# *Log or linear? Distinct intuitions of the number scale in Western and Amazonian indigene cultures*

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## Abstract

The mapping of numbers onto space is fundamental to measurement and to mathematics. Is this mapping a cultural invention, or a universal intuition shared by all humans regardless of culture and education? We probed number-space mappings in the Mundurucu, an Amazonian indigene group with a reduced numerical lexicon and little or no formal education. At all ages the Mundurucu mapped symbolic and non-symbolic numbers onto a logarithmic scale, while Western adults used a linear mapping with small or symbolic numbers, and a logarithmic mapping when numbers were presented nonsymbolically under conditions that discouraged counting. Thus, the mapping of numbers onto space is a universal intuition, and this initial intuition of number is logarithmic. The concept of a linear number line appears to be a cultural invention that fails to develop in the absence of formal education.

*What then is mathematics if it is not a unique, rigorous, logical structure? It is a series of great intuitions carefully sifted, and organized by the logic men are willing and able to apply at any time*

-- Morris Kline, *Mathematics: The loss of certainty (p. 312)*

The mapping of numbers onto space plays an essential role in mathematics, from measurement and geometry to the study of irrational numbers, Cartesian coordinates, the real number line and the complex plane (*1, 2*). How does the human mind gain access to such abstract mathematical concepts? Constructivist theories view mathematics as a set of cultural inventions that are progressively refined in the history of mathematics and are slowly acquired during childhood and adolescence (*3*). However, the mental construction of mathematical may have deeper foundations. Mathematical objects may find their ultimate origin in basic intuitions of space, time and number that have been internalized through millions of years of evolution in a structured environment, and that emerge early in ontogeny, independently of education (*2*, *4*). Here, we present evidence that reconciles these two points of view: our results suggest that all humans share the intuition that number maps onto space, but that culture-specific experiences alter the form of this mapping.

Previous psychological and neuroimaging research supports the view that a sense of number is present in humans and many other species at an early age, and with a reproducible substrate in the bilateral intraparietal sulcus (*5*-*8*). This region is remarkably close or even overlapping with areas engaged in the coding of spatial dimensions such as size, location and gaze direction (*9*-*11*). Interactions between numerical and spatial codes in parietal cortex may therefore occur at this level. Indeed, in human adults, the mere presentation of an Arabic numeral automatically elicits a spatial bias in both motor responding and attention orienting

(*11-13*). Brain-lesioned patients show corresponding impairments in comparing and bisecting line segments and numbers (*14*), and some people even report a vivid experience of seeing numbers at fixed locations on an idiosyncratic spatially contiguous 'number form' (*15, 16*).

Recent experiments document a remarkable shift in the child's conception of how numbers map onto space (*17-19*). When asked to point towards the correct location for a spoken number word onto a line segment labelled with 0 at left and 100 at right, even kindergarteners understand the task and behave non-randomly, systematically placing smaller numbers at left and larger numbers at right. They do not distribute the numbers evenly, however, and instead devote more space to small numbers, imposing a compressed, logarithmic mapping. For instance they might place number 10 near the middle of the 0-100 segment. This compressive response fits nicely with animal and infant studies that demonstrate that numerical perception obeys Weber's law, a ubiquitous psychophysical law whereby increasingly larger quantities are represented with proportionally greater imprecision, compatible with a logarithmic internal representation with fixed noise (*7, 20, 21*). A shift from logarithmic to linear mapping occurs later in development, between 1$^{st}$ and 4$^{th}$ grade depending on experience and the range of numbers tested (*17-19*).

All of these observations, however, were made in Western subjects who all had access to mathematical education and culture at an early age. Prior to formal schooling, Western children may acquire the number line concept from Arabic numerals seen on elevators, rulers, books, etc. Thus, existing studies do not reveal which aspects of the number-space mapping constitute a basic intuition that would continue to exist in the absence of a structured mathematical language and education. In particular, we do not know if the log-to-linear shift would occur spontaneously in the course of brain maturation, or whether it requires exposure to critical educational material or culture-specific devices such as rulers or graphs.

To address these issues, we gathered evidence from psychological experimentation in the Munducuru, an Amazonian indigene culture with little access to education (*22, 23*). Previous research has established that, although their lexicon of number words is reduced and they have little or no access to rulers, measurement devices, graphs or maps, the Munduruc entertain sophisticated concepts of both number and space, albeit in an approximate and non-verbal manner (*22, 23*). We therefore asked whether they conceive of these two domains as being related by a systematic mapping and, if so, what form this number-space mapping takes.

A total of 33 Mundurucu adults and children were tested individually in a number-space task (figure 1)(*24*). On each trial, a line segment was displayed on a computer screen, with one dot at left and ten dots at right (or, in a separate block, 10 and 100 dots respectively). Then other numbers were presented in random order, in various forms (sets of dots, sequences of tones, spoken Mundurucu words, spoken Portuguese words). For each number, the participant pointed to a screen location and this response was recorded by a mouse click, without feedback. Only two training trials were presented, with sets of dots whose numerosity corresponded to the ends of the scale (e.g. one and ten). The participants were told that these two stimuli belonged to their respective ends, but that other stimuli could be placed at any location. Because training did not involve intermediate numbers, performance on all subsequent trials served to reveal whether the participants would spontaneously use a systematic mapping, and if so, whether their mapping would be compressive or linear.

The Mundurucu's mean responses revealed that they understood the task. Although some participants tended to use only the endpoints of the scale (see *24*), most used the full response continuum and adopted a consistent strategy of mapping consecutive numbers onto consecutive locations (figure 2). There was a significant positive correlation between stimulus number and mean response location, regardless of the modality in which the numbers were presented. The task was easy when the stimuli were sets of dots similar to the reference labels placed at the endpoints (numbers 1-10, $r^2$=92.6%, 8 d.f.; numbers 10-100, $r^2$=91.9%, 8 d.f.). However, the Mundurucu continued to use a systematic number-space mapping with untrained stimuli that only shared with the reference labels an abstract concept of number: sequences of tones 1-10 ($r^2$=92.5%, 8 d.f.), spoken Mundurucu number words ($r^2$=91.8, 6.d.f.), and Portuguese number words ($r^2$=91.1%, 8 d.f.), although a small proportion of random responses tended to slightly flatten the curves. Note that the Mundurucu stimuli included complex expressions that are very rarely uttered, such as "pũg põgbi ebadipdip bodi" [approximate translation: "one handful (and) four on the side"]. The results suggest that the Mundurucu partially understand the quantity to which these expressions refer.

Crucially, however, linear regression did not provide the best model of participants' responses. Rather, for all modalities of presentation, the curves were negatively accelerated. A multiple regression procedure evaluated the contribution of a logarithmic regressor, over and above the linear regressor. The logarithmic compression effect was significant for all stimulus modalities, although it was only marginal with Portuguese words (one-tailed p=0.04; see significance levels and regression weights on figure 2). Additional analyses allowed us to exclude interpretations in terms of linear responding with different slopes for small and large numbers, parallax error, experimenter bias, or bimodal responding (see *24*). The Mundurucu seem to hold intuitions of numbers as a log scale where the middle of the interval 1 through 10 is 3 or 4, not 5 or 6.

Previous number-space mapping experiments with Western subjects included only symbolic numerals, whereas the present experiment included non-symbolic visual and auditory numerosities. Thus, it was important to verify whether these novel stimuli were rated linearly or logarithmically in educated Western subjects. As shown in figure 2, American adults rated linearly all numerals presented in English and in Spanish as well as the sets of 1-10 dots, which could easily be counted. However, they exhibited a significant logarithmic component with sets of 10-100 dots and with sequences of tones. When the two groups of participants were compared directly, the Mundurucu showed a greater compressive non-linearity than the American subjects only with sets of 1-10 dots (p=0.003) and with numerals in the first language (p=0.033). This finding concurs with previous data suggesting that Western subjects estimate large numerosities in an approximate and compressive manner (*25, 26*). Their judgments are linear only when the numbers are presented in a symbolic manner or as small sets whose numerosity can be precisely assessed.

The Mundurucu population is heterogeneous, and some of our participants, particularly the children, had received a little education. To examine the impact of this variable, we calculated, for each participant, an index of non-linearity in the number-space mapping: the weight of the log regressor in a multiple regression of the data on linear and log regressors. For this analysis, we pooled over the trials with dots 1-10 and number words, but excluded those with dots 10-100 and tones for which Western subjects showed some non-linearity. The index confirmed a highly significant non-linearity in Mundurucu participants ($t=6.20$, 34 d.f., $p<10^{-6}$). In American participants, performance did not deviate from linearity ($p=0.08$) and differed markedly from that of the Mundurucu (Welch $t=4.37$, 48.6 d.f., $p<0.0001$). Crucially, the Mundurucu's non-linearity remained significant even when restricting the analysis to adults ($t=4.34$, 23 d.f., $p=0.0002$), to monolingual speakers ($t=5.36$, 29 d.f., $p<10^{-5}$), or to uneducated participants ($t=2.60$, 7 d.f., $p=0.035$; see figures S7-S10 for a graphic depiction of subgroup performance)(*24*). T-tests, linear and rank-order regression analyses showed no effect of gender, age, education or bilingualism. There was only a trend towards a reduced non-linearity as a function of age (Kendall tau = -0.23, $p=0.055$). While this observation suggests that older Mundurucu may evolve towards a greater understanding of the linear number line, it should be noted that in Western children, the mapping becomes linear over the range 10-100 by the 1st or 2nd grade (*17-19*), while in our data, even the oldest

Mundurucu adults (age>40) continued to show a highly significant non-linearity over the range 1-10 ($t=3.36$, 11 d.f., $p=0.006$).

Finally, we analyzed the special case of Portuguese numerals. Although overall performance was logarithmic, subdivision by education level indicated that logarithmic responding held for participants with 1-2 years of education ($t=3.15$, 16 d.f., $p=0.006$; figure S9), but not for those with no education at all or with more education. In uneducated participants, performance with Portuguese numerals was highly variable and weakly correlated with number ($r^2=39.0$, $p=0.053$; figure S8), suggesting that many of these subjects simply did not know the meaning of Portuguese numerals. For the most educated group, on the other hand, performance was strictly linear ($r^2=94.5\%$, $p<10^{-5}$; figure S10). Excluding participants with no education, we found that greater education significantly changed the responses to Portuguese from logarithmic to linear ($t=2.48$, 16.6 df, $p=0.024$) but left responses to Mundurucu numerals and dot patterns unchanged ($p>0.5$), thus yielding a significant interaction ($p=0.008$). Strikingly, within the more educated group, performance varied significantly with number notation ($t=3.12$, 9 d.f., $p=0.012$), as it was linear for Portuguese numerals but logarithmic for Mundurucu numerals and dot patterns 1-10.

Overall, these results reveal both universal and culture-dependent facets of the sense of number. After a minimal instruction period, even members of a remote culture with reduced vocabulary and education readily understand that number can be mapped onto a spatial scale. The exact form of this mapping switches dramatically from logarithmic to linear, however, depending on the ages at which people are tested, the education that they received, and the format in which numbers are presented,.

In light of the performance of Amazonian adults, it is clear that the mental revolution in Western children's number line does not result from a simple maturation process: logarithmic thinking persists into adulthood for the Mundurucu, even for very small numbers in the range 1-10, whether presented as dots, tones or spoken Mundurucu words. What are the sources of this universal logarithmic mapping? Research on the brain mechanisms of numerosity perception have revealed a compressed numerosity code, whereby individual neurons in parietal and prefrontal cortex exhibit a Gaussian tuning curve on a logarithmic axis of number (*27*). As first noted by Gustav Fechner, such a constant imprecision on a logarithmic scale can explain Weber's law – the fact that larger numbers require a proportional larger difference in order to remain equally discriminable. Indeed a recent model suggests that the tuning properties of number neurons can account for many details of elementary mental arithmetic in humans and animals (*21*) In the final analysis, the logarithmic code may have been selected during evolution for its compactness: like an engineer's slide rule, a log scale provides a compact neural representation of several orders of magnitude with fixed relative precision.

It is not yet known which critical educational or cultural experience turns this initial representation into a linear scale. When a cultural difference in conceptual representation is observed in a remote population, Whorf's hypothesis is often invoked (*28*), according to which language determines the organization of thought. In the present case, however, the Whorfian explanation fails, since neither linguistic competence *per se* (present in all Mundurucu), nor numerical vocabulary and verbal counting (present in most Mundurucu, including some monolingual speakers and young children, see 24) suffice to induce the log-to-linear shift (*17-19*). Speculatively, two factors underlying the shift may be experience with measurement, whereby a fixed numerical unit is applied to different spatial locations, and experience with addition and subtraction, ultimately yielding the intuition that all consecutive numbers are separated by the same interval +1. The most educated Mundurucu eventually understand that linear scaling, which allows measurement and invariance over addition and subtraction, is central to the Portuguese number word system. At the same time, they still do not extend this principle to the Mundurucu number words, where perceptual similarity between quantities is still seen as the most relevant property of numbers. The system of Mundurucu number words may be a cultural device that does not emphasize measurement or invariance by addition/subtraction as a defining feature of number, contrary to Western numerals.

The simultaneous presence of linear and compressed mental representations of numbers is probably not unique to the Mundurucu. In American children, logarithmic mapping does not disappear all at once, but vanishes first for small numbers and much later for larger numbers 1-1000 (up to 4th or 6th grade in some children) (*17-19*). In fact, a logarithmic representation may remain dormant in all of us for very large numbers or whenever we approximate numbers (*29*), including prices (*30*). Thus, log and linear scales may be deeply embedded in all of our mental activities (*31*).

## *Figure legends*

### Figure 1.

Number mapping task with numbers 1-10. A horizontal segment, labelled with a set of one dot on the left and a set of 10 dots on the right, was constantly present on screen. Numbers were presented visually as sets of dots or auditorily as sequences of tones (see *24*), Mundurucu numerals or Portuguese numerals . For Mundurucu numerals, a rough translation into Arabic numerals is provided (e.g. “pũg põgbi xex xep bodi” ≈ “one handful (and) two on the side” ≈ 7; “xex xep põgbi” ≈ “two handfuls” ≈ 10). For each stimulus, participants pointed to a place on the line, and the experimenter clicked it with the computer mouse, which made a small bar appear.

### Figure 2.

Average location of numbers on the horizontal segment, separately for Mundurucu participants (left column) and for American participants (right column). Data are mean +/- standard error of the mean. Graphs of performance broken down by age group and education are available as supplementary material (*24*).

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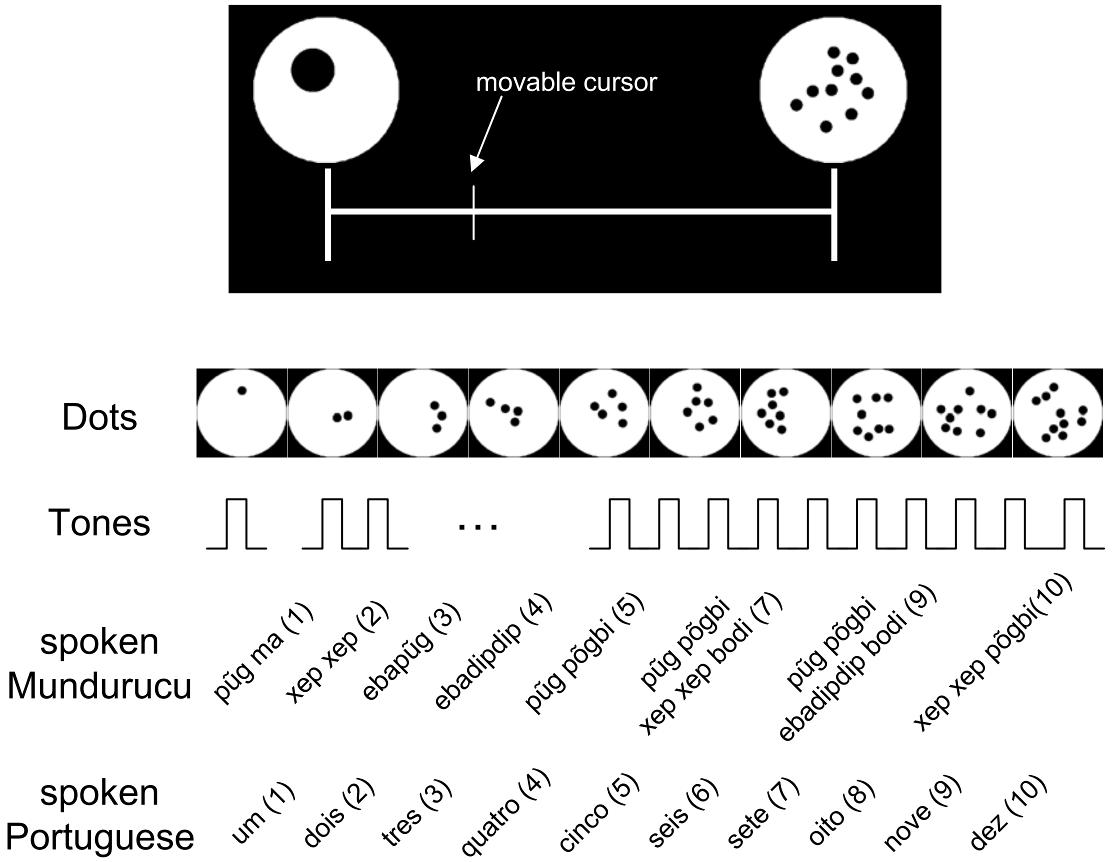
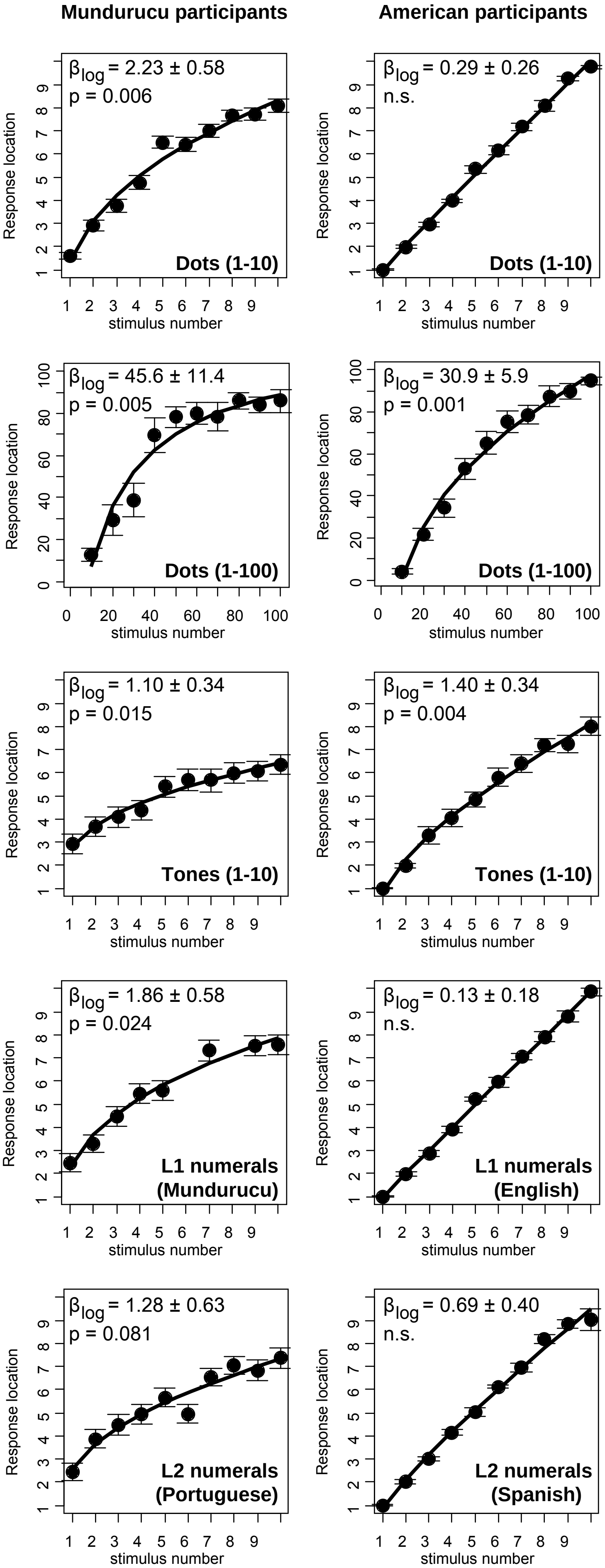



Dehaene et al, Figure 1




Dehaene et al, Figure 2