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# OBSERVABILITY, REDUNDANCY, RELIABILITY AND INTEGRATED DESIGN OF MEASUREMENT SYSTEMS

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**Abstract.** The problem of sensor placement is considered. The quality of diagnosis depends on the number and location of measurements. We first deal with this dependence by analysing the observability. The concepts of degree of redundancy is introduced. The criterion of reliability is useful for designer to enhance the reliability of the measurement system. When some variables are required for the control of the process, the reliability of the measurement system may be computed taking into account the reliability of all sensors. In order to improve the diagnosis, we propose an original and computational tool for designer to conceive the automatic and optimal sensor placement. Our objective is to design the measurement system meeting the following constraints: observability of variables required for the controlling and the maintenance, fulfilment of different degrees of redundancy imposed for some variables and fulfilment of criterion of sensor cost or/and reliability.

**Key Words:** Degree of redundancy, observability, reliability, canonical matrix, cycle analysis.

## 1. INTRODUCTION

It is important for process control improvement to integrate the design of the measurement system while conceiving the process itself. The main problem to define a measurement system concerns the selection of variables to be measured; especially the state estimation of the process is influenced by this choice. Indeed, estimation techniques are only applicable for observable systems; one of the fundamental steps then consists in isolating observable parts of the process. Publications on observability concepts are numerous; (Vaclavek 1969) is one of the firsts to undertake the linear system study. (Mah et al., 1976) and (Gomolka et al., 1992) develop a technique based on the graph theory. One can also quote the works performed for bilinear systems in the domain of data reconciliation (Kretsovalis et al., 1988; Crowe 1989; Maquin et al., 1989 and Ragot et al., 1991).

Some synthesis works deal with this topic; indeed, in order to carry out a diagnosis, a minimum number of information provided from the sensors of the process is required. If the amount of information is not sufficient, a number of supplementary sensors is proposed so as to complete "poor" parts in information of the process (Vaclavek 1969; Ragot et al., 1986; Darouach et al., 1986 and Maquin et al., 1987). The conception of measurement system for bilinear models has also been published (Ragot et al., 1992). Thus, the observability concept has provided one of the criteria for the improvement of the measurement systems. More recently, optimization study for the global reliability of the measurement system taking into account simultaneously observability and sensor reliability has been studied (Turbatte et al., 1993).

Our purpose is focused on the conception of sensor placement by considering the reliability of sensors. Based on the concepts of observability, redundancy and reliability, this communication presents an automatic sensor placement technique allowing to minimize the global cost of the instrumentation. More precisely, given the list of variables required for the control, the maintenance and the safety of the process (in the following, we only use the word control instead), we seek the optimal sensor placement that meets the following

objectives: 1) guaranteeing the observability of variable required for the control 2) guaranteeing degrees of redundancy of some variables 3) ensuring minimal cost of the instrumentation.

The proposed algorithm is based on the analysis of cycles of the graph associated with a process. It presents an interest as much for its facility of implementation as for its rapidity with regard to matrix methods (Darouach et al., 1986 and Maquin et al., 1987). Having concern for the simplicity, we limit this communication to the linear system study. It is organized as follows: the section 2 is relative to the process description; the section 3 presents the analysis of the process according to the concepts of observability, redundancy and reliability; using the analysis results of section 3, the fourth section proposes an algorithm to solve the problem of sensor placement and presents some numerical results.

## 2. DESCRIPTION OF A PROCESS

A process can be described by equations linking different variables; in a linear system, these equations may be represented by a graph made up of  $m$  arcs and  $n$  nodes. The complete set of equations of the process can be written as:

$$M X = 0 \quad (1)$$

where  $M$  represents the  $(n,m)$  node incidence matrix of rank  $n$ , and  $X$  the  $(m,1)$  vector of variables of the system. Considering  $L$ , the set of variables of the process, it is divided into two following subsets:  $L_1$  is the subset of variables required for the control and  $L_2$  the subset of variables non required for the control. In the case of a process equipped with sensors, measured and unmeasured variables can be distinguished into the two lists  $X_m$  and  $X_{\bar{m}}$ .

As an example, let us consider the process network of figure 1; it can be described by the node incidence matrix  $M$  (table 1), where the first row contains the numbers of variables, and the first column the numbers of equations (nodes).

The subsets of variables required and non required for the control respectively are:

$$L_1 = \{1, 4, 6, 9, 10\} \quad L_2 = \{2, 3, 5, 7, 8\}$$

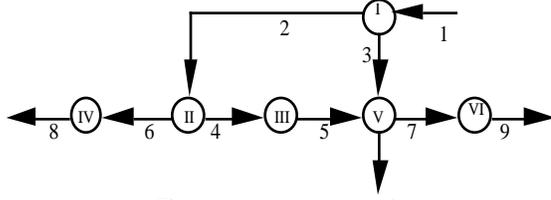


Fig. 1. A process network

	1	2	3	4	5	6	7	8	9	10
I	1	-1	-1	.	.	.	.	.	.	.
II	.	1	.	-1	.	-1	.	.	.	.
III	.	.	.	1	-1	.	.	.	.	.
IV	.	.	.	.	1	.	.	-1	.	.
V	.	.	1	.	1	.	-1	.	.	-1
VI	.	.	.	.	.	.	1	.	-1	.

Table 1. Node incidence matrix M

From this process network, the construction of a cyclic graph is possible by connecting arcs corresponding to its feeds (entries) and its products (exits) to a so-called environment node. This graph is interesting because it allows all the necessary cycles for the implementation of algorithms that we will present to be determined (Berge 1989).

### The cycle matrix

It is important to note that a process graph is independent of its measurement system. Especially, cycles are the same whether the process is supplied with sensors or not. The research of cycles depends on the determination of a spanning tree of the graph. The literature proposes several methods among which the method of Gauss-Jordan is retained. This one uses linear equation combinations (rows) and variable exchanges (columns) in order to obtain a canonical form of the node incidence M (n.m) called the canonical node incidence matrix or the fundamental cutset matrix (Maquin et al., 1987; Gomolka et al., 1992):

$$M_C = [I_C \mid M_{CS}] \quad (2)$$

where  $I_C$  is the identity matrix (n.n) whose columns correspond to the branches of the spanning tree, and  $M_{CS}$  the singular matrix (n.(m-n)) whose columns correspond to the chords. The fundamental cycle matrix  $C_F$  ((m-n). m) is defined by

$$C_F = [C_{FS} \mid I_F] \quad (3)$$

$$(C_{FS})^T = -M_{CS} \quad (4)$$

If we are solely interested in the occurrence of a variable in a cycle, the absolute value (of terms) of the matrix  $C_F$  is used and noted  $C_{FA}$ . The matrix containing all the cycles generated from the fundamental cycles will be noted  $C_T$ .

**Theorem 1:** a cycle containing k different chords results from the ring sum of k fundamental cycles containing these k chords.

**Proof:** If we consider the expressions (2) and (3), the variables of the identity submatrix  $I_F$  columns correspond to the chords. Then, k distinct fundamental cycles are composed of k distinct chords, therefore k different arcs. The ring sum of these k fundamental cycles is the same operation as the deletion of common arcs of these cycles. As these k chords are distinct, there are no possible deletion of these k chords.

### Algorithm of cycle generation

Let us denote  $c = m-n$ , the number of fundamental cycles of a connected graph and  $C_c^k$  the binomial number of k elements among c. According to the theorem 1, each of the c fundamental cycles contains a chord. Cycles formed by the addition of k distinct fundamental cycles contain k

chords. The number of these cycles is  $C_c^k$ . To obtain all the cycles, we use the recursive procedure:

- by using the fundamental cycle matrix, we generate cycles containing 2 chords,
- then, we generate cycles containing 3 chords from cycles containing 2 chords and fundamental cycles,
- more generally, we generate cycles containing k chords from cycles containing (k-1) chords and fundamental cycles.

The total number of cycles is given by Nbc:

$$Nbc = C_c^1 + C_c^2 + \dots + C_c^c = \sum_{i=1}^c C_c^i \quad (5)$$

This algorithm will be useful both for the analysis of a process equipped with sensors and for the conception of a measurement system.

## 3. ANALYSIS OF AN INSTRUMENTED PROCESS

In this section, we introduce the concepts of observability, degree of redundancy of a variable and reliability.

### 3.1. Matrix of all the cycles of a graph

For an instrumented process, we define four lists whether a variable is measured or not and according to its membership to  $L_1$  and  $L_2$ :

- $X_{1m}$ : measured variables required for the control,
- $X_{1\bar{m}}$ : unmeasured variables required for the control,
- $X_{2m}$ : measured variables non required for the control,
- $X_{2\bar{m}}$ : unmeasured variables non required for the control.

These four lists allow the columns of the matrix M to be rearranged as follows:

<-----measured----->		>-----unmeasured-----<	
$X_{1m}$	$X_{2m}$	$X_{1\bar{m}}$	$X_{2\bar{m}}$
$M_1$	$M_2$	$M_3$	$M_4$

Table 2. Node incidence matrix M

Then, this matrix is restructured. Using Gauss-Jordan elimination, we first construct the greatest identity matrix  $I_{C2}$  with unmeasured variables. Then, if the rank of this matrix is not equal to n, we reiterate the operation with measured variables, which generates the matrix  $I_{C1}$ . The matrix  $M_5$  concerns the remaining unmeasured variables. The submatrices  $M_6$ ,  $M_7$  and  $M_8$  are constituted of remaining measured variables. Thus, we obtain the canonical matrix  $M_C$  with the following form:

me	$\bar{m}$	$\bar{m}\bar{e}$	$m\bar{e}$	me
$I_{C1}$	0	0	0	$M_8$
0	$I_{C2}$	$M_5$	$M_6$	$M_7$

Table 3. Canonical matrix  $M_C$

For each list  $L_i$  ( $i = 1, 2$ ), we define:

- $X_{ime}$ : variables corresponding to the identity submatrix  $I_{C1}$  and the submatrices  $M_8$  and  $M_7$
- $X_{i\bar{m}}$ : variables corresponding to the identity submatrix  $I_{C2}$
- $X_{i\bar{m}\bar{e}}$ : variables corresponding to the submatrix  $M_5$
- $X_{im\bar{e}}$ : variables corresponding to the submatrix  $M_6$

The cycle matrix  $C_{FA}$  is easily obtained, using the relationships (3) and (4).

me	$\bar{m}$	$\bar{m}\bar{e}$	$m\bar{e}$	me
0	$ M_5^T $	$I_{F1}$	0	0
0	$ M_6^T $	0	$I_{F2}$	0
$ M_8^T $	$ M_7^T $	0	0	$I_{F3}$

Table 4. Fundamental cycle matrix

For the process of the figure 1 where  $X_m = \{3, 4, 5, 8, 9, 10\}$  and  $X_{\bar{m}} = \{1, 2, 6, 7\}$ , the cycle matrix, where measured variables are in bold type, is presented in the

table 5 (the fundamental cycle matrix corresponds to the four first rows of this matrix).

4	9	1	6	2	7	8	10	3	5
.	.	1	1	1	.	1	.	.	.
.	1	.	.	.	1	.	1	.	.
1	1	1	.	1	1	.	.	1	1
.	1	1	1	1	1	1	1	.	.
1	1	.	1	1	1	.	.	1	1
.	.	1	.	.	.	.	1	1	1
1	.	1	.	1	.	.	1	1	1
.	.	.	1	1	.	1	1	1	1
1	.	.	1	.	.	1	1	.	1
1	.	1	1	.	.	1	.	1	1
1	1	.	.	1	1	.	1	1	1
1	1	1	1	.	1	1	1	1	1

Table 5. Cycle matrix  $C_T$

### 3.2. Observability concepts

The observability analysis consists in classifying variables according to four categories. A deducible and measured variable (redundant) is a measured variable whose value can still be deduced from measurements of the other variables of an equation when the sensor measuring this variable fails. A measured and non deducible variable is a measured variable whose value cannot be deduced from other variables of an equation when the sensor measuring this variable fails. An unmeasured and deducible variable is an unmeasured variable whose value can be obtained from other variables of an equation. The equation in which a deducible variable occurs is a deduction equation. An unmeasured non deducible variable is an unmeasured variable appearing only in equations comprising two unmeasured variables at least.

Measured variables that are deducible or not and unmeasured deducible variables are observable variables. The canonical matrix structure of the table 3 leads to the variable classification by researching redundancy and deduction equations. However, we are going to show that the use of cycles facilitates this classification.

#### Rules of observability

The observability analysis is based on the study of cycles, using the four next rules:

Rule I: a measured variable is deducible if, and only if, it only belongs to cycles where at least two variables are measured.

Rule II: a measured variable is not deducible if, and only if, it at least belongs to a cycle where it is the only measured variable.

Rule III: an unmeasured variable is deducible if, and only if, it only belongs to cycles comprising one measured variable at least.

Rule IV: an unmeasured variable is not deducible if, and only if, it at least belongs to a cycle where no variable is measured.

#### Observability algorithm

Using these four rules, we propose the following algorithm for the observability study:

- 1) computing the cycle matrix  $C_T$
  - 2) computing cycles comprising from 0 to 2 measured variables among cycles of  $C_T$ , (the knowledge of cycles with more than two measurements is not necessary according to the rule I)
  - 3) detecting cycles which do not contain measured variables: unmeasured variables belonging to these cycles are unmeasured and non deducible variables (rule IV). The other unmeasured variables of the system are unmeasured but deducible variables.
  - 4) detecting cycles containing only one measured variable: measured variables belonging to these cycles are measured and non deducible variables (rule II). The other measured variables of the system are deducible measured variables.
- The cycle matrix comprising a maximum number of two measured variables is obtained from the cycle matrix  $C_T$  (table 5) and is presented in the table 6:

4	9	1	6	2	7	8	10	3	5
.	.	1	1	1	.	1	.	.	.
.	1	.	.	.	1	.	1	.	.
.	1	1	.	.	1	.	.	1	.
.	.	1	.	.	.	.	1	1	.

Table 6. Matrix of cycles comprising two measured variables at the most per cycle.

After having analysed this matrix, we can classify the variables in the following lists according to their observability state:

$$\begin{aligned} X_{m\bar{e}} &= \emptyset & X_{m\bar{e}} &= \{1, 2, 6, 7\} \\ X_{m\bar{e}} &= \{8\} & X_{me} &= \{3, 4, 5, 9, 10\} \end{aligned}$$

### 3.3. Degree of redundancy

**Definition 1:** a redundant variable of degree k is an observable variable whose value remains deducible in case of simultaneous failures of any k sensors.

Especially, a minimal observable variable is a variable of zero redundancy degree; a variable belonging to a redundancy equation is redundant of degree 1 at least.

**Definition 2:** an unmeasured variable is pseudo-redundant (the difference with a redundant variable is that a pseudo-redundant variable is not measured) of degree 1 if it at least belongs to an equation where all the other variables are redundant or pseudo-redundant of degree 1.

By applying the next rules (V and VI), one can more generally deduce the redundancy degree of a measured variable, and similarly the pseudo-redundancy degree of an unmeasured variable:

Rule V: a measured variable is redundant of degree k if, and only if, it only belongs to cycles where at least (k+1) variables are measured.

Rule VI: an unmeasured variable is pseudo-redundant of degree k if, and only if, it only belongs to cycles where at least (k+1) variables are measured.

The redundancy degree of a variable is found by computing the minimum number of measured variables contained in cycles (among these of  $C_T$ ) to which this variable belongs. If this number is equal to k, then the redundancy degree is (k-1).

Again, consider the process of the figure 1. From the cycle matrix (table 5), we obtain the following results:

- the measured variable 8 that belongs to the only cycle (the first) containing one measurement is a minimal observable variable.
- the unmeasured variables 1, 2 and 6 belong to the only cycle where one variable is measured. They are pseudo-redundant variables of degree 0 or minimal observable variables.
- the measured variables 3, 9 and 10 are redundant of degree 1 because they belong to cycles whose minimum number of measured variables is two.
- the measured variables 4 and 5 are redundant of degree 2 because they belong to cycles whose minimum number of measured variables is three.

According to the preceding results, values of the variables 3, 9 and 10 are available even if any one sensor fails. Values of the variables 4 and 5 remain always available in case of simultaneous failures of any two sensors.

In practice, the obtaining of the redundancy degrees of variables is interesting since that allows the security of the process functioning to be guaranteed.

### 3.4. Reliability of a measurement system

The measurement system function is to provide necessary information for the control of the process. Then, this function is not fulfilled if the measurement of a required variable for the control is no longer available. The reliability of an instrumentation system is defined as the probability that information required for the control are available through measurements or deduction during the time interval [0, t]. Then, it is advisable to compute the

number of sensor failures conserving the observability of the variables required for the control (list  $L_1$ ).

### Assumptions

*H0*: sensors are considered irreparable.

*H1*: there is no breakdown of common cause.

*H2*: the failure rate  $\lambda$  of a sensor is independent of the time, which allows the reliability of a sensor to be defined analytically by using for example, the law of Poisson:

$$r(t) = \exp(-\lambda t) \quad (6)$$

*H3*: initially, the system is observable.

By definition, the reliability of a sensor  $i$  is the probability  $r_i(t)$  that no failure occurs during the time interval  $[0, t]$ . More generally, the reliability of a system constituted of  $p$  sensors with respective reliabilities  $r_i(t)$  may be expressed by:

$$R(t) = f(r_1(t), r_2(t), \dots, r_p(t)) \quad (7)$$

We also define the MTTF function (Mean Time To Failure) by:

$$MTTF = \int_0^{\infty} R(t) dt \quad (8)$$

The calculation of the MTTF may or may not take into account the redundancy equations. If we do not consider the model, the failure of a sensor measuring a value required for the control cannot be admitted. The reliability  $R_0(t)$  is the probability that all sensors well function. If the model is taken into account, some sensor failures can be tolerated. Indeed, one might estimate the value of a variable whose sensor has failed through redundant equations owing to the model. Practically, it is necessary to determine:

- the maximum number  $v$  of admissible failed sensors conserving the observability of the variables required for the control.
- the probability  $R_i(t)$  so that the variables required for the control are available when  $i$  sensors ( $i=1, \dots, v$ ) fail.

Finally, the reliability of the measurement system is expressed by:

$$R(t) = \sum_{i=0}^v R_i(t) \quad (9)$$

It is obvious from (9) that the reliability increases if analytic relationships of the process are taken into account. It is shown that there is a relationship linking the observability, the degree of redundancy and the reliability.

The computation of the reliability necessitates to enumerate the cases where the failure of sensors can be tolerated. The value of a variable whose sensor fails is no longer known if, among all cycles to which this variable belongs, there is at least one cycle which do not contain measured variables (Rule IV). The availability of the value of a variable therefore depends on the number of measured variables in the cycle possessing the less ones; the more this cycle has measured variables, the more the measurement system can tolerate sensor breakdowns.

The computation of the maximal number  $v$  of admissible failed sensor is based on the following theorem.

**Theorem 2:** in a connected graph of  $(n+1)$  nodes, the maximal number of deducible variables through others is  $n$ ; these variables are carried by the branches of the considered spanning tree.

**Proof:** considering the expression (2) and the canonical matrix (table 3) whose regular part variables are carried by the branches of the spanning tree. It is obvious that these variables are deducible through the knowledge of variable values of the singular part of the matrix.

### 3.5. Classification of variables required for the control

The canonical matrix structure (table 3) allows redundancy and deduction equations of variables required for the control to be determined (subset  $L_1$ ). Indeed, variables are classified in the following classes ( $i=1,2$ ):

$X_{1me}$ : deducible measured variables of  $L_1$

$X_{1\bar{m}}$ : unmeasured variables of  $L_1$

$X_{2me}$ : unmeasured deducible variables of  $L_1$

$X_{2\bar{m}\bar{e}}$ : unmeasured non deducible variables of  $L_1$

$X_{1\bar{m}\bar{e}}$ : measured non deducible variables of  $L_1$

The unmeasured deducible variables of  $X_{1\bar{m}}$  belong to the rows of the submatrix  $I_{C2}$ . These rows correspond to null rows of  $M_5$ . From the canonical matrix (table 3), variables of the list  $X_{2me}$  are obtained by rearranging  $M_5$  to make its null rows appear. As the system is initially observable (according to *H3*), non deducible variables of  $X_{1\bar{m}\bar{e}}$  do not exist. Since we are interested in the observability of variables of the list  $L_1$ , the equations containing variables of the list  $L_2$  as well as the columns corresponding to variables of the lists  $X_{2me}$ ,  $X_{2\bar{m}\bar{e}}$  are suppressed. We obtain the following reduced matrix:

	(a)	(b)	(c)	(d)	(e)	(f)	(g)
	$X_{1me}$	$X_{2me}$	$X_{1\bar{m}\bar{e}}$	$X_{1\bar{m}\bar{e}}$	$X_{2\bar{m}\bar{e}}$	$X_{1me}$	$X_{2me}$
1	$I_{C11}$					$M_{81}$	
2		$I_{C12}$					
3			$I_{C211}$		$M_{61}$	$M_{71}$	

Table 8. Reduced canonical matrix  $M_C$

Finally, columns of bloc (g) are reordered to obtain zero submatrix in the zone (g1):

	(a)	(b)	(c)	(d)	(e)	(f)	(ga)	(gb)
	$X_{1me}$	$X_{2me}$	$X_{1\bar{m}\bar{e}}$	$X_{1\bar{m}\bar{e}}$	$X_{2\bar{m}\bar{e}}$	$X_{1me}$	$X_{2me}$	$X_{2me}$
	$I_{C11}$					$M_{811}$		
		$I_{C12}$						$M_{812}$
			$I_{C211}$		$M_{61}$	$M_{711}$		$M_{712}$

Table 9. Reduced canonical matrix  $M_C$

If we consider the theorem 2 and the "horizontal bands" (1) and (3) of the table 8, the knowledge of the values of variables corresponding to the columns of (d), (e), (f), (ga) and (gb) allows the deduction of necessary variables (columns of (a) and (c)) in case of simultaneous failure of sensors measuring variables corresponding to the columns of (a) and (b). The number  $v$  is equal to  $(m_a + m_b)$  where  $m_a$  et  $m_b$  are respective numbers of columns of (a) and (b). If the submatrix  $M_{712}$  of dimension  $(n_3, m_{gb})$  is a null matrix ( $n_3$  is the total number of lines of the matrix  $I_{C211}$ ), the equations (2) of the table 8 will no longer be useful for the deduction of necessary variables. Then, the number  $v$  is equal to  $(m_a + m_b + m_{gb})$ .

### 3.6. Reliability under constraint

The problem consists in determining the number of failures conserving the observability of variables required for the control (list  $L_1$ ). If equations of the process are not considered, the probability that all sensors function is equal to:

$$R_0(t) = r(t)^p \quad (10)$$

On the other hand, if we take into account these equations, the probability that a sensor fails while  $(p-1)$  others ( $p > 1$ ) well function is:  $(1-r(t))r(t)^{p-1}$ . If  $\alpha_1$  is the number of possibilities to consider a failed sensor while ensuring the observability of necessary variable, the probability that the system is operational while any one sensor fails is

$$R_1(t) = \alpha_1 (1-r(t))r(t)^{p-1} \quad (11)$$

More generally, we have:

$$R_i(t) = \alpha_i (1-r(t))^i r(t)^{p-i} \quad (12)$$

where the coefficient  $\alpha_i$  is the total number of configurations that admit a number  $i$  of failed sensors

while ensuring the observability of variables required for the control of the process.

The problem consists in computing  $R_i(t)$ . We therefore compute the coefficients  $\alpha_i$  for  $i = 0, \dots, v$ . According to the definition of the coefficient  $\alpha_i$ , cycles comprising only variables non required for the control (list  $L_2$ ) have not to be counted. Furthermore, we eliminate cycles whose number of measured variables is higher than  $v$ . These cycles are not useful for the proposed analysis in the case where the number of simultaneous failures is less or equal to  $v$ . We finally obtain a so-called  $C_{T_v}$  cycle matrix whose analysis comprises three steps:

**step 1:** extracting among cycles of the matrix  $C_{T_v}$  those whose number of measured variables varies from 1 to  $i$  (cycles with  $k$  measured variables such that  $k$  is higher than  $i$  will have one measured variable at least when  $i$  sensors fail simultaneously, which is not interesting for the research of non admitted combinations),

**step 2:** forming  $C_p^1$  combinations of  $i$  failed sensors among  $p$  sensors. For each combination, variables measured by failed sensors are considered as unmeasured ones. We then calculate the number of measured variables for each cycle. According to the observability rule III, if all cycles include a measured variable at least, the considered combination is admissible,

**step 3:** deducing  $\alpha_i$  which is equal to the number of admissible combinations, then  $R_i(t)$ .

Consider the example of the figure 1 and the following lists:

$$L_1 = \{1, 4, 6, 9, 10\} \quad L_2 = \{2, 3, 5, 7, 8\}$$

$$X_m = \{3, 4, 5, 8, 9, 10\} \quad X_{\bar{m}} = \{1, 2, 6, 7\}$$

The greatest admissible number of failed sensors is equal to 2. Cycles of the graph are presented in the table 6. The matrix  $C_{T_2}$  whose cycles contain 2 measured variables at the most, is given in the table 10:

4	9	1	6	2	7	8	10	3	5	N
.	.	1	1	1	.	1	.	.	.	1
.	1	.	.	.	1	.	1	.	.	2
.	1	1	.	.	1	.	.	1	.	2
.	.	1	.	.	.	.	1	1	.	2

Table 10. Matrix of cycles  $C_{T_2}$ .

Let us note  $\lambda$  the common failure rate of all sensors. The coefficient  $\alpha_0$  is equal to 1. Among six possibilities to consider a sensor failing, only the possibility to consider the failure of the sensor measuring the variable 8 is not admitted (according to the cycle 1). Thus,  $\alpha_1$  is equal to 5. Combinations including the variable 8 are not retained. Besides, next combinations are not accepted either: (9,10), (9,3), (10,3) according to the last three cycles. We have 7 admissible combinations for  $\alpha_2$ . Therefore, the function of reliability is:

$$R(t) = e^{-6\lambda t} + 5(1 - e^{-\lambda t})e^{-5\lambda t} + 7(1 - e^{-\lambda t})^2 e^{-4\lambda t}$$

Then, the value of the MTTF is  $0.45/\lambda$

## 4. DESIGN OF MEASUREMENT SYSTEM

### 4.1. Position of the problem and objective

We are provided with the subset of variables required for the control of the process (list  $L_1$ ). A weight, that is proportional to the cost of sensor, is assigned to each variable. Among the variables of the subset  $L_1$ , some are important for the functioning security; these are "high availability" variables. This demand may be ensured by analytic redundancy. According to the previous rules and especially the rule V, we can obtain a redundancy degree  $k$  for a high availability variable solely if the minimum number of measurable variables of all cycles to which this variable belongs is greater or equal to  $k+1$ . Necessary variables (subset  $L_1$ ) are assumed measurable. Therefore, the purpose is the selection of variables to be measured

with regard to the criterion of the minimum sensor cost so as to ensure:

- the minimal observability of variables required for the control ( $L_1$ ),
- redundancy degrees of variables of the sets  $Ld_k$ .

Having concern for the simplicity and without loss of generality, the value of  $k$  is limited to 1.

### 4.2. Algorithm

It comprises three steps: the first consists to create the matrix of all cycles, the second to select an optimal set of variables to be measured with respect to the criterion of cost and minimal observability of necessary variables (list  $L_1$ ), the third to choose supplementary variables to be measured in order to ensure redundancy degree 1 of variables of the set  $Ld_1$  while minimizing the corresponding sensor cost and taking into account measured variables given by the second step.

#### step 1: useful cycle matrix

From the lists  $L_1$  and  $L_2$ , the procedure of researching a spanning tree is the same as that of section 3. The measured and unmeasured variables might be replaced by those of the lists  $L_1$  and  $L_2$  respectively. Then, we deduce the fundamental cycle matrix  $C_{FA}$  and the cycle matrix  $C_T$ .

#### step 2: minimal observability of variables required for the control.

According to the rule V, we need to measure a variable at least per cycle containing a variable of  $L_1$ . We simplify this research by eliminating, among cycles of  $C_T$ , those containing only variables of the subset  $L_2$ . The resulting matrix  $C_1$  of dimension  $(n_{c1}, m_{c1})$  is obtained. During this step, we first compute all possible sets of variables to be measured  $L_{int}$ , then select one,  $L_{s1}$  which satisfies the cost criterion.

#### step 3: redundancy of variables of the set $Ld_1$ .

According to the rule V, we have to measure two variables at least per cycle containing a variable of  $Ld_1$ . First, all possible sets of variables to be measured are computed. Before selecting a list satisfying the cost criterion, we take into account the proposed measured variables of the precedent list  $L_{s1}$ . Then, we select the list  $L_{s2}$  fulfilling the cost criterion.

Finally, the set of variables to be measured fulfilling the objective is the union of the lists  $L_{s1}$  and  $L_{s2}$ .

## 4.3. Results

Consider the process network of the figure 1 with  $L_1 = \{1, 4, 6, 9, 10\}$  and  $L_2 = \{2, 3, 5, 7, 8\}$ , we wish to define the measurement system with a redundancy degree 1 for variables of the list  $Ld_1 = \{1, 9\}$ . To each variable is associated a weight representing the cost of the sensor (table 11).

Variable	Cost	Variable	Cost
1	3	6	4
2	3	7	8
3	4	8	7
4	1	9	2
5	9	10	5

Table 11. Costs associated with variables.

**step 1:** the cycle matrix is the same as that presented in the table 6. It only includes cycles containing at least a variable of the list  $L_1$ ; let us note it by  $C_1$ .

**step 2:** the whole possible measurement lists for the selection (lists of intersection  $L_{in}$ ) is not presented here due to their sizes. Taking into account the constraint of minimum cost, the following variables are to be measured to guarantee the minimal observability of the list  $L_1$ :  $L_{s1} = \{1, 2, 4, 9\}$

**step 3:** the variables 1, 2, 4 and 9 are considered as measured ones, the matrix  $C_{11}$  contains cycle number 2, 8, 11 and 12 of the matrix  $C_1$  (because they contain less than two measured variables). Besides, we have suppressed columns of  $C_1$  corresponding to variables 1 and 9 (list  $Ld_1$ ). The resultant matrix  $C_{11}$  is the following.

4	6	2	7	8	10	3	5
.	.	.	1	.	1	.	.
.	1	1	.	1	1	1	.
1	1	.	.	1	1	.	1

Table 12. Cycle matrix  $C_{11}$

We need to measure one variable of each list of the lists (corresponding to a cycle of  $C_{11}$ ): {7,10}, {10,3}, {6,2,8,10,3}, {4,6,8,10,5}. Finally, the supplementary variable to be measured is the variable 10.

In conclusion, the variables to be measured that meet the objective while minimizing the sensor cost are {1, 2, 4, 9, 10}. The corresponding total cost is 14. The reader may verify that cycles (of the matrix  $C_1$ ) containing variables 1 and 9 comprise at least two measured variables. We can calculate the reliability of this measurement system with the help of the algorithm of the section 3. The evolution of the reliability is represented by  $R(t)$ . The calculation is carried out with an identical failure rate for each sensor being equal to  $1.25 \cdot 10^{-4}$ . The calculation of MTTF is in fact represented by the area of the surface limited between the curve  $R(t)$  and the time axis. In the figure 3, the curve in solid line represents the reliability  $R(t)$  of the system with the measured variable list {1, 2, 4, 9, 10}. As a comparison, the curve in dashed line represents the reliability of the system obtained with the measured variable list {1, 2, 4, 9}, that only ensures the minimal observability of variables required for the control.

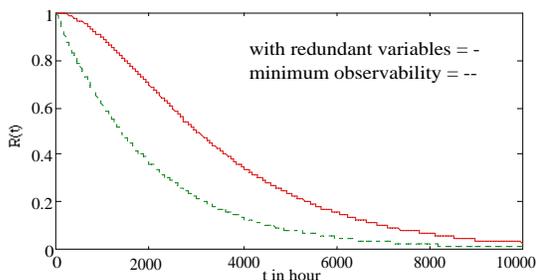


Fig. 3. Evolution of the reliability.

For the measurement system ensuring only the minimal observability of variables required for the control, we obtain a value of MTTF being equal to  $0.25/1.25 \cdot 10^4$ . The measurement system satisfying the objective gives a better value of MTTF with  $0.45/1.25 \cdot 10^4$ . We observe that the system ensuring the redundancy degree 1 of variables 1 and 9 is more reliable than that not providing redundancy equations.

If the cost of sensors is not taken into account, several configurations may satisfy the objectives of minimal observability and degrees of redundancy but present a less important reliability with a same number of sensors. Let us consider a measurement system (number 1) involving the measurements of the variables {1, 2, 4, 9, 10} and the second one (number 2) involving the measurements of variables {1, 4, 6, 9, 10}. It is easy to verify that the second measurement system meets the previous requirements. All the variables required for the control are observable and variables 1 and 9 are redundant of degree 1. The figure 4 shows the reliability of the two measurement systems.

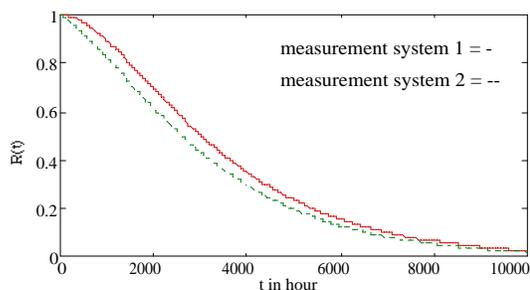


Fig. 4. Comparison of the evolution of the reliability. From this figure, it is obvious that the reliability of the measurement system 2 is inferior than that of measurement system 1 and so is the MTTF ( $0.4/1.25 \cdot 10^4$ , for the

measurement system 2) although the number of measurements is the same.

## 5. CONCLUSION

We have presented an analysis of the observability, the redundancy degree and the reliability; this technique is based on the analysis of cycles of the graph associated with the process. This analysis has been applied to the conception of the measurement system of a process. The proposed solution gives the optimum number of sensors and their location in order to: obtain (1) the minimal observability of variables required for the control, (2) redundancy degrees of variable of high degree of availability while minimizing the cost of measurements. In the case where several solutions provide a same minimum cost, we can use a complementary criterion taking into account the reliability. The use of cycles is all the more interesting for the conception of measurement system as it allows one to obtain a list of measurements easily, according to one or several given criteria. The solution of a multicriterion problem including precision of estimation, sensitivity to a modification of precision, observability, redundancy degree and reliability is a future direction of research.

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