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Abtract:

A phenomenon called « piping » often occurs in hydraulics works (earthdams, dykes), involving the formation and the progression of a continuous tunnel between the upstream and the downstream side. It is one of the main cause of failure. Starting from the bulk equations for diphasic flow with diffusion and from the jump equations with erosion, we propose a simplified model for dense piping flows with erosion. Numerical simulation with constant input and output pressure shows that the influence of the particles concentration can be signifiant in the beginning of the process, resulting in the enlargment of the hole at the exit.

Résumé :

Un phénomène appelé renard hydraulique survient souvent sur les ouvrages hydrauliques (barrages, digues), conduisant à la formation et au développement d'un tunnel continu entre l'amont et l'aval. C'est l'une des causes principales de rupture. A partir des équations d'écoulement diphasique avec diffusion, et des équations de saut avec érosion, nous proposons un modèle simplifié pour les écoulement denses de conduit avec érosion. Les résultats numériques avec pression constante en entrée et en sortie montrent que l'influence de la concentration en particles solides peut être significative au début de l'évolution, conduisant à un élargissement du trou en sortie.

Key-words :

piping erosion; dense flow; turbulent flow

1 Introduction

Erosion of soil resulting from piping, seepage or overtopping is the main cause of serious hydraulic work (dykes, dams) failure, in terms of the risk of flooding areas located downstream. The present study concerns the first process: the enlargement of a crack, which leads to an erosion process known as "piping" in soil mechanics. Piping erosion often occurs in hydraulics works, involving the formation and the progression of a continuous tunnel between the upstream and the downstream side.

A large literature on soil erosion exists in the field of hydraulics and river engineering (Chanson, 1999), and in the field of poromechanics and petroleum engineering (Vardoulakis *et al.*, 1995). In the field of geomechanics, Bonelli *et al.* (2006) have proposed a model for dilute piping flows with erosion. We focus here on the dense flow case.

2 Two-phase flow equations with interface erosion

The soil is eroded by the flow, which then carries away the eroded particles. As long as the particles are small enough in comparison with the characteristic length scale of the flow, this two-phase flow can be said to be a continuum. We take Ω to denote the volume of the two-phase mixture and Γ the fluid/soil interface. For the sake of simplification, sedimentation and deposition processes are neglected, and the soil is taken to be saturated. The mass conservation equations for the water/particles mixture and for the mass of the particles as well as the balance equation of momentum of the mixture within Ω can be written as follows in a Eulerian framework (Nigmatulin, 1990):

$$\frac{\partial\rho}{\partial t}+\vec{\nabla}\cdot(\rho\vec{u})=0\,,\quad \frac{\partial\rho Y}{\partial t}+\vec{\nabla}\cdot(\rho Y\vec{u})=-\vec{\nabla}\cdot\vec{J}\,,\quad \frac{\partial\rho\vec{u}}{\partial t}+\vec{\nabla}\cdot(\rho\vec{u}\otimes\vec{u})=\vec{\nabla}\cdot\boldsymbol{\sigma}\,.$$

In these equations, ρ is the density of the mixture, depending on the particles mass fraction Y, $\vec{u}$ is the mass-weighted average velocity, $\vec{J}$ is the mass diffusion of the flux of particles, and $\boldsymbol{\sigma}$ is the Cauchy stress tensor in the mixture.

As there is a process of erosion, a mass flux crosses the interface Γ. Let us take $\vec{n}$ to denote the normal unit vector of Γ oriented outwards from the soil, and $\dot{v}_\Gamma$ to denote the normal velocity of Γ. The jump equations over Γ are

$$\left[\!\left[\rho(\vec{v}_\Gamma-\vec{u})\cdot\vec{n}\right]\!\right]=0\,,\quad \left[\!\left[\rho Y(\vec{v}_\Gamma-\vec{u})\cdot\vec{n}\right]\!\right]=\left[\!\left[\vec{J}\cdot\vec{n}\right]\!\right],\quad \left[\!\left[\rho\vec{u}(\vec{v}_\Gamma-\vec{u})\cdot\vec{n}\right]\!\right]=-\left[\!\left[\boldsymbol{\sigma}\cdot\vec{n}\right]\!\right],$$

where $[\![a]\!]=a_s-a_w$ is the jump of any physical variable a across the interface, and a_s and a_w stands for the limiting value of a on the solid and fluid sides of the interface, respectively. The soil is assumed to be homogeneous, rigid and devoid of seepage. The co-ordinate system depends on the soil in question. The total flux of eroded material (both particles and water) crossing the interface is therefore $\dot{m}=-\rho_s\vec{v}_\Gamma\cdot\vec{n}$, where ρ_s is the density of the saturated soil.

Erosion laws dealing with soil surface erosion by a tangential flow are often written in the form of threshold laws such as (Chanson, 1999):

$$\dot{m}=k_{er}(\left|\tau_w\right|-\tau_c)\text{ if }\left|\tau_b\right|>\tau_c,\ 0\text{ otherwise}\,,$$

where τ_c is the critical shear stress involved in the erosion, k_{er} is the coefficient of soil erosion, and τ_w is the tangential shear stress at the interface defined as follows:

$$\left|\tau_w\right|=\sqrt{(\boldsymbol{\sigma}\cdot\vec{n})^2-(\vec{n}\cdot\boldsymbol{\sigma}\cdot\vec{n})^2}\Big|_w\,.$$

3 Application to dense piping flows with erosion

The use of above equations is extended here to the study of two-phase piping flow with erosion, by introducing a spatial integration over a cylinder Ω with radius R (initial value R_0) and length L (Fig. 1). The dilute flow case has been adressed by Bonelli *et al.* (2006): this assumption is relevant only when the pipe is not “too long” (i.e when $L\bar{u}k_{er}\ll R_0$ where $\bar{u}$ is the average longitudinal velocity). This hypothesis is not necessary here.

We simplify above equations in a boundary layer theory spirit in order to obtain the Reduced Navier Stokes/Prandtl equations (Lagree & Lorthois, 2005). We therefore integrate the obtained system on a cross section. For simplification, some assumptions are made: the tangential velocities are supposed continuous across Γ (no-slip condition on the interface), and the concentration is uniform in a section. Of course, the concentration is not uniform, and it influences most probably the velocity profil. However, to our knowledge, radial profil of velocity and concentration in turbulent pipe flow with erosion remains to be investigated.

We take ρ^p and ρ^w to denote the solid and water density, respectively, and ϕ to denote the volume fraction of the solid phase. Volume fraction and mass fraction are linked as follows: $\rho^p\phi = \rho Y$. The mixture density is therefore $\rho = \phi(\rho^p - \rho^w) + \rho^w$. In particular, the saturated soil density is $\rho_s = (\rho^p - \rho^w)\phi_s + \rho^w$ where is $\phi_s = 1 - n$ is the compacity of the soil, while n is the porosity.

The following system is obtained:

$$\frac{\partial R}{\partial t} = \frac{\dot{m}}{\rho_s}, \quad \frac{\partial \overline{u}}{\partial x} + 2\frac{\overline{u}}{R}\frac{\partial R}{\partial x} = 0\,, \tag{1}$$

$$\frac{\partial \overline{\phi}}{\partial t} + \beta\overline{u}\frac{\partial \overline{\phi}}{\partial x} = \frac{2\dot{m}}{R\rho_s}(\phi_s - \overline{\phi}), \quad \overline{\rho}\left(\frac{\partial \overline{u}}{\partial t} + \beta\overline{u}\frac{\partial \overline{u}}{\partial x}\right) = \frac{2}{R}(\tau_w - \beta'\overline{u}\dot{m}) - \frac{\partial p}{\partial x}. \tag{2}$$

where $\overline{a}$ denotes the mean value of a across any section, and

$$\beta = \frac{\overline{u^2}}{\overline{u}^2}, \quad \beta' = 1 + (\beta - 1)\left(1 - \frac{\overline{\rho}}{\rho_s}\right).$$

Although turbulence models of considerable sophistication are now commonly used in single-phase fluid flows, the same cannot be said for the two-phase flow involving heavy particles of varying concentration. We use a strong assumption for the behaviour law:

$$\tau_w = -\overline{\rho} f_w(R_e) f_m(\overline{\phi})\overline{u}^2\,.$$

The influence of the concentration is taken from the quadratic rheological law for hyperconcentration of Julien (1995), while that of the Reynolds number is taken from Barenblatt (1997):

$$f_m(\overline{\phi}) = 1 + c_B\frac{\rho^d}{\overline{\rho}}\left(\frac{d^d}{l_m}\right)^2\lambda(\overline{\phi}),\ \lambda(\overline{\phi}) = \left[\left(\frac{\phi_s}{\overline{\phi}}\right)^{1/3} - 1\right]^{-1},$$

$$f_w(R_e) = \left(\frac{2^{\alpha}\alpha_e(1+\alpha_e)(2+\alpha_e)}{e^{3/2}(\sqrt{3}+5\alpha_e)}\right)^{2/(1+\alpha_e)},\ \alpha_e = \frac{3}{2\ln R_e},$$

where c_B is the Bagnold coefficient, d^p is the (eroded) soil particles diameter, and l_m is the mixing length taken to be $l_m = c_l R$. The coefficient β is evaluated with the radial velocity profil proposed by Barenblatt (1997) for turbulent pipe flows:

$$\beta = \frac{(1+\alpha_e)(2+\alpha_e)^2}{4(1+2\alpha_e)}.$$

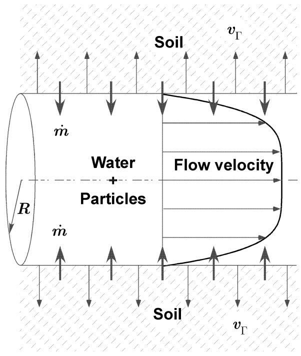

FIG. 1 – Axisymmetrical flow with soil erosion and transport of the particles.

4 Numerical results

The system Eqs. (1), (2) is solved numerically with unknowns $(\overline{\phi}, R, p, \overline{u})$. Initial conditions and boundary conditions are

$$R(0,x) = R_0,\ \overline{\phi}(0,x) = 0,\ \overline{u}(0,x) = 0,\ p(0,x) = 0\,,$$

$$\overline{\phi}(t,0) = 0,\ p(t,0) = p_{in}(t),\ p(t,L) = 0\,.$$

These boundary conditions require a specific numerical procedure, of which the formulation is beyond the scope of the present paper. We consider a unit step of input pressure $p_{in}(t) = 0$ if $t < 0$, p_{in}^* if $t > 0$. Erosion occurs if $P_0 > \tau_c$ where $P_0 = R_0 p_{in}^* \,/\, (2L)$ is the initial driving pressure. Numerical values of parameters are given in table I. These values correspond to a pipe in a highly erosive soil (see e.g. Wan & Fell, 2004), of a large earthdam.

Fig. 2 shows the temporal evolutions of (ϕ, R, p). The concentration reaches high values (close to ϕ_s), and then decreases. The radius has an exponential-like evolution with time. The pressure is somewhat influenced by the concentration: the average value of p increases a bit, then decreases a bit, and then stays at a constant mean value. The velocity is given by the total mass conservation equation, which can be re-written as $\overline{u} = \overline{u}_0 (R \,/\, R_0)^{1/2}$ where $\overline{u}_0$ is the initial velocity. Fig. 3 gives (ϕ, R, p) in relation to x for successive times until t =20 mn. The concentration profiles shows the accumulation of particles until the maximum value (close to ϕ_s), then the slow decrease to a dilute flow. High values of output concentration lead to enlargment of the output hole. This effect is progressively erased when dilute flow occurs. The pressure profile is affected during the dense flow period, and then shows a classical linear-like decrease.

Fig. 4 shows the three-dimensionnal plot of the hole at time t =9 mn. Taking into account the many simplifying assumptions, namely those concerning the radial profiles of concentration and velocity, this nice picture shows that this simple model is able to capture some of the main features of the piping erosion.

R_0	L	p_{in}^*	c_B	d^p	c_l	ρ^p	ρ^w	ϕ_s	τ_c	k_{er}
3 mm	100 m	3.33 MPa	0.01	1 mm	0.1	2700 kg/m^3	1000 kg/m^3	0.5	10 Pa	10^{-2} s/m

TABLE. 1 – Numerical values of parameters.

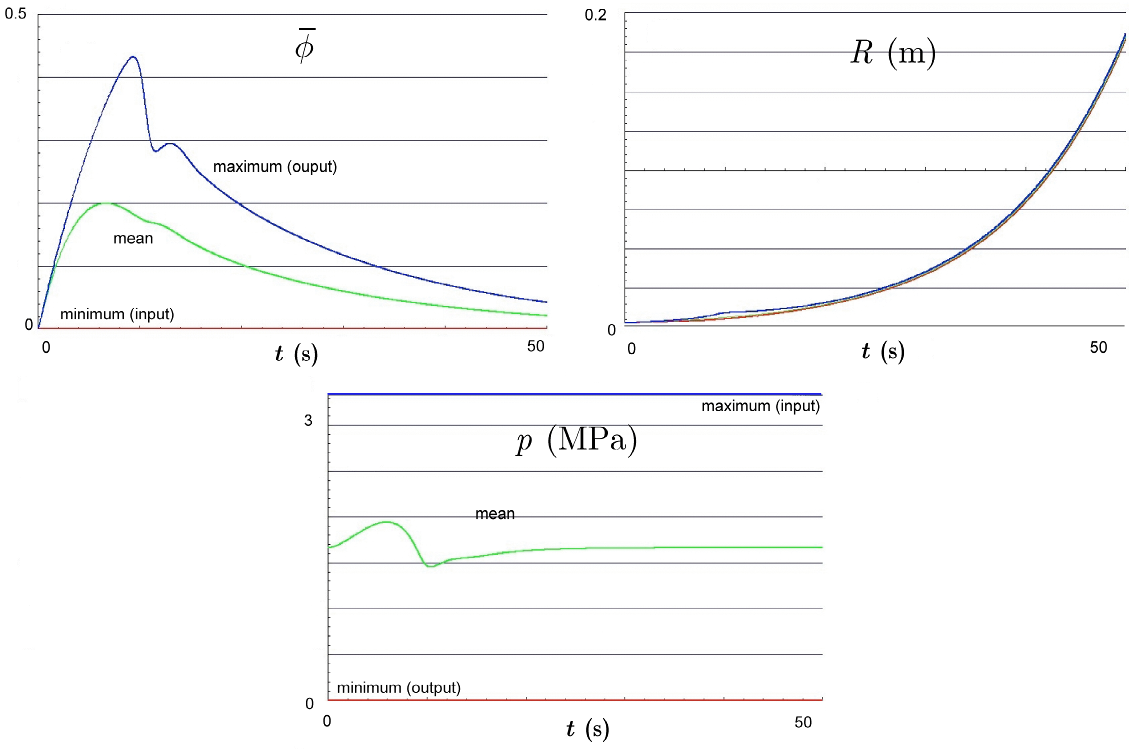

FIG. 2 – Temporal evolutions of $(\overline{\phi}, R, p)$, minimum, mean and maximum values.

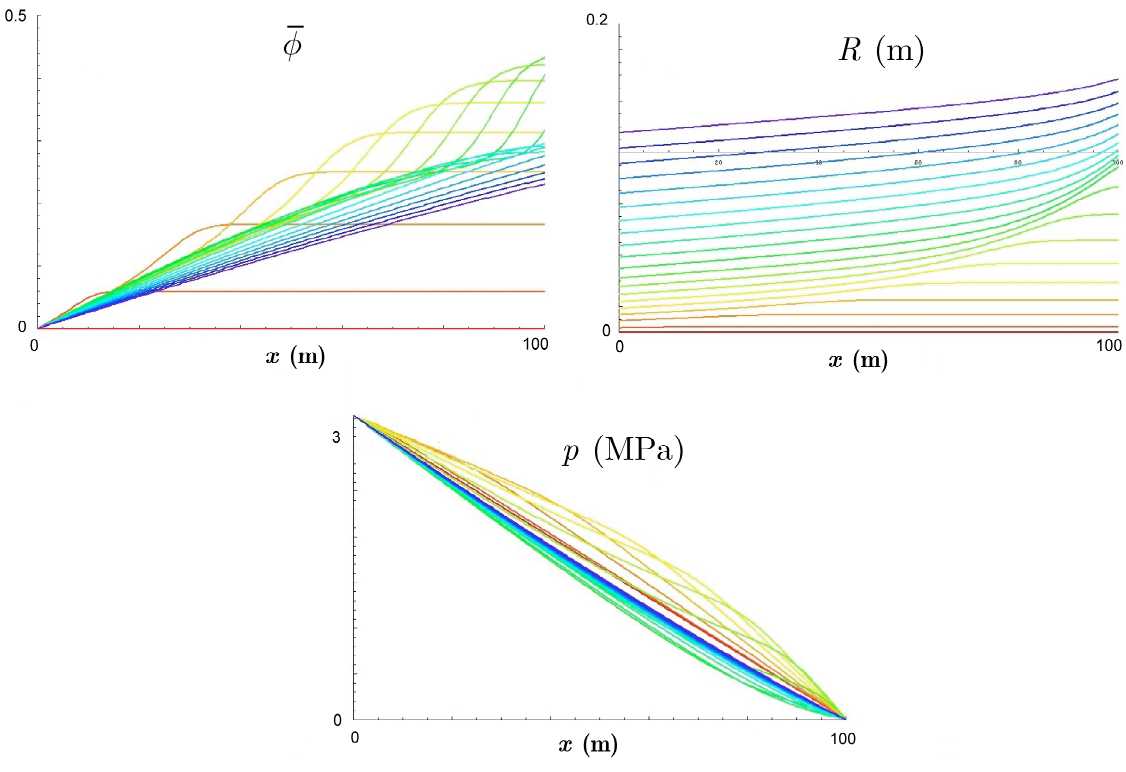

FIG. 3 – Spatial profiles of $(\overline{\phi}, R, p)$.

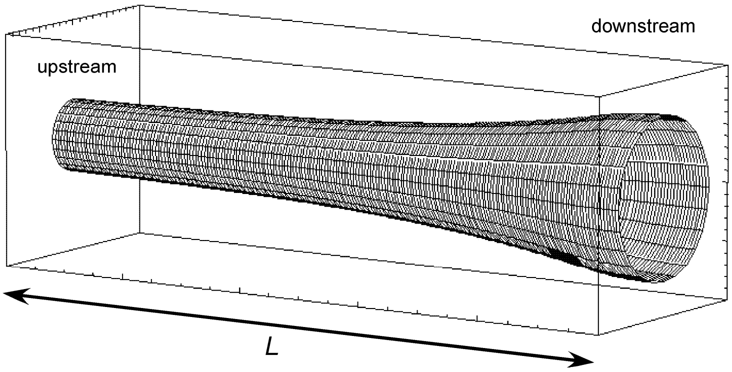

FIG. 4 – Three-dimensionnal plot of the hole

6 Conclusions

We propose a simplified model for piping dense flows with erosion, able to capture some of the main features of the piping erosion. Numerical simulation under constant input and output pressure condition shows that the influence of the particles concentration can be signifiant in the beginning of the process, resulting in the enlargment of the hole at the exit. However, such a modelling needs informations concerning radial profiles of velocity and concentration in turbulent pipe flow with erosion. To our knowledge, this issue has not been addressed experimentaly, nor numerically.

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