Detection of the boundaries of time-frequency patterns with a reassignment-based method

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Abstract

A boundary detection of time-frequency patterns of the spectrogram based on the reassignment vector field is proposed. Given that the spectrogram tends to spread the time-frequency patterns, whereas reassignment moves the spread energy back to a pattern's point, reassignment vectors aim at the pattern. Looking for time-frequency areas with homogeneous reassignment vector angles leads to a boundary detection.

1. Introduction

Reassignment, originally proposed by Kodera *et al.* in 1976 [KVG76], reintroduced in 1993 by Augier and Flandrin [AF93], is a non-linear method which creates a new time-frequency representation by moving the spectrogram values away from their computation place. Reassignment focuses component energy, by moving each (t, f) location to the local gravity center of the signal distribution around (t, f). This new location may be equivalently defined as the (t_g, f_i) site, where t_g and f_i are the estimated group delay and instantaneous frequency respectively. Reassignment is thus a use of the phase information of the Short Time Fourier Transform (STFT).

We obtain a reassignment vector field $((t_g - t), (f_i - f))$, associated to a given spectrogram, which describes how time-frequency locations are reassigned. A time-frequency segmentation based on the reassignment method has already been proposed by Chassande-Mottin *et al.* in 1998 [CM98], which splits the time-frequency plane in different basins of attraction, where attractors are the (t,f) locations invariant to reassignment.

Reassignment is known to be efficient for the analysis of frequency modulation, but is inefficient for the analysis of wide band signals, consequently for time-frequency regions where only noise is present.

In this paper, we consider the reassignment vector field only as a source of information about the time-frequency structure of the signal. Spectrogram is known to spread spectral patterns. Signal energy is spread and jut out from the ideal signal pattern. Reassignment will move this energy back to the pattern. Consequently, contiguous time-frequency locations around the pattern boundaries will be associated to reassignment vectors with approximately same angles aiming at the pattern. By searching over the time-frequency representation for local homogeneity of the reassignment vector angle, we will be able to determine the boundaries of the patterns.

In the first section, the reassignment principle is recalled. The second section presents the algorithm of boundaries detection. The last one points out perspectives for future works.

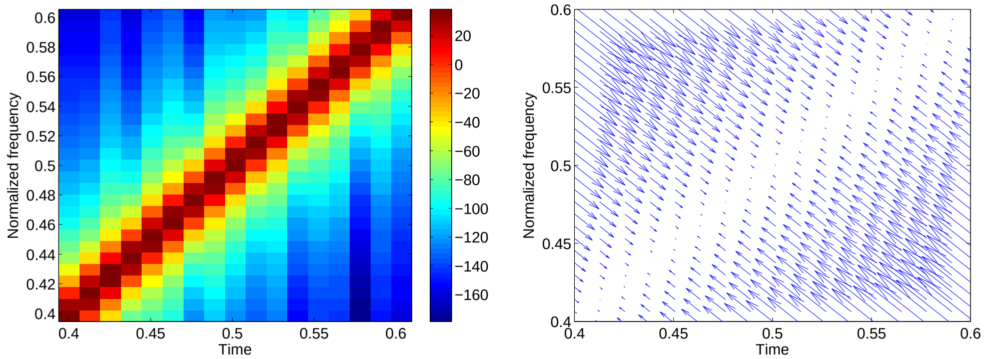

FIGURE 1. Example of a linear chirp $x(t) = e^{i2\pi\beta t^2/2}$, $\beta = 1$. On the left, the spectrogram of a linear chirp, on the right, the associated theoretical reassignment vector field. The energy contained in the time-frequency locations are moved on the line $f = \beta t$.

2. Reassignment principle

In order to reassign the spectrogram computed from the STFT $F_x^{(h)}(t,f)$ of the signal $x(t)$ defined as

$$F_x^{(h)}(t,f) = \int_{-\infty}^{+\infty} x(t-\tau)h^*(\tau)e^{-2\pi f\tau}d\tau, \tag{2.1}$$

where $h(t)$ is the window function, we introduce two auxiliary STFT, based on the windows $T(t) = t \times h(t)$ and $D(t) = (d/dt)h(t)$. These three STFT give the new location $(t_g(t,f), f_i(t,f))$ of the (t,f) spectrogram value $|F_x^{(h)}(t,f)|^2$ estimated by [AF93]

$$(t_g(t,f), f_i(t,f))^{\mathrm{T}} = \left(t - \mathrm{Re}\left\{\frac{F_x^{(T)}(t,f)}{F_x^{(h)}(t,f)}\right\}, f - \frac{1}{2\pi}\mathrm{Im}\left\{\frac{F_x^{(D)}(t,f)}{F_x^{(h)}(t,f}\right\}\right)^{\mathrm{T}}, \tag{2.2}$$

where $F_x^{(T)}$ and $F_x^{(D)}$ are the STFT computed with windows $T(t)$ and $D(t)$ respectively. We define the reassignment vector $\mathbf{r}(t,f)$ to each (t,f) location such as

$$\mathbf{r}(t,f) = (t_g(t,f) - t, f_i(t,f) - f)^{\mathrm{T}}. \tag{2.3}$$

FIG. 1 shows an example of such a reassignment vector field for a linear chirp $x(t) = e^{i2\pi\beta t^2/2}$, $\beta = 1$. Theoretically, the reassignment vector field computed with a Gaussian window is [CM98]

$$\mathbf{r}(t,f) = \frac{\sqrt{2}}{2}(f - \beta t, \beta t - f)^{\mathrm{T}}, \tag{2.4}$$

that means that all reassignment vectors are aiming at the instantaneous frequency line $f = \beta t$.

3. Boundaries detection

On a pattern boundary, all reassignment vectors aim at the pattern. Locally, we consider that the boundaries variations are smooth enough to have almost parallel reassignment vectors.

We propose thus an algorithm looking for the time-frequency locations where associated reassignment vectors and all of its neighbours have close angles.

Due to the discretization of the spectrogram, reassignment vectors associated to con-

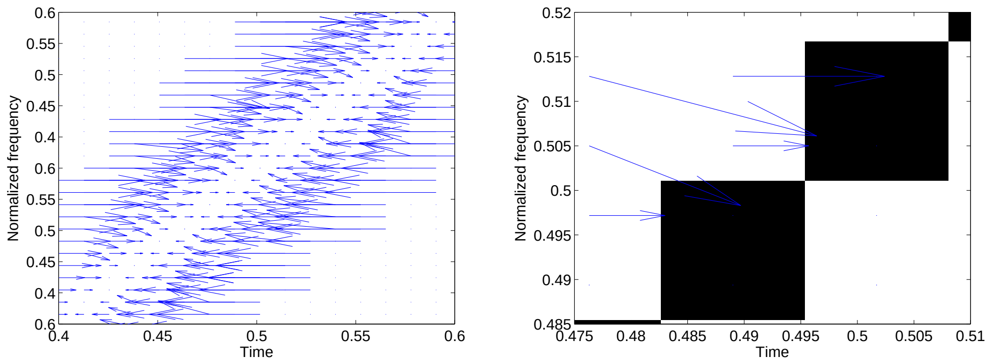

FIGURE 2. On the left, the real computed reassignment vector field of FIG. 1. On the right, a zoom on some reassignment vectors. The black squares indicate the highest values of the spectrogram, where the energy is moved.

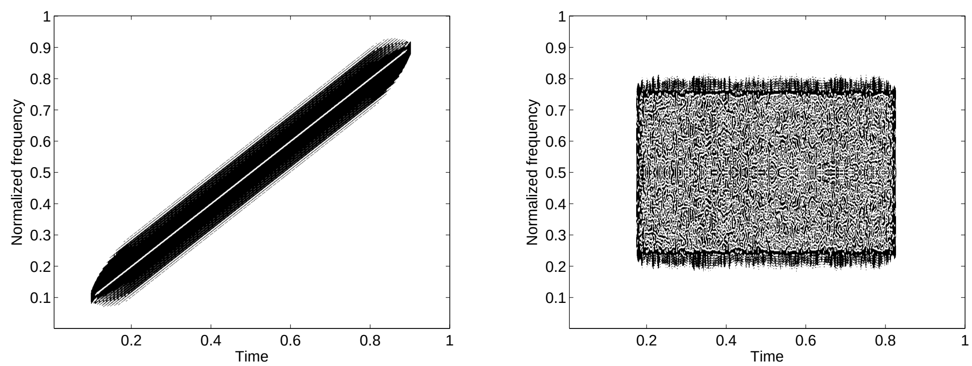

FIGURE 3. Results of the boundaries detection. On the left, the result for a deterministic linear chirp. On the right, the result for a frequency-filtered white Gaussian noise.

tiguous time-frequency locations don't have equal angles. FIG. 2 shows this effect on the linear chirp of FIG. 1. The spectrogram energy is reassigned to the nearest discrete location corresponding to the theoretical line $f = \beta t$. Two neighbour reassignment vectors may be aiming at two different points, leading to different angles.

Consequently, we would say that reassignment vectors having close angles means that the difference between the considered angles is lower than a given threshold θ_0. This threshold allows on the first hand to avoid the discretization problem and on the other hand to follow the variations of the pattern's boundaries. We take arbitrarily threshold $\theta_0 = \pi/4$. The detection algorithm returns an indicator denoted $\det(t, f)$ such as

$$\det(t,f) = \begin{cases} 1 \text{ if } |\text{angle}(\mathbf{r}(t,f)) - \text{angle}(\mathbf{r}(t',f'))| \leq \theta_0 \quad \forall (t',f') \in N_{t,f}, \\ 0 \text{ otherwise,} \end{cases} \tag{3.1}$$

where angle($\mathbf{r}$) is the angle of vector $\mathbf{r}$ and $N_{t,f}$ the considered l_1xl_2 neighbourhood around (t, f).

FIG. 3 gives two examples for a 3x3 neighbourhood. The first is the result on a deterministic linear chirp windowed in time, the second is obtained on a filtered Gaussian noise. The STFT were computed with a 127 points Hanning window, an overlap of 120 points and 512 computed frequencies.

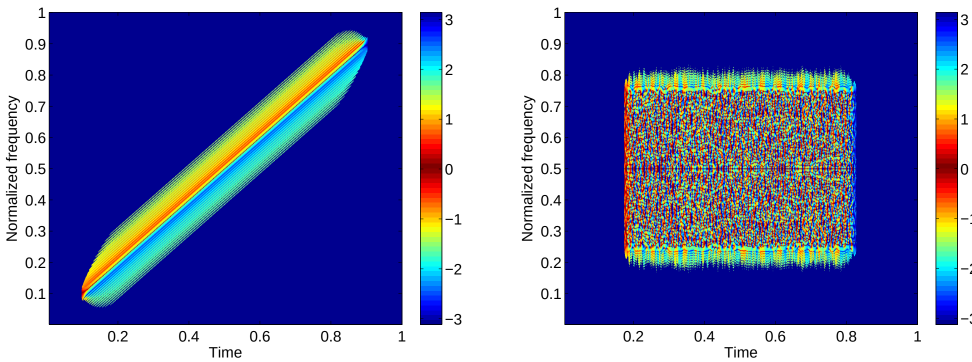

FIGURE 4. Values of the reassignment vectors angles, for the signals used in FIG. 3. The colorbar displays the same color for π and $-\pi$, in dark blue. Reassignment vectors aiming down and aiming up have the same color in light green.

4. Conclusion and Perspectives

We proposed a detection algorithm of boundaries of time-frequency patterns. First results on synthetic signals are promising.

In future work, we propose to consider the correlation between the time-frequency coefficients, in order to determine a suitable neighbourhood's size for the research of similar angles in the reassignment vectors field, currently fixed to 3x3.

A second idea will be to filter the reassignment vector field before detection. In the context of a signal embedded in a white Gaussian noise, we want to remove the small local variations due to the noise, and keep the variations due to the spectrogram spreading. A median filter on the angle of the reassignment vectors field seems to be an interesting choice.

Another goal is to consider the value of the reassignment vectors angles, in order to construct a closed contour. As shown on FIG. 4, there is a continuous variation of this angle along the pattern's contour, all pointing at the pattern.

Finally, we want to take advantage of the information of boundaries detection in a time-frequency segmentation, based on the statistical features of the STFT [MHM06,MM06]. Close signal patterns may be segmented in a single class. A boundary detection may indicate what are the patterns containing more than one component.

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