes the equilibrium quantity produced by each of the $(n - 1)$ firms that accepted initially the license offer r and $q^{NL}(c - \epsilon + r, c - \epsilon + \bar{r})$ is the equilibrium quantity produced by the unsuccessful challenger who produces at marginal cost $c - \epsilon + \bar{r}$. If $\bar{r}(r)$ is the royalty rate that maximizes $P(r, \bar{r})$ with respect to \bar{r} , a licensing contract involving a royalty rate r will be accepted by all the downstream firms if and only if:

$$\pi(c - \epsilon + r, c - \epsilon + r) \geq \theta \pi(c - \epsilon + \bar{r}(r), c - \epsilon + r) + (1 - \theta) \pi(c - \epsilon, c - \epsilon) \quad (4)$$

Since $\bar{r}(r) \leq \epsilon$ we have $\pi(c - \epsilon + \bar{r}(r), c - \epsilon + r) \geq \pi(c, c - \epsilon + r)$ which entails that constraint (4) is (weakly) more stringent than (1). More precisely, a royalty rate r could be accepted if the patentholder commits to refuse a license to a challenger or license him at $\bar{r} = \epsilon$, but not accepted if he cannot commit. This implies that the maximum royalty rate the patentholder can make the n firms pay is (weakly) smaller when renegotiation of a licensing contract (after patent validation) is introduced. This is illustrated in the case of Cournot competition with linear demand.

2.4 Cournot competition with linear demand

Assume that the downstream firms compete à la Cournot in an homogeneous market where the demand is given by $Q = \max(a - p, 0)$ where $a > c$. If x is the constant marginal cost of a firm and $y \leq x$ the (common) marginal cost of the remaining $(n - 1)$ firms, denote by $\pi(x, y)$ the equilibrium profit of the firm with a cost x when confronted to $(n - 1)$ competitors with a cost y . The firm having a higher marginal cost $x > y$ is active on the market if and only if $x < \frac{a+(n-1)y}{n}$. When this condition is met, we obtain $\pi(x, y) = \frac{(a-nx+(n-1)y)^2}{(n+1)^2}$. From this expression we derive: $\pi_1(x, y) = -\frac{2n}{n+1} \frac{a-nx+(n-1)y}{n+1} < 0$; $\pi_2(x, y) = \frac{2(n-1)}{n+1} \frac{a-nx+(n-1)y}{n+1} > 0$; $\pi_1(x, y) + \pi_2(x, y) = -\frac{2}{n+1} \frac{a-nx+(n-1)y}{n+1} < 0$; $\pi_{11}(x, y) = \frac{2n^2}{(n+1)^2} > 0$; $\pi_{12}(x, y) = -\frac{2n(n-1)}{(n+1)^2} < 0$; $\pi_{22}(x, y) = \frac{2(n-1)^2}{(n+1)^2} > 0$. Note that in this case the function $x \rightarrow \pi(x, x)$ is convex ($\frac{\partial^2 \pi(x, x)}{\partial x^2} = \pi_{11}(x, x) + 2\pi_{12}(x, x) + \pi_{22}(x, x) = \frac{2}{(n+1)^2} > 0$).

2.4.1 Determination of the acceptable royalty rates

When $\epsilon < \frac{a-c}{n-1}$, we have $\pi(c, c - \epsilon) > 0$. Therefore, if $\epsilon < \frac{a-c}{n-1}$, a licensing contract with a royalty rate r is accepted by all firms if and only if $r \leq r_1(\theta)$ where $r_1(\theta)$ is the unique positive solution in r to equation (1) which, in the case of Cournot competition with linear demand, is equivalent to the following equation:

$$(a - c + \epsilon - r)^2 = \theta[a - c - (n - 1)(\epsilon - r)]^2 + (1 - \theta)(a - c + \epsilon)^2 \quad (5)$$

When $\epsilon \geq \frac{a-c}{n-1}$, we have $\pi(c, c - \epsilon) = 0$. We determine the value \hat{r} such that the inequality $\pi(c, c - \epsilon + r) > 0$ is equivalent to $r > \hat{r}$. A simple calculation leads to $\hat{r} = \epsilon - \frac{a-c}{n-1}$. Therefore, a licensing contract based on a royalty rate $r \leq \hat{r}$ is accepted if and only if $r \leq r_2(\theta)$ where $r_2(\theta)$ is the unique solution in $r \in [0, \epsilon]$ to the following equation:

$$[a - c + \epsilon - r]^2 = (1 - \theta)[a - c + \epsilon]^2$$

The positive solution of this equation is given by $r_2(\theta) = [1 - \sqrt{1 - \theta}](a - c + \epsilon)$. This expression can be used to determine the patent strength threshold $\hat{\theta} = 1 - [\frac{n(a-c)}{(n-1)(a-c+\epsilon)}]^2$ such that $r_2(\hat{\theta}) = \hat{r}$.

Thus, if $\epsilon \leq \frac{a-c}{n-1}$, the maximum royalty rate the patentholder can make all the downstream firms accept is $r(\theta) = r_1(\theta)$ for all $\theta \in [0, 1]$, while if $\epsilon \geq \frac{a-c}{n-1}$, the maximum

royalty rate the patentholder can make all the downstream firms accept is given by:

$$r(\theta) = \begin{cases} r_2(\theta) = [1 - \sqrt{1 - \theta}](a - c + \epsilon) & \text{if } 0 \leq \theta \leq \hat{\theta} \\ r_1(\theta) & \text{if } \hat{\theta} \leq \theta \leq 1 \end{cases}$$

Note that the royalty rate $r_2(\theta) = [1 - \sqrt{1 - \theta}](a - c + \epsilon)$ is convex in θ and that $r_2'(0) = \frac{1}{2}(a - c + \epsilon) > \epsilon$ for any non-drastic innovation, i.e. $\epsilon \leq a - c$. Since $r_2(\theta)$ is convex over $[0, \hat{\theta}]$, we can state that $r_2(\theta) \geq r_2(0) + \theta r_2'(0) = \theta r_2'(0)$ which entails that $r(\theta) = r_2(\theta) \geq \theta \epsilon$ for $\epsilon \leq a - c$ and $\theta \in [0, \hat{\theta}]$. Thus, the maximum royalty rate $r(\theta)$ acceptable by all firms is higher than the benchmark value $r^e(\theta) = \theta \epsilon$ for $\theta \leq \hat{\theta}$.¹²

2.4.2 Optimal choice of the royalty rate by the patentholder

Consider an innovation $\epsilon \geq \frac{a-c}{n-1}$ covered by a patent of strength $\theta \leq \hat{\theta}$. The patentholder's licensing revenues when the n firms accept to pay a royalty rate r are equal to $nrq(c - \epsilon + r, n) = nr \frac{(a-c+\epsilon-r)}{n+1}$, which is a concave function in r . Note that the condition $\epsilon \leq \tilde{r}_n(\epsilon)$ where $\tilde{r}_n(\epsilon) = \arg \max[nrq(c - \epsilon + r, n)] = \frac{a-c+\epsilon}{2}$ is equivalent to $\epsilon \leq a - c$, i.e. the innovation is non-drastic. Moreover, the maximum royalty rate when $(n - 1)$ firms buy the license is given by $\tilde{r}_{n-1}(\epsilon) = \arg \max[\theta(n - 1)rq(c - \epsilon + r, n - 1)] = \frac{a-c+n\epsilon}{2n}$, so that in the linear case we have $\tilde{r}_n(\epsilon) > \tilde{r}_{n-1}(\epsilon)$ for any $\epsilon > 0$. Note also that $r(\theta)$ is convex over $\theta \in [0, 1]$ because $\pi(x, x)$ is convex in x over $[c - \epsilon, c]$. Since $r(\theta) = r_2(\theta) \geq \theta \epsilon$ for $\theta \in [0, \hat{\theta}]$, corollary (9) and its proof entail that $r^*(\theta) = \min(r(\theta), \tilde{r}_n(\epsilon))$ and litigation does not occur in this case.

The patentholder's royalty rate choice can be described as follows.

- For an innovation ϵ such that $\frac{a-c}{n} < \epsilon \leq a - c$, we have $\epsilon \leq \tilde{r}_n(\epsilon)$. Since $r(\theta) \leq \epsilon$ then $r(\theta) \leq \tilde{r}_n(\epsilon)$ and consequently $r^*(\theta) = r(\theta) = [1 - \sqrt{1 - \theta}](a - c + \epsilon)$ for all $\theta \in [0, \hat{\theta}]$: the royalty rate the patentholder will set is equal to the maximum royalty rate that all downstream firms accept.

- For an innovation ϵ such that $\epsilon > a - c$, we have $\epsilon > \tilde{r}_n(\epsilon)$. Consequently, for such a drastic innovation, the patentholder will choose to set the royalty rate to the value $r(\theta)$ if and only if this value does not exceed $\tilde{r}_n(\epsilon)$. The unique solution in θ to the equation $r(\theta) = \tilde{r}_n(\epsilon)$ is given by $\tilde{\theta}_n = \frac{3}{4}$. Note that $\tilde{\theta}_n < \hat{\theta}$ is true if and only if $\epsilon > \frac{n+1}{n-1}(a - c)$. Thus two subcases emerge:

¹²The linear demand case illustrates thus proposition 3 in the case where $\pi(x, x)$ is convex and $\eta(\epsilon) < 1$.

i/ if $a - c < \epsilon \leq \frac{n+1}{n-1}(a - c)$ then the patentholder's optimal choice of royalty rate is given by $r^*(\theta) = r(\theta) = [1 - \sqrt{1 - \theta}](a - c + \epsilon)$ for all $\theta \in [0, \hat{\theta}]$;

ii/ if $\epsilon > \frac{n+1}{n-1}(a - c)$ then

$$r^*(\theta) = \begin{cases} [1 - \sqrt{1 - \theta}](a - c + \epsilon) & \text{if } 0 \leq \theta \leq \frac{3}{4} \\ \frac{a - c + \epsilon}{2} & \text{if } \frac{3}{4} \leq \theta \leq \hat{\theta} \end{cases}$$

2.4.3 Renegotiation

Suppose now that the possibility to renegotiate a licensing contract with an unsuccessful challenger is introduced. Denote firm n the challenging firm and \bar{r} the per-unit royalty rate at which a license is offered if the challenge fails. Cournot competition between $(n - 1)$ firms (indexed by $i = 1, 2, \dots, n - 1$) whose marginal cost is $c - \epsilon + r$ and firm n whose marginal cost is $c - \epsilon + \bar{r}$ leads to the following equilibrium outputs:

$$q_i(r, \bar{r}) = \begin{cases} \frac{a - c + \epsilon - 2r + \bar{r}}{n + 1} & \text{if } i = 1, \dots, n - 1 \\ \frac{a - c + \epsilon - n\bar{r} + (n - 1)r}{n + 1} & \text{if } i = n \end{cases}$$

For a given r , the value of the royalty rate \bar{r} that maximizes the patentholder's licensing revenue is the solution to the following program:

$$\max_{\bar{r} \in [0, \epsilon]} P(r, \bar{r}) = (n - 1)r \frac{a - c + \epsilon - 2r + \bar{r}}{n + 1} + \bar{r} \frac{a - c + \epsilon - n\bar{r} + (n - 1)r}{n + 1}$$

Suppose that the innovation is non-drastic, i.e. $\epsilon < a - c$. The unique unconstrained maximum of the concave function $\bar{r} \rightarrow P(r, \bar{r})$ is given by the FOC $\frac{\partial P(r, \bar{r})}{\partial \bar{r}} = \frac{a - c + \epsilon + 2(n - 1)r - 2n\bar{r}}{n + 1} = 0$. The maximum of the function $P(r, \bar{r})$ over the interval $\bar{r} \in [0, \epsilon]$ is reached at

$$\bar{r}(r) = \min \left(\epsilon, \frac{(n - 1)}{n}r + \frac{a - c + \epsilon}{2n} \right) = \min \left(\epsilon, r + \frac{1}{n} \left(\frac{a - c + \epsilon}{2} - r \right) \right)$$

Since $\epsilon < a - c$, we have $\frac{a - c + \epsilon}{2} > \epsilon$. Therefore, $r \in [0, \epsilon] \implies \frac{a - c + \epsilon}{2} - r \geq 0$ and consequently $\bar{r}(r) \geq r$. Hence a firm which refuses a licensing contract and unsuccessfully challenges the patent's validity will get a new licensing offer with a higher royalty rate than the royalty paid by licensees that have accepted the initial licensing contract.¹³

Moreover, the condition $\bar{r}(r) < \epsilon$ is fulfilled if and only if $r < \left(\frac{2n-1}{2(n-1)}\right)\epsilon - \frac{a-c}{2(n-1)} \equiv \varphi$,

¹³ It is obvious that the patentholder's position is stronger after the patent has been upheld by the court than before.

which is positive whenever $\epsilon > \frac{a-c}{2n-1}$. For such a royalty rate r , we have $\pi(c - \epsilon + \bar{r}(r), c - \epsilon + r) = \left[\frac{a-c+\epsilon-n\bar{r}(r)+(n-1)r}{n+1} \right]^2$, and the condition expressing that all firms accept the licensing contract r is:

$$\pi(c - \epsilon + r, c - \epsilon + r) \geq \theta \pi(c - \epsilon + \bar{r}(r), c - \epsilon + r) + (1 - \theta) \pi(c - \epsilon, c - \epsilon)$$

Replacing $\bar{r}(r)$ by its value, one obtains:

$$\frac{(a - c + \epsilon - r)^2}{(n + 1)^2} \geq \theta \left[\frac{a - c + \epsilon}{2(n + 1)} \right]^2 + (1 - \theta) \left[\frac{a - c + \epsilon}{(n + 1)} \right]^2 = \frac{4 - 3\theta}{4} \left[\frac{a - c + \epsilon}{(n + 1)} \right]^2$$

This inequality is satisfied if and only if:

$$r \leq (a - c + \epsilon) \left(1 - \frac{\sqrt{4 - 3\theta}}{2} \right)$$

Hence a royalty rate $r < \varphi$ is accepted by all firms if and only if the previous inequality holds. Denoting $\bar{\theta}$ the unique solution in θ to the equation $(a - c + \epsilon) \left(1 - \frac{\sqrt{4 - 3\theta}}{2} \right) = \varphi$, we can then state that for $\theta \leq \bar{\theta}$, the maximum royalty rate accepted by all firms when post-trial license offer is possible is given by:

$$r^p(\theta) = (a - c + \epsilon) \left(1 - \frac{\sqrt{4 - 3\theta}}{2} \right)$$

Straightforward computations lead to $\frac{dr^p}{d\theta}(0) = \frac{3}{8}(a - c + \epsilon)$. It is easy to show that $\frac{dr^p}{d\theta}(0) < \epsilon$ for any $\epsilon \in \left] \frac{3}{5}(a - c), a - c \right[$. Consequently for such intermediate innovations, $r^p(\theta) < \theta\epsilon$ for sufficiently small values of θ . Note that for such innovations, the condition $\epsilon > \frac{a-c}{2n-1}$ is satisfied since $\frac{3}{5}(a - c) > \frac{a-c}{2n-1}$ for any $n \geq 2$.

These results are summarized in the following proposition:

Proposition 10 *In a Cournot model with an homogeneous product and a linear demand $Q = \max(a - p, 0)$ the maximum per-unit royalty rate that induces a unique perfect subgame equilibrium in which all firms choose to buy a license of a patented technology that reduces the marginal cost by $\epsilon \in \left] \frac{3}{5}(a - c), a - c \right[$ is given by $r^p(\theta) = (a - c + \epsilon) \left(1 - \frac{\sqrt{4 - 3\theta}}{2} \right)$ for a patent strength θ smaller than a threshold $\bar{\theta} \in]0, 1[$. The royalty $r^p(\theta)$ is sustained by a renegotiated royalty $\bar{r}(r^p(\theta)) < \epsilon$, and is smaller than the expected benchmark royalty $\theta\epsilon$ for a sufficiently weak patent.*

This example illustrates the role of post-trial renegotiation in licensing an uncertain patent. An individual challenge becomes less risky when it is possible to renegotiate *ex post* a new royalty after the issue of the trial. Consequently, the patentee loses some of its market power in determining *ex ante* the per-unit royalty rate that deters litigation at equilibrium. For this reason refusing a license to an unsuccessful challenger should not be allowed.

3 Fixed fee licensing schemes

In this section, licensing contracts offered by the patentholder P to the n downstream firms involve a fixed fee only. The modelling leads to the same three-stage game as in the per-unit royalty licensing scheme, simply replacing the royalty rate by a fixed fee in the licensing contract offered by the patentholder.

Denote $\pi^L(k)$ (respectively $\pi^{NL}(k)$) the equilibrium profit of a downstream firm producing at a constant marginal cost $c - \epsilon$ (respectively c) in an industry of n firms, out of which k firms produce at marginal cost $c - \epsilon$ and the remaining $n - k$ firms produce at marginal cost c .

We introduce the following assumption which states that a licensee's profit when all firms buy the license is higher than a non-licensee's profit whatever the number of licensees.

$$\text{A8: } \pi^{NL}(k) < \pi^L(n) \text{ for all } k < n.$$

We start with a preliminary result describing what happens at equilibrium when not all firms accept the up-front fee.

Lemma 11 *Consider a Nash equilibrium of stage 2. If not all firms accept the licensing contract in this equilibrium then there is at least one firm (among those who do not accept the contract) that challenges the patent validity.*

Proof. Let us show that a situation where only $k < n$ firms accept the contract and none of the remaining $n - k$ firms challenges the patent validity cannot be a Nash equilibrium of stage 2. If one of these firms challenges the patent validity it gets an expected profit of $\theta\pi^{NL}(k) + (1 - \theta)\pi^L(n)$, whereas it gets a profit equal to $\pi^{NL}(k)$ if no firm challenges the patent validity. From A8 it follows that $\theta\pi^{NL}(k) + (1 - \theta)\pi^L(n) > \pi^{NL}(k)$ which means that a downstream firm who does not accept the licensing contract is always better off challenging the patent validity. ■

In order to derive the demand function for licenses, we introduce the following assumption:

A9: For all k between 0 and $n - 1$,

$$\pi^L(k) - \pi^L(k + 1) \geq \pi^{NL}(k - 1) - \pi^{NL}(k)$$

According to this assumption, a licensee's incremental profit is at least equal to a non-licensee's incremental profit when the number of licensees is reduced by one unit. A more precise interpretation of this assumption is given below.

3.1 Demand function for licenses: stage 2

The following proposition gives the demand for licenses at the Nash equilibrium of stage 2 as a function of the value of the up-front fee F chosen by the patentholder P in stage 1:

Proposition 12 Denote $F_n(\theta) = \theta (\pi^L(n) - \pi^{NL}(n - 1))$ and $F_k = \pi^L(k) - \pi^{NL}(k - 1)$ for all $k \leq n - 1$.

If $F \leq F_n(\theta)$ then the unique Nash equilibrium of stage 2 is the situation where all downstream firms accept the licensing contract.

If $F_n(\theta) < F \leq F_{n-1}$ then the Nash equilibria of stage 2 are the situations where $n - 1$ downstream firms accept the licensing contract and one firm does not.

For any k between 0 and $n - 2$, if $F_{k+1} < F \leq F_k$ then the Nash equilibria of stage 2 are the situations where k downstream firms accept the licensing contract and the remaining $n - k$ firms do not.

If $F > F_1$ then the unique Nash equilibrium of stage 2 is the situation where all downstream firms reject the licensing contract.

To avoid the multiple equilibria problem that arises when F is equal to one of the threshold values F_k we assume that a downstream firm which is indifferent between accepting the license offer made by the patentholder and refusing it chooses to accept it. Hence, we can define the number $k(F, \theta)$ of firms that accept at equilibrium the

license offer F made by the patentholder:

$$k(F, \theta) = \begin{cases} n & \text{if } F \leq F_n(\theta) \\ n-1 & \text{if } F_n(\theta) < F \leq F_{n-1} \\ \dots & \dots \\ k & \text{if } F_{k+1} < F \leq F_k \\ \dots & \dots \\ 0 & \text{if } F > F_1 \end{cases}$$

Note that $k(F, \theta)$ depends on θ only through the threshold $F_n(\theta)$. More precisely, if we denote $F_n(1) = F_n$ we have $F_n(\theta) = \theta F_n$ and $F > F_n(\theta)$ implies $k(F, \theta) = k(F)$.

3.2 Choice of the fixed fee: stage 1

The patentholder will choose F so as to maximize its licensing revenues anticipating the number of downstream firms that will accept the license offer. If the up-front fee F is such that all firms accept the offer then the patentholder's licensing revenues are equal to nF . If the up-front fee is such that there is at least one firm that does not accept the offer then litigation occurs and the patentholder gets licensing revenues equal to $k(F)F$ only when the patent validity is upheld by the court. This happens with probability θ which entails that the expected licensing revenues of the patentholder when F induces a number of licensees k smaller than n are equal to $\theta k(F)F$. The expected licensing revenues of the patentholder as a function of the up-front fee F can be summarized as follows:

$$P(F, \theta) = \begin{cases} nF & \text{if } F \leq F_n(\theta) \\ \theta(n-1)F & \text{if } F_n(\theta) < F \leq F_{n-1} \\ \dots & \dots \\ \theta kF & \text{if } F_{k+1} \leq F \leq F_k \\ \dots & \dots \\ 0 & \text{if } F > F_1 \end{cases}$$

Since the demand function of licenses is stepwise, the maximization of $P(F, \theta)$ with respect to F will lead to one (or several) of the thresholds $F_n(\theta)$ and $F = F_k$, $k \leq n-1$. In other words, the maximization program $\max_{F \geq 0} P(F, \theta)$ is equivalent to the maximization program

$$\max_{F \in \{F_1, \dots, F_k, \dots, F_{n-1}, F_n(\theta)\}} P(F, \theta)$$

Since $F_n(\theta) = \theta F_n$, the expected licensing revenues $P(F, \theta)$ for a value of F belonging to the set $\{F_1, \dots, F_k, \dots, F_{n-1}, F_n(\theta)\}$ is given by:

$$P(F, \theta) = \begin{cases} n(\theta F_n) & \text{if } F = \theta F_n \\ \theta(n-1)F_{n-1} & \text{if } F = F_{n-1} \\ \dots & \dots \\ \theta k F_k & \text{if } F = F_k \\ \dots & \dots \\ \theta F_1 & \text{if } F = F_1 \end{cases}$$

This shows that for any $\theta \neq 0$, maximizing $P(F, \theta)$ over the set $\{F_1, \dots, F_k, \dots, F_{n-1}, F_n(\theta)\}$ is equivalent to maximizing $P(F, 1)$ over the set $\{F_1, \dots, F_k, \dots, F_{n-1}, F_n\}$ in the sense that if the maximum of $P(F, 1)$ is reached at F_k then the maximum of $P(F, \theta)$ is reached at F_k if $k < n$ and at $F_n(\theta) = \theta F_n$ if $k = n$. Hence, we have the following result:

Proposition 13 *If the maximum of $P(F, 1)$ is reached at $F^* = F_n$ then the patentholder offers a licensing contract with an up-front fee $F^*(\theta) = F_n(\theta) = \theta F_n$ that induces a number of licensees equal to the total number of downstream firms. If the maximum of $P(F, 1)$ is reached at $F^* = F_k$ with $k < n$ then the patentholder offers a licensing contract with an up-front fee $F^* = F_k$ that induces a number of licensees equal to k .*

This proposition entails the following two results:

Corollary 14 *The equilibrium number of licensees k^* does not depend on the patent strength θ*

Proof. The previous proposition shows that the choice of the fixed fee by the patentholder does not depend on θ . Since the number of licensees is determined by the value of the up-front fee fixed by the patentholder it follows that the equilibrium number of licensees does not depend on the patent strength θ . ■

Corollary 15 *The equilibrium expected licensing revenues of the patentholder under an up-front fee regime, denoted $P_F^*(\theta) = P(F^*(\theta), \theta)$, are proportional to the patent strength, i.e.:*

$$P_F^*(\theta) = \theta P_F^*(1)$$

Proof. If the patentholder offers a licensing contract involving an up-front fee $F = F_n(\theta) = \theta F_n$ then its equilibrium licensing revenues are $P_F^*(\theta) = n(\theta F_n) = \theta(nF_n) = \theta P_F^*(1)$. If the patentholder offers a licensing contract involving an up-front fee $F = F_k$ where $k < n$, then its equilibrium licensing revenues are $P_F^*(\theta) = \theta(kF_k) = \theta P_F^*(1)$. ■

The results of this section lead to the conclusion that licensing an uncertain patent by means of an up-front fee is not affected by the uncertainty, in the sense that the number of licensees does not depend on the patent strength and the patentholder's licensing revenues are exactly proportional to the patent strength. These results are very different from those obtained with a per-unit royalty rate (previous section) or with a two-part tariff as in Farrell and Shapiro (2007). This leads to a first conclusion: licensing weak patents is very sensitive to the chosen licensing scheme. We must now compare the licensing revenues collected through these schemes.

4 Royalty rate vs. fixed fee

In this section we show that, at least under some circumstances, the patentholder prefers to use a royalty rate rather than an up-front fee in licensing contracts. Denote $P_r^*(\theta) = P(r^*(\theta))$ the optimal patentholder's profit when the per-unit royalty licensing scheme is used.

Proposition 16 *If the patentholder gets higher licensing revenues when using the royalty rate scheme than with the fixed fee scheme when patent validity is perfect, i.e. $\theta = 1$, it will also prefer to use a royalty rate rather than a fixed fee when the patent's validity is uncertain, i.e. $\theta < 1$.*

Proof. This follows immediately from the fact that $P_r^*(\theta) \geq \theta P_r^*(1)$ whereas $P_F^*(\theta) = \theta P_F^*(1)$. Therefore, if $P_r^*(1) \geq P_F^*(1)$ then $P_r^*(\theta) \geq \theta P_r^*(1) \geq \theta P_F^*(1) = P_F^*(\theta)$ which means that the patentholder's licensing revenues are higher when the royalty rate mechanism is used. ■

This proposition gives only a sufficient condition for royalty rate contracts to be preferred over up-front fee contracts when the innovation is covered by an uncertain patent. If royalties are preferred to fixed fees when $\theta = 1$, the former will be also preferred to the latter when $\theta < 1$. However, this condition is far from necessary as the following example shows: fixed fees may be preferred when $\theta = 1$ whereas royalties are preferred for small values of θ .

Example: Cournot competition with a linear demand

We know from Kamien and Tauman (1986) that with in a perfect patent setting, i.e. $\theta = 1$, the patentholder's licensing revenues are higher with an up-front fee than with a royalty rate.¹⁴ We show hereafter that this ranking does not hold anymore when the patent is uncertain: the patentholder may prefer to use the royalty rate mechanism rather than the fixed fee mechanism.

We consider innovations of intermediate magnitude, i.e. $\frac{a-c}{n-1} < \epsilon < a - c$ protected by weak patents, i.e. $\theta \in]0, \hat{\theta}[$ with $\hat{\theta} = 1 - [\frac{n(a-c)}{(n-1)(a-c+\epsilon)}]^2$. We do so because in this case, we know the analytical expression of the royalty rate the patentholder will set, i.e. $r(\theta) = r_2(\theta) = [1 - \sqrt{1 - \theta}](a - c + \epsilon)$. This allows us to compute the quantity produced at equilibrium by each downstream firm: $q(c - \epsilon + r(\theta), n) = \frac{\sqrt{1-\theta}(a-c+\epsilon)}{n+1}$. The equilibrium licensing revenues derived from the royalty $r(\theta) = r_2(\theta)$ are thus given by:

$$P_r^*(\theta) = nr(\theta)q(c - \epsilon + r(\theta), n) = \frac{n(\sqrt{1-\theta} - 1 + \theta)(a - c + \epsilon)^2}{n + 1}$$

Kamien and Tauman (1986, proposition 2) gives the patentholder's profit expression when $\theta = 1$. Using this expression and corollary (15), we derive the value of the patentholder revenues for any $\epsilon \in]\frac{a-c}{n-1}, a - c[$:

$$P_F^*(\theta) = \begin{cases} \frac{2\theta n}{(n+1)^2} \epsilon^2 \left[\frac{a-c}{2\epsilon} + \frac{n+2}{4} \right]^2 & \text{if } \frac{a-c}{n-1} < \epsilon \leq \frac{2(a-c)}{n} \\ \frac{\theta n(n+2)}{(n+1)^2} \epsilon (a - c) & \text{if } \frac{2(a-c)}{n} \leq \epsilon < a - c \end{cases}$$

Let us compare $P_r^*(\theta)$ and $P_F^*(\theta)$. First note that $P_r^*(\theta)$ is linear in θ while $P_F^*(\theta)$ is concave in θ . Second, these functions take the same value for $\theta = 0$. We can then state that a sufficient condition for $P_r^*(\theta)$ to be greater than $P_F^*(\theta)$ for all $\theta \in [0, \hat{\theta}]$ is that $P_r^*(\hat{\theta}) \geq P_F^*(\hat{\theta})$. The left-hand side of this inequality is given by $P_r^*(\hat{\theta}) = nr(\hat{\theta})q(c - \epsilon + r(\hat{\theta})) = n\hat{r}q(c - \epsilon + \hat{r}) = \frac{n^2}{(n-1)(n+1)} (\epsilon - \frac{a-c}{n-1})(a - c)$ while the right-hand side depends on whether ϵ is such that $\frac{a-c}{n-1} \leq \epsilon \leq \frac{2(a-c)}{n}$ or $\frac{2(a-c)}{n} \leq \epsilon \leq a - c$.

Let us examine the subcase $\frac{2(a-c)}{n} \leq \epsilon \leq a - c$. When this condition is satisfied, we have $P_F^*(\hat{\theta}) = \frac{n(n+2)}{(n+1)^2} \epsilon (a - c) \left(1 - [\frac{n(a-c)}{(n-1)(a-c+\epsilon)}]^2 \right)$. Comparing $P_r^*(\hat{\theta})$ to $P_F^*(\hat{\theta})$ amounts then to compare $\frac{n}{n-1} (\epsilon - \frac{a-c}{n-1})$ to $\frac{n+2}{n+1} \epsilon \left(1 - [\frac{n(a-c)}{(n-1)(a-c+\epsilon)}]^2 \right)$. A sufficient (and necessary) condition to have $P_r^*(\hat{\theta}) \geq P_F^*(\hat{\theta})$ is $(\frac{n}{n-1} - \frac{n+2}{n+1}) \epsilon - \frac{n}{n-1} \frac{a-c}{n-1} \geq -\frac{n+2}{n+1} \epsilon [\frac{n(a-c)}{(n-1)(a-c+\epsilon)}]^2$. The

¹⁴Sen (2005) shows that this result holds only when the number of firms in the downstream industry is not too high.

left-hand side of this inequality is clearly increasing in ϵ while it is straightforward to show that the right-hand side is decreasing in ϵ . Therefore, to show that the previous inequality holds for any $\epsilon \in \left[\frac{2(a-c)}{n}, a-c \right]$, it is sufficient to show that it holds for $\epsilon = \frac{2(a-c)}{n}$. Taking the inequality for $\epsilon = \frac{2(a-c)}{n}$ and simplifying by $(a-c)$, we get $\frac{n}{n-1} \left(\frac{2}{n} - \frac{1}{n-1} \right) > \frac{2(n+2)}{n(n+1)} \left(1 - \frac{n}{(n-1)\left(1+\frac{2}{n}\right)} \right)^2$ which can be shown after some algebraic manipulations to be equivalent to $\frac{n}{n-1} > \frac{2}{n+1}$, which is obviously true. Thus, the condition $P_r^*(\hat{\theta}) \geq P_F^*(\hat{\theta})$ holds for an innovation such that $\frac{2(a-c)}{n} \leq \epsilon \leq a-c$. We can then state the following result:

Proposition 17 *If the downstream firms compete à la Cournot in a market where the demand is linear, then for an innovation ϵ of intermediate magnitude, i.e. such that $\frac{2(a-c)}{n} \leq \epsilon \leq a-c$, covered by a relatively weak patent, i.e. such that $\theta \leq \hat{\theta}$, the patentholder gets higher licensing revenues using a royalty rate rather than an up-front fee, whereas if the patent were perfect the inverse would be true.*

It is important to note that the per-unit royalty licensing scheme is preferred in this probabilistic right framework only because the patent is uncertain, while a possible preference for this licensing scheme in the framework of a perfect protection rests on a completely different reason, mainly related to the size of the downstream industry (Sen, 2005).

5 Conclusion

The consequences of licensing uncertain patents have been examined in this paper by addressing the following question: to what extent licensing a patent that has a positive probability to be invalidated if it is challenged favors the patentholder when confronted to potential users in an oligopolistic downward industry? Our results show that the answer to Farrell and Shapiro's question "How strong are weak patents?" is very sensitive to the choice of the licensing scheme. Two licensing schemes have been examined: the per-unit royalty rate and the up-front fee. The most salient result is that these two mechanisms lead to opposite consequences. While licensing uncertain patents by means of a royalty rate allows in general the patentholder to reap some extra profit relative to the expected profit after the court resolution of the patent validity, a fixed fee regime discards completely this possibility. Under a fixed fee the patentholder obtains exactly

its expected revenue. These results mainly arise from letting the number of licensees depend on the price of the license chosen by the patentholder, either a per-unit royalty rate or an up-front fee. The second important result is that under the per-unit royalty licensing regime the holder of a weak patent may prefer to sell at the same royalty rate as if the patent was certain, taking thus the risk of triggering a litigation on patent validity. However the justification of this result is completely different from Farrell and Shapiro (2007). It is precisely when the royalty rate acceptable by all the firms in the downward industry is too low that the holder of a weak patent prefers to sell at the royalty rate that maximizes its licensing revenues as if the patent was certain. Moreover we have shown that even if fixed fees are preferred when the patent is very strong, royalties may be more profitable if the patent is uncertain, particularly if it is weak. The classical properties of licensing certain patents may thus be reversed in the uncertain patent framework. We have also explored different policy levers affecting the patentholder's market power when using a per-unit royalty rate. We showed that its market power may be reduced in two ways: First, by preventing the patentholder's refusal to sell a license to an unsuccessful challenger. Second, by favoring collective challenges of patents' validity, particularly when competition intensity in the downstream market is so high that individual incentives to challenge a patent are weak.

One important question concerns the patent quality problem. Since the patent system involves a two-tier process combining patent office examination and challenge by a court of the validity of the granted patent, there are two possible approaches to this problem.

The first approach is to find some ways to encourage third parties to bring to a court pieces of evidence in order to challenge the validity of presumably weak patents (post-grant opposition in Europe or post-grant reexamination in the United States). Giving more incentives to potential licensees to challenge a patent validity is necessary insofar as the free riding aspect weakens individual incentives. In this perspective, two policy levers are suggested by our model: the renegotiation of the licensing contract with an unsuccessful challenger and the cooperative approach among potential licensees to collectively accept or refuse a licensing contract. Incentives to renegotiate could be encouraged by not allowing a patentee to refuse a license to an unsuccessful challenger. Allowing a joint decision for accepting or refusing a licensing contract may also reduce the patentholder's market power.

The second approach to the patent quality problem is to improve the screening

process inside the patent office itself through the strengthening of the patentability standards, turning back the Lemley's "rational ignorant patent office principle" (Lemley, 2001). This second approach could be interesting, particularly when the patent strength is no more common knowledge but a private information parameter (Chiou, 2008). The patent office could thus propose to any applicant a menu involving the choice of either paying an extra fee to obtain a thorough examination process at the patent office signalling thus a high patent quality or paying a lower fee to simply obtain a "standard" examination process that may signal the weakness of the patent. Designing an efficient mechanism to implement such a procedure is left for future investigation.

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APPENDIX

Proof of Lemma 1

Condition (1) is equivalent to:

$$\pi(c - \epsilon + r, c - \epsilon + r) - \theta\pi(c, c - \epsilon + r) - (1 - \theta)\pi(c - \epsilon, c - \epsilon) \geq 0$$

A3 implies that $\pi(c - \epsilon + r, c - \epsilon + r)$ is strictly decreasing in r , and A2 implies that $\pi(c, c - \epsilon + r)$ is increasing in r . It follows that $g(r) = \pi(c - \epsilon + r, c - \epsilon + r) - \theta\pi(c, c - \epsilon + r) - (1 - \theta)\pi(c - \epsilon, c - \epsilon)$ is strictly decreasing in r and continuous (by A1). Furthermore, $g(0) = \pi(c - \epsilon, c - \epsilon) - \theta\pi(c, c - \epsilon) - (1 - \theta)\pi(c - \epsilon, c - \epsilon) = \theta(\pi(c - \epsilon, c - \epsilon) - \pi(c, c - \epsilon)) \geq 0$ and $g(\epsilon) = \pi(c, c) - \theta\pi(c, c) - (1 - \theta)\pi(c - \epsilon, c - \epsilon) = (1 - \theta)(\pi(c, c) - \pi(c - \epsilon, c - \epsilon)) \leq 0$. Therefore, there exists a unique solution to the equation $g(r) = 0$ and this solution, denoted $r(\theta)$, belongs to the interval $[0, \epsilon]$. Moreover, since g is strictly decreasing, the condition $g(r) \geq 0$ is equivalent to $r \leq r(\theta)$.

Proof of Proposition 2

Differentiating the equation $\pi(c - \epsilon + r^c(\theta), c - \epsilon + r^c(\theta)) = \theta\pi(c, c) + (1 - \theta)\pi(c - \epsilon, c - \epsilon)$ with respect to θ , we get $\frac{dr^c(\theta)}{d\theta} = \frac{\pi(c, c) - \pi(c - \epsilon, c - \epsilon)}{(\pi_1 + \pi_2)(c - \epsilon + r^c(\theta), c - \epsilon + r^c(\theta))}$. Both the numerator and the denominator are negative which implies that $r^c(\theta)$ is increasing.

(i) Since $\pi(c, c) - \pi(c - \epsilon, c - \epsilon) < 0$ (A4), $\frac{dr^c(\theta)}{d\theta}$ is decreasing in θ over $[0, 1]$ (i.e. $r^c(\theta)$ is concave) if and only if $(\pi_1 + \pi_2)(c - \epsilon + r^c(\theta), c - \epsilon + r^c(\theta))$ is decreasing in θ over $[0, 1]$. Since $r^c(\theta)$ is continuous and strictly increasing from $r^c(0) = 0$ to $r^c(1) = \epsilon$, the latter condition is equivalent to $(\pi_1 + \pi_2)(x, x)$ is decreasing in x over $[c - \epsilon, c]$, which means that $x \rightarrow \pi(x, x)$ is concave over $[c - \epsilon, c]$. In this case, $r^c(\theta) \geq \theta r^c(1) + (1 - \theta)r^c(0) = \theta\epsilon$.

(ii) can be shown in a similar way.

Proof of Proposition 4

We first show that if $r < \epsilon$ it is impossible to have an equilibrium in which the number k of firms accepting the offer is strictly less than $n - 1$. If this was true then one of the $n - k \geq 2$ firms that have not accepted the licensing contract could get a higher expected profit by deviating unilaterally and accepting the contract. Indeed, if it deviates then litigation will still occur because there will remain at least one firm refusing the license offer. This would result in the deviating firm having a marginal cost $c - \epsilon + r$ instead of c in case the patent is upheld, while still having a marginal cost equal to $c - \epsilon$ if the patent is invalidated by the court. Hence, the number of firms accepting the license offer $r < \epsilon$ at equilibrium is at least equal to $n - 1$. This remains true for $r = \epsilon$ under the assumption that a firm accepts the offer when indifferent between accepting or refusing it. Furthermore, if $r \leq r(\theta)$, condition (1) shows that an equilibrium cannot involve $k = n - 1$ licensees. Thus i/ is proven.

If $r > r(\theta)$, an outcome in which one firm refuses the license offer while the others accept it is a Nash equilibrium: condition (1) shows that the firm refusing the offer gets a higher profit than if it had accepted it, and it has been shown that the remaining firms do not benefit from refusing the license since the patent will be challenged anyway. This proves ii/.

Part iii/ of the proposition is straightforward.

Proof of Lemma 5

Let $k \in \{n - 1, n\}$. Since the function $r q(c - \epsilon + r, k - 1)$ is concave in r and reaches its maximum at $\tilde{r}_k(\epsilon)$ then it is increasing over $[0, \tilde{r}_k(\epsilon)]$. Consequently, the following

holds:

$$\epsilon \leq \tilde{r}_k(\epsilon) \iff \frac{\partial}{\partial r} (rq(c - \epsilon + r, k))|_{r=\epsilon} \geq 0 \iff q(c, k) + \epsilon \frac{\partial}{\partial r} (q(c - \epsilon + r, k))|_{r=\epsilon} \geq 0$$

Let us compare $\frac{\partial}{\partial r} (q(c - \epsilon + r, n))|_{r=\epsilon}$ and $\frac{\partial}{\partial r} (q(c - \epsilon + r, n - 1))|_{r=\epsilon}$. It is clear that $q(c, n) = q(c, n - 1)$: both expressions refer to the individual output of a firm in a symmetric oligopoly consisting of n firms producing at marginal cost c . Thus, using assumption A5, we get: $\frac{q(c - \epsilon + r, n - 1) - q(c, n - 1)}{r - \epsilon} \leq \frac{q(c - \epsilon + r, n) - q(c, n)}{r - \epsilon}$ for all $r < \epsilon$. Taking the limit of both sides as $r \rightarrow \epsilon$, we obtain $\frac{\partial}{\partial r} (q(c - \epsilon + r, n))|_{r=\epsilon} \geq \frac{\partial}{\partial r} (q(c - \epsilon + r, n - 1))|_{r=\epsilon}$. Hence, $q(c, n) + \epsilon \frac{\partial}{\partial r} (q(c - \epsilon + r, n))|_{r=\epsilon} \geq q(c, n - 1) + \epsilon \frac{\partial}{\partial r} (q(c - \epsilon + r, n - 1))|_{r=\epsilon}$. Therefore, the following chain of implications holds:

$$\begin{aligned} \epsilon \leq \tilde{r}_{n-1}(\epsilon) &\implies q(c, n - 1) + \epsilon \frac{\partial}{\partial r} (q(c - \epsilon + r, n - 1))|_{r=\epsilon} \geq 0 \\ &\implies q(c, n) + \epsilon \frac{\partial}{\partial r} (q(c - \epsilon + r, n))|_{r=\epsilon} \geq 0 \implies \epsilon \leq \tilde{r}_n(\epsilon) \end{aligned}$$

Proof of Proposition 6

Assume that $\epsilon \leq \tilde{r}_{n-1}(\epsilon)$. By lemma (5), the inequality $\epsilon \leq \tilde{r}_n(\epsilon)$ holds as well. In this case the maximum of $P^*(r)$ over $[0, r(\theta)]$ is reached at $r(\theta)$, and its maximum over $]r(\theta), \epsilon]$ is reached at ϵ . Therefore, we must compare $nr(\theta)q(c - \epsilon + r(\theta), n)$ to $(n - 1)\theta\epsilon q(c, n - 1)$. Consider a royalty rate $r \in [0, \epsilon]$. The inequality $nrq(r, n) \geq (n - 1)\theta\epsilon q(c, n)$ is fulfilled if and only if $\frac{rq(c - \epsilon + r, n)}{\epsilon q(c, n)} \geq \frac{n-1}{n}\theta$. Since the function $r \rightarrow \frac{rq(c - \epsilon + r, n)}{\epsilon q(c, n)}$ is strictly increasing and continuous in r and takes the value 0 for $r = 0$ and 1 for $r = \epsilon$, there exists a unique solution to the equation $\frac{rq(c - \epsilon + r, n)}{\epsilon q(c, n)} = \frac{n-1}{n}\theta$, which is denoted by $s(\theta)$. The condition $\frac{rq(c - \epsilon + r, n)}{\epsilon q(c, n)} \geq \frac{n-1}{n}\theta$ can then be written as $r \geq s(\theta)$. Hence the inequality $nr(\theta)q(c - \epsilon + r(\theta), n) \geq (n - 1)\theta\epsilon q(c, n)$ amounts to $r(\theta) \geq s(\theta)$. The convexity of $s(\theta)$ can be derived from the concavity of $w : r \rightarrow rq(c - \epsilon + r, n)$ and its increasingness over $[0, \epsilon]$: differentiating twice the equation $\frac{w(s(\theta))}{w(\epsilon)} = \theta \frac{n-1}{n}$, we get $w''(s(\theta))(s'(\theta))^2 + w'(s(\theta))s''(\theta) = 0$ which leads to $s''(\theta) = -\frac{w''(s(\theta))(s'(\theta))^2}{w'(s(\theta))} > 0$. The property $s(0) = 0$ is immediate and the property $s(1) < \epsilon$ derives from $\frac{n-1}{n} < 1$.

Proof of Proposition 7

Assume that $\tilde{r}_{n-1}(\epsilon) < \epsilon \leq \tilde{r}_n(\epsilon)$. In this case, the maximum of $P(r)$ over $[0, r(\theta)]$ is reached at $r(\theta)$. Define $\tilde{\theta}_{n-1}$ as the unique solution in θ to the equation $r(\theta) = \tilde{r}_{n-1}(\epsilon)$

(it is straightforward to check that such a solution exists in $[0, 1]$ and is unique). Two subcases must be distinguished:

- Subcase 1: $\theta \leq \tilde{\theta}_{n-1}$: The maximum of $P(r)$ over $]r(\theta), \epsilon]$ is then reached at $\tilde{r}_{n-1}(\epsilon)$. Determining the royalty rate that maximizes $P(r)$ over $[0, \epsilon]$ amounts then to the comparison of $nr(\theta)q(c - \epsilon + r(\theta), n)$ and $(n-1)\theta\tilde{r}_{n-1}(\epsilon)q(c - \epsilon + \tilde{r}_{n-1}(\epsilon), n-1)$. The former is greater than the latter if and only if $r(\theta)$ is greater than $v(\theta)$ defined as the unique solution in r to the equation $nrq(c - \epsilon + r, n) = (n-1)\theta\tilde{r}_{n-1}(\epsilon)q(c - \epsilon + \tilde{r}_{n-1}(\epsilon), n-1)$. The existence, uniqueness, increasingness and convexity with respect to θ of such a solution can be established in a similar way to that of $s(\theta)$. The function $v(\theta)$ satisfies as well the properties $v(0) = 0$ and $v(1) < \epsilon$. The first inequality is straightforward to show and the second one derives from $n\epsilon q(c, n) > n\tilde{r}_{n-1}(\epsilon)q(c - \epsilon + \tilde{r}_{n-1}(\epsilon), n)$ (which holds because $\tilde{r}_{n-1}(\epsilon) < \epsilon \leq \tilde{r}_n(\epsilon)$) and $n\tilde{r}_{n-1}(\epsilon)q(c - \epsilon + \tilde{r}_{n-1}(\epsilon), n) > (n-1)\tilde{r}_{n-1}(\epsilon)q(c - \epsilon + \tilde{r}_{n-1}(\epsilon), n-1)$. Indeed these two inequalities result in $n\epsilon q(c, n) > (n-1)\tilde{r}_{n-1}(\epsilon)q(c - \epsilon + \tilde{r}_{n-1}(\epsilon), n-1) = nv(1)q(c - \epsilon + v(1), n)$ and consequently lead to $\epsilon > v(1)$.

- Subcase 2: $\theta > \tilde{\theta}_{n-1}$: The upper bound of $P(r)$ over $]r(\theta), \epsilon]$ is then reached at $r(\theta)^+$. From the expression of $P(r)$, it is clear that $P(r(\theta)) > P(r(\theta)^+)$. Hence, the maximum of $P(r)$ over $[0, \epsilon]$ is reached at $r(\theta)$. Consequently litigation is always deterred in this subcase.

Proof of Proposition 8

Assume that $\epsilon > \tilde{r}_n(\epsilon)$. By lemma (6) the inequality $\epsilon > \tilde{r}_{n-1}(\epsilon)$ holds as well. Analogously to $\tilde{\theta}_{n-1}$, define $\tilde{\theta}_n$ as the unique solution in θ to the equation $r(\theta) = \tilde{r}_n(\epsilon)$. Three subcases are distinguished:

- Subcase 1: $\theta \leq \min(\tilde{\theta}_{n-1}, \tilde{\theta}_n)$: The maximum of $P(r)$ over $[0, r(\theta)]$ is then reached at $r(\theta)$ and its maximum over $]r(\theta), \epsilon]$ is reached at $\tilde{r}_{n-1}(\epsilon)$. Hence the analysis conducted in subcase 1 in the proof of proposition 7 applies here.

- Subcase 2: $\tilde{\theta}_{n-1} < \theta < \tilde{\theta}_n$: The maximum of $P(r)$ over $[0, r(\theta)]$ is then reached at $r(\theta)$ and its maximum over $]r(\theta), \epsilon]$ is reached at $r(\theta)^+$. Therefore the maximum of $P(r)$ over $[0, \epsilon]$ is reached at $r(\theta)$ (see subcase 2 in the proof of proposition 7) which implies that litigation is deterred. Note that this subcase is not relevant if the inequality $\tilde{\theta}_{n-1} < \tilde{\theta}_n$ does not hold.

- Subcase 3: $\theta \geq \tilde{\theta}_n$: The maximum of $P(r)$ over $[0, r(\theta)]$ is then reached at $\tilde{r}_n(\epsilon)$. This is sufficient to state that the maximum of $P(r)$ over $[0, \epsilon]$ is reached at $\tilde{r}_n(\epsilon)$. This follows from the fact that the function $r \rightarrow nrq(c - \epsilon + r, n)$ reaches its *unconstrained*

maximum at $\tilde{r}_n(\epsilon)$ and $nrq(c - \epsilon + r, n) > \theta(n - 1)rq(c - \epsilon + r, n - 1)$ for any $r \in [0, \epsilon]$. The latter inequality results from assumption A7: $nq(c - \epsilon + r, n) = Q(c - \epsilon + r, n) \geq Q(c - \epsilon + r, n - 1) \geq (n - 1)q(c - \epsilon + r, n - 1)$.

Proof of Corollary 9

Assume that $r(\theta) > \theta\epsilon$. Since $s(\theta)$ is a convex function such that $s(0) = 1$ and $s(1) < \epsilon$ then $s(\theta) \leq \theta\epsilon$ for all $\theta \in [0, 1]$. Consequently a sufficient condition for the inequality $r(\theta) \geq s(\theta)$ to hold is that $r(\theta) \geq \theta\epsilon$. The same conclusion applies for the convex function $v(\theta)$. Given this, the first part of the corollary follows immediately from the three previous propositions.

Using the three previous propositions, it is straightforward to check that under the conditions $r(\theta) \geq s(\theta)$ and $r(\theta) \geq v(\theta)$ (which hold when $r(\theta) > \theta\epsilon$), the optimal royalty rate set by the patentholder simplifies as follows: $r^*(\theta) = \min(r(\theta), \tilde{r}_n(\epsilon))$. Using the inequality $r(\theta) > \theta\epsilon$, we get $r^*(\theta) > \min(\theta\epsilon, \tilde{r}_n(\epsilon)) > \min(\theta\epsilon, \theta\tilde{r}_n(\epsilon)) = \theta \min(\epsilon, \tilde{r}_n(\epsilon)) = \theta r^*(1)$. Hence for all $\theta \in [0, 1]$, $\theta r^*(1) < r^*(\theta) = \min(r(\theta), \tilde{r}_n(\epsilon))$. Since the function $P(r) = nrq(c - \epsilon + r, n)$ is concave in r over $[0, r(\theta)]$ then $P(\theta r^*(1)) > \theta P(r^*(1))$ and since it reaches its maximum at $\tilde{r}_n(\epsilon)$, it is increasing over $[0, r^*(\theta)]$ which entails that $P(r^*(\theta)) > P(\theta r^*(1))$. From the two previous inequalities, we obtain that $P(r^*(\theta)) > \theta P(r^*(1))$.

Proof of Proposition 12

The situation where the n firms accept the licensing contract F is a Nash equilibrium if and only if:

$$\pi^L(n) - F \geq \theta \pi^{NL}(n - 1) + (1 - \theta) \pi^L(n)$$

which can be rewritten as:

$$F \leq \theta (\pi^L(n) - \pi^{NL}(n - 1)) \tag{6}$$

that is

$$F \leq F_n(\theta)$$

A situation where $n - 1$ firms accept the licensing contract and one firm does not is a Nash equilibrium (of stage 2) if and only if:

$$\theta \pi^{NL}(n - 1) + (1 - \theta) \pi^L(n) \geq \pi^L(n) - F \tag{7}$$

and

$$\theta[\pi^L(n-1) - F] + (1-\theta)\pi^L(n) \geq \theta\pi^{NL}(n-2) + (1-\theta)\pi^L(n) \quad (8)$$

Condition (7) means that the one firm that does not accept the licensing contract and challenges the patent's validity does not find it optimal to unilaterally deviate by accepting the licensing contract. Condition (8) means that none of the $n-1$ firms which accept the licensing contract find it optimal to unilaterally deviate by refusing the contract. When the number of firms accepting the contract is strictly less than n , litigation will occur (lemma 11) which entails that the firms accepting the contract pay the fixed fee F only if the patent validity is upheld, which happens with probability θ . With the complementary probability $1-\theta$, the patent is invalidated and all the firms get the same profit namely $\pi^L(n)$. It is straightforward to show that conditions (7) and (8) are equivalent to the following double inequality:

$$\theta[\pi^L(n) - \pi^{NL}(n-1)] \leq F \leq \pi^L(n-1) - \pi^{NL}(n-2)$$

that is

$$F_n(\theta) \leq F \leq F_{n-1}$$

Note that the inequality $\theta[\pi^L(n) - \pi^{NL}(n-1)] < \pi^L(n-1) - \pi^{NL}(n-2)$ follows immediately from A9 for $\theta = 1$ and is *a fortiori* satisfied for $\theta < 1$.

A situation where $k \leq n-2$ firms accept the licensing contract and the remaining do not is a Nash equilibrium of the stage 2 subgame if and only if:

$$\theta(\pi^L(k) - F) + (1-\theta)\pi^L(n) \geq \theta\pi^{NL}(k-1) + (1-\theta)\pi^L(n) \quad (9)$$

and

$$\theta\pi^{NL}(k) + (1-\theta)\pi^L(n) \geq \theta(\pi^L(k+1) - F) + (1-\theta)\pi^L(n) \quad (10)$$

Condition (9) means that none of the k firms accepting the licensing contract finds it optimal to unilaterally deviate by refusing the contract and condition (10) means that none of the $n-k$ firms refusing the licensing contract finds it optimal to unilaterally deviate by accepting the contract. It is easy to see that conditions (9) and (10) can be combined into the following double inequality that does not depend on θ :

$$\pi^L(k+1) - \pi^{NL}(k) \leq F \leq \pi^L(k) - \pi^{NL}(k-1)$$

that is:

$$F_{k+1} \leq F \leq F_k$$

Note that the inequality $\pi^L(k+1) - \pi^{NL}(k) \leq \pi^L(k) - \pi^{NL}(k-1)$ follows from A9. Thus, the role of assumption A9 is to guarantee that the set of values of F belonging to the interval $[F_{k+1}, F_k]$ is not empty.

A situation where no firm accepts the licensing contract is a Nash equilibrium if and only if:

$$\theta \pi^{NL}(0) + (1 - \theta) \pi^L(n) \geq \theta (\pi^L(1) - F) + (1 - \theta) \pi^L(n)$$

which can be rewritten as:

$$\pi^{NL}(0) \geq \pi^L(1) - F$$

or equivalently as:

$$F \geq \pi^L(1) - \pi^{NL}(0) = F_1$$