Do Redistributive Pension Systems Increase Inequalities and Welfare? *

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Abstract

Using a capital-skill complementarity technology, we analytically show that an increase in the direct redistributivity of Pay-As-You-Go (PAYG) pension systems has a positive impact on wages and on wage inequalities. We also show that life expectancy inequalities play an important role in the achievement of these results. Then, we calibrate our model and we find that, if life expectancy inequalities are sufficiently high, a more redistributive pension system increases the wealth and the welfare of every agent of the economy. Moreover, such a policy decreases the tax rate of the pension system.

Keywords: Inequality, Pension System, Redistribution, Capital-Skill Complementarity

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1 Introduction

Most of the economic literature related to pension systems has studied the macroeconomic impact of an increasing size of pension systems[1]. However, only a few papers deal with the impact of the structure of pension systems. In this paper, the term "structure" means that a pension system can be Beveridgian, Bismarkian or a mix of the two. As in Casamatta *et al.* (2000), a pension system has a pure Beveridgian structure if each agent receives the same pension. Conversely, if pensions completely depend on activity wages then the pension system is Bismarkian. The structure of pension systems determines their redistributive properties[2]. The more the pension system is Beveridgian, the more it redistributes resources among the population. France, Germany and Italy have a Bismarkian structure. Canada, the Netherlands and New-Zeland are essentially Beveridgian. Finally, Japan, the United-Kingdom and the United States have mixed pension systems (Sommacal 2006, Casamatta *et al.* 2000). Countries are different from one another because of this intra-generational component, so it can be relevant to consider the macroeconomic impacts of a policy which changes the structure of pension systems.

In this paper, we analyze the impact of a policy which increases the Beveridgian part of pension systems. We find that life expectancy inequalities play a significant role in the study of the impact of the structure of pension systems.

There is a growing empirical literature which analyzes these life expectancy inequalities[3]. Mesrine (1999) studies the inequalities of length of life according to socio-professional groups in France[4]. The most striking feature of his paper is that a worker has a probability

[1] See Docquier and Paddison (2003), or Casarico and Devillanova (2007). These results are notably questioned by Groezen *et al.* (2007), Lambrecht *et al.* (2005) or Le Garrec (2005).

[2] Here and in the rest of this paper, the term "redistribution" means "direct redistribution", i.e. the redistribution of wealth through the indexation of pensions on activity wages.

[3] See Attanasio and Emmerson (2001), Bommier *et al.* (2003) or Adams *et al.* (2003) for a survey.

[4] These inequalities also depend on other factors like sex or geographical localization. For example, in France the life expectancy of women is 84.1, while that of men is only 77.2 (INSEE 2006). Moreover, Rican

to die between 35 and 65 almost twice higher than that of an executive manager. Furthermore, their life expectancy at 35 is 38 and 44 respectively. The same qualitative results are observed in the United-States (Panis and Lillard 1995, Deaton and Paxson 2000). Finally, Robert-Bobbée and Cadot (2007) show that this inequality is also observed for elderly people. For agents who are 86, the ones with highest education level can expect to live 20% longer than the ones with lowest education level.

In this paper, we consider an overlapping generations economy in which agents live for two periods. We assume that there is a fixed fraction of skilled agents in the population[5]. As the empirical literature suggests, skilled agents have a longer life expectancy than unskilled ones. Both offer their labor inelastically when they are young[6] (first period of life), and both retire at the very beginning of their second period of life. The government levies a tax rate on activity wages in order to finance a Pay-As-You-Go (PAYG) pension system. This pension system has a mixed structure, i.e. it has a Beveridgian and a Bismarkian component. Finally, we assume that firms use a capital-skill complementarity technology. It means that the elasticity of substitution between capital and unskilled labor is higher than that between capital and skilled labor (Krusell *et al.* 2000). This assumption has been empirically observed[7] and it implies that an increase in the capital per capita level increases wage inequalities (Duffy *at al.* 2004).

We show analytically that if the Beveridgian part of pension systems increases, then it has a positive impact on capital per capita. Given the technology of firms, it means that this redistributive policy has a positive impact on wages and on wage inequalities. The

and Salem (1999) show that there are strong disparities according to the localization of people in France.

[5]It means that we assume that the structure of pension systems has no impact on occupational choices.

[6]In doing so we do not model the burden of income taxation on labor supply. We make the same assumption as in Feldstein (1985) given that:

> "The primary cost of providing social security benefits is the welfare loss that results from reductions in private saving" (Feldstein 1985, pp.303).

[7]See Griliches (1969), Fallon and Layard (1975), Krusell *et al.* (2000) or Duffy *et al.* (2004).

impact on other macroeconomic variables cannot be determined *a priori*. It explains why we calibrate our model. If life expectancy inequalities are sufficiently high, we show that a more redistributive pension system increases the wealth and the welfare of each agent of the economy. Moreover, such a policy decreases the tax rate of the pension system.

Hachon (2008) makes the same exercise with a continuum of agents endowed with different productivity levels. Our paper differs from this in two ways. Firstly, we use more general utility and production functions in our basic framework. Thus, we show that the structure of pension systems has an impact on wage inequalities. Secondly, because there are only two groups of agents in our paper, it is easier for us to emphasize that the life expectancy differential is what matters to determine the macroeconomic impact of our redistributive policy.

Two kinds of papers can be related to ours. In the first kind of papers, authors study the macroeconomic impact of a change in the structure of pension systems. Sommacal (2006) studies the macroeconomic impacts of a more redistributive unfunded pension system. He uses a defined-contribution pension system[8] with endogenous labor supply and imperfect substituability between two kinds of labor: skilled and unskilled labor. He finds that an increasing redistributivity of pension systems decreases output but also the wealth of each agent of the economy. Compared to this study, we make three different assumptions. We assume an exogenous labor supply, a defined-benefit pension system and a capital-skill complementarity technology. We show that conclusions are completely different in this new framework.

The second kind of papers studies the impacts of a change in the size of pension systems when firms use a capital-skill complementarity technology. Casarico and Devillanova (2007)

[8] A pension system has a defined-benefit structure if it is the tax rate which adjusts itself to changes in the economic and demographic environment. Conversely, it has a defined-contribution organization if it is the replacement rate which adjusts itself.

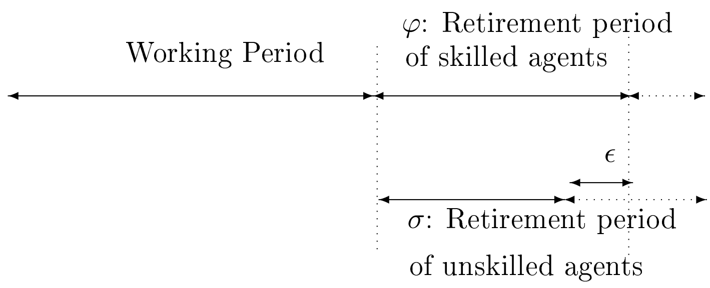

Figure 1: Life of agents

find that an increase in the size of pension systems has a negative impact on capital accumulation, on the share of the educated population, on output and on wage inequalities.

Our paper is organized as follows. In section 2 we present our model and our main assumptions. In section 3 we study the dynamic of our model and its properties. In section 4 we solve our model numerically. Section 5 includes some concluding remarks.

2 The Model

We assume an overlapping generations economy in which agents live for two periods. In the first period of their life, agents work. In the second one, they are retired. In order to have a tractable model which includes a capital-skill complementarity technology, we assume that there are only two groups of agents: skilled and unskilled agents. We assume that there are eN_t skilled agents and $(1-e)N_t$ unskilled agents at each period t, with $1 > e > 0$. There is no uncertainty. Skilled and unskilled agents only live a fraction φ and σ respectively, of their second period of life. It is the same assumption as in Gorski *et al.* (2007). We denote by ϵ the mortality differential between these two groups of agents. It means that $\epsilon = \varphi - \sigma$. We assume that $\epsilon \geq 0$. For $\epsilon = 0$ there are no inequalities of length of life, and for $\epsilon = \varphi$ uneducated agents live only their first period of life. Figure 1 represents the life cycle of an agent.

2.1 Consumers

Each consumer of the economy belongs either to the group of educated agents or to the group of uneducated agents. All agents have the same preferences. These preferences are intertemporally separable and they have the Constant-Intertemporal-Elasticity-of-Substitution form[9]:

$$U_t^i = U(c_t^i) + \beta T^i U\left(\frac{d_{t+1}^i}{T^i}\right) \tag{1}$$

It is such that:

$$U(x) = \begin{cases} \frac{x^{1-\eta}}{1-\eta} & \text{if } \eta > 0 \text{ but } \eta \neq 1 \\ \ln(x) & \text{if } \eta = 1 \end{cases}$$

β denotes the pure time preference factor. U_t^i denotes the utility level of an agent of type i (with $i \in \{s, u\}$) born at the beginning of period t. The superscript s means that an agent is skilled, whereas the superscript u means that an agent is unskilled. c_t^i (d_t^i) denotes the consumption of a young (old) agent. T^i denotes the length of life of an agent of type i. It increases the weight that an agent attaches to his future utility. But at the same time, it decreases the consumption flow of the second period of life[10][11]. We have:

$$T^i = \begin{cases} \varphi & \text{if } i = s \\ \sigma & \text{if } i = u \end{cases}$$

[9] Andersen (2008) uses the same utility function.

[10] See d'Autume (2003).

[11] Using the budget constraints and the market conditions defined below, it is straightforward to show that T^i can also denote the probability of dying of an agent of type i. In that case, it is sufficient to assume that the educated and the uneducated population are sufficiently large and that there is a perfectly competitive annuity market for each group of agents.

The budget constraints of an agent of type i are:

$$c_t^i = w_t^i(1-\tau_t) - S_t^i \tag{2}$$

$$d_{t+1}^i = R_{t+1}S_t^i + p_{t+1}^i \tag{3}$$

with R_{t+1} the interest factor, τ_t the tax rate used to finance a PAYG pension system. S_t^i denotes the savings of a young agent of type i at period t. p_{t+1}^i denotes the pension that an agent of type i receives when he is retired. Every agent maximizes (1) with respect to S_t^i given the budget constraints (2) and (3). We obtain:

$$S_t^i = \frac{T^i(\beta R_{t+1})^{1/\eta} w_t^i(1-\tau_t)}{T^i(\beta R_{t+1})^{1/\eta} + R_{t+1}} - \frac{p_{t+1}^i}{T^i(\beta R_{t+1})^{1/\eta} + R_{t+1}} \tag{4}$$

Without a pension system, i.e. for $\tau = p^i = 0$, saving is an increasing function of the length of life. The longer the length of life is, the more agents value their future consumption, and thus, the more they save. Moreover, if τ or if p^i increases, then saving decreases. Firstly, because it decreases the net wage of the first period of life of agents. Secondly, because it increases the revenues from the second period of life of agents.

2.2 Firms

We assume that the technology of firms has the following form:

$$Y_t = F(L_t^u, G(K_t, L_t^s)) \tag{5}$$

with $F(.,.)$ and $G(.,.)$ two homogenous functions of degree 1. L_t^s (L_t^u) denote the quantity of skilled (unskilled) labor used in the production function. $F()$ and $G()$ are such that[12]:

$$F_i > 0 \text{ and } G_i > 0$$

$$F_{ii} < 0 \text{ and } G_{ii} < 0$$

$$F_{ij} > 0 \text{ and } G_{ij} > 0 \text{ with } i \neq j$$

We assume a perfect competition on the final good market and on inputs markets. It implies that, at equilibrium, wages and capital return are:

$$w_t^u = F_1((1-e), G(k_t, e)) \equiv w^u(k_t) \tag{6}$$

$$w_t^s = F_2((1-e), G(k_t, e))G_2(k_t, e) \equiv w^s(k_t) \tag{7}$$

$$R_t = F_2((1-e), G(k_t, e))G_1(k_t, e) \equiv R(k_t) \tag{8}$$

with $k_t = K_t/N_t$. Given the properties of the functions $F()$ and $G()$, it can be shown that:

$$\frac{\partial R(k_t)}{\partial k_t} < 0$$
$$\frac{\partial w^u(k_t)}{\partial k_t} > 0$$

A priori, we cannot determine the sign of the derivative of $w^s(k_t)$ with respect to k_t. As in Duffy *et al.* (2004), in the rest of this paper we make the following assumption:

Assumption 1: $\dfrac{\partial w^s(k_t)/\partial k_t}{w^s(k_t)} k_t > \dfrac{\partial w^u(k_t)/\partial k_t}{w^u(k_t)} k_t.$

This assumption necessarily implies that $\partial w^s(k_t)/\partial k_t > 0$. It means that the elasticity of the wages of skilled agents with respect to capital per capita is higher than the one

[12] g_i denotes the derivative of $g()$ with respect to its *ith* argument. g_{ij} denotes the derivative of g_i with respect to its *jth* argument.

concerning the wages of unskilled agents. Appendix 2 shows that this assumption is correlated with the capital-skill complementarity technology. In this paper, Iw denotes the wage inequality ratio. It is such that:

$$I_{w,t} = \frac{w_t^s}{w_t^u} \tag{9}$$

Given assumption 1, this wage inequality ratio is an increasing function of k_t.

Furthermore, we make the following assumption[13]:

Assumption 2: There exists a threshold value $\tilde{k}$ sufficiently small, such that $w^u(k_t) < w^s(k_t)$, $\forall k_t \geq \tilde{k}$ with $\tilde{k} \geq 0$. We have $\tilde{k} = 0$ if $w^s(0) \geq w^u(0)$, and $\tilde{k} > 0$ if $w^s(0) < w^u(0)$.

Even if this assumption seems obvious, the general form we use does not necessarily implies that $w^u(k_t) < w^s(k_t)$, $\forall k_t \geq 0$. However, given assumption 1, we can reasonably assume that there exists a small threshold value $\tilde{k}$ such that the wage level of skilled agents is higher than the one of unskilled agents.

2.3 Government

We assume a PAYG pension system. The revenues of this system come from a proportional tax on wages: τ_t. These revenues are used to provide a pension for elderly people. Their pension depends on the wages of young agents having the same productivity as them, and on the average wage of the economy. Their respective weighting is λ and $(1 - \lambda)$. The first part of this pension represents the Bismarkian component, whereas the second part represents the Beveridgian component of this system (Casamatta *et al.*, 2000). λ measures

[13] This assumption is reasonable as long as unskilled labor is not scarce.

the indexation of pensions on the activity wages of agents. If $\lambda = 0$, each agent receives the same pension and the pension system is completely Beveridgian. Conversely, if $\lambda = 1$ the level of pensions only depends on the wage of agents and the pension system is purely Bismarkian. **The smaller λ is, the more this pension system is redistributive.**[14].

Consumers receive only a fraction ν (with $0 < \nu \leq 1$) of this weighted average, and only during their second period of life T^i. ν denotes the average replacement rate of the pension system.

The pension level of an agent of type i is:

$$p^i_{t+1} = \nu \left(\lambda w^i_{t+1} + (1-\lambda)\bar{w}_{t+1}\right) T^i \tag{10}$$

The budget constraint of the government can be written:

$$\tau_t N_t \overline{w}_t = N_{t-1}\left[e\nu(\lambda w^s_t + (1-\lambda)\overline{w}_t)\varphi + (1-e)\nu(\lambda w^u_t + (1-\lambda)\overline{w}_t)\sigma\right] \tag{11}$$

with $\overline{w}_t$ the average wage of the economy. It is obtained by:

$$\overline{w}_t = e w^s_t + (1-e) w^u_t \tag{12}$$

Some simple manipulations imply that the tax rate can be expressed as:

$$\tau_t = \frac{\nu}{1+n}\left[\varphi - \epsilon(1-e)\left(\lambda\frac{w^u(k_t)}{\overline{w}(k_t)} + 1 - \lambda\right)\right] \tag{13}$$

If $\epsilon = 0$, the tax rate is simply equal to the product between the replacement rate and the old-age dependency ratio of the economy (d'Autume, 2003). The second component

[14] In this paper the term "redistributivity" only concerns the direct redistribution of pension systems and not the effective redistribution of pension systems. The effective redistribution, which is the difference between tax paid and amount received, can be very different because of life expectancy inequalities.

between brackets is the ratio between the pensions not paid to unskilled agents because of their lower length of life, and the average wage of the economy. *Ceteris paribus*, under the reasonable assumption that $w_t^u < w_t^s$, $\forall t$, the tax rate is an increasing function of λ. This result is very intuitive. Indeed, educated agents have the longer length of life. Therefore, an increase in λ (i.e. a decrease in the redistributivity of the pension system) increases the indexation of pensions on wages. It implies that the pension of skilled agents increases. Moreover, they benefit from these pensions for a longer period of time than other agents. Consequently, the tax rate has to increase to finance these additional expenditures.

Another interesting point is that the tax rate τ_t is an increasing function of k_t. Indeed, under assumption 1, it is straightforward to show that $w^u(k_t)/\overline{w}(k_t)$ is a decreasing function of k_t. It means that the relative cost not paid to unskilled agents, because of their lower length of life, decreases with the level of capital per capita.

3 The Equilibrium and its Properties

All markets clear at each period t if:

$$L_t^s = eN_t \tag{14}$$

$$L_t^u = (1-e)N_t \tag{15}$$

$$K_{t+1} = eN_tS_t^s + (1-e)N_tS_t^u \tag{16}$$

The dynamic of the economy is obtained using equations (4), (6), (7), (8), (10), (12), (13), (14), (15) and (16).

It is straightforward to show that we obtain:

$$LHS(k_{t+1},\lambda) \equiv$$

$$(1+n)k_{t+1} + e\frac{p^s(k_{t+1},\lambda)}{\varphi(\beta R(k_{t+1}))^{1/\eta} + R(k_{t+1})} + (1-e)\frac{p^u(k_{t+1},\lambda)}{\sigma(\beta R(k_{t+1}))^{1/\eta} + R(k_{t+1})} =$$

$$(1-\tau(k_t,\lambda))[e\frac{\varphi(\beta R(k_{t+1}))^{1/\eta}}{\varphi(\beta R(k_{t+1}))^{1/\eta} + R(k_{t+1})}w^s(k_t)+$$

$$(1-e)\frac{\sigma(\beta R(k_{t+1}))^{1/\eta}}{\sigma(\beta R(k_{t+1}))^{1/\eta} + R(k_{t+1})}w^u(k_t)] \equiv RHS(k_{t+1},k_t,\lambda) \quad (17)$$

Since the dynamic of the economy is complicated we make two further assumptions[15]:

Assumption 3: The equilibrium trajectory is unique and increasing. It can be written: $k_{t+1} = \Psi(k_t)$, with $\Psi_1(k_t) > 0$.

Assumption 4: There exists at least one non-trivial stable steady state (k^s_{SS}) such that $k^s_{SS} > \tilde{k}$.

These two assumptions are sufficient to establish the following proposition.

Proposition 1: *An increase in λ has a negative impact on every stable steady state (k^s_{SS}). Consequently, it also decreases wage inequalities.*

Proof: See appendix 1.□

Proposition 1 shows that a more redistributive pension system (a decrease in λ) in-

[15]We show in the next section that the following assumptions are checked for reasonable values of parameters.

creases the level of capital per capita. Indeed, a decrease in λ has two kinds of effects on saving. The first one is to decrease the *tax rate* τ for a given level of capital per capita. Then, the net wage of every consumer increases. It has a positive impact on saving. The second one is a *pension effect*. Unskilled agents decide to decrease their saving because they benefit from a more redistributive pension system. But at the same time, skilled agents increase their saving because pensions are less indexed on activity wages. The increase in saving of skilled agents overcompensates the decrease of the one of unskilled agents because skilled agents live for a longer period of time.

Given our technology and under assumption 1, a decrease in λ also implies that wage inequalities (I_w) increase.

Let us now consider the impact of this redistributive policy on the wealth level and on the welfare level of every agent. These analytical results are obtained at steady state to simplify the exposition. Every derivative is thus a comparison between steady states.

The wealth level of an agent of type i, with $i \in \{s, u\}$, can be written:

$$W^i(\lambda) = w^i(k(\lambda))(1 - \tau(k(\lambda), \lambda)) + \frac{p^i(k(\lambda), \lambda)}{R(k(\lambda))} \tag{18}$$

with $k'(\lambda) < 0$. The net impact of a decrease in λ on $W^i()$ is:

$$-\frac{dW^i(\lambda)}{d\lambda} = \underbrace{-\frac{dw^i(k)}{dk}\frac{dk}{d\lambda}(1 - \tau(k, \lambda))}_{A>0} + \underbrace{w^i(k)\left(\frac{\partial \tau(k, \lambda)}{\partial k}\frac{dk}{d\lambda} + \frac{\partial \tau(k, \lambda)}{\partial \lambda}\right)}_{B\lesseqgtr 0}$$
$$\underbrace{-\frac{1}{R(k)}\left(\frac{\partial p^i(k, \lambda)}{\partial k}\frac{dk}{d\lambda} + \frac{\partial p^i(k, \lambda)}{\partial \lambda}\right)}_{C>0 \text{ if } i=u,\ C\lesseqgtr 0 \text{ if } i=s} + \underbrace{p^i(k, \lambda)\frac{\frac{dR(k)}{dk}\frac{dk}{d\lambda}}{(R(k))^2}}_{D>0} \tag{19}$$

Element A is positive because of the positive impact of the redistributive pension system on the wage level, through capital accumulation. Element B can be positive or negative. Indeed, as mentioned above, a more redistributive pension system has a direct negative impact on the tax rate because the pension system redistributes resources in favour of agents having a short life expectancy. However, as this policy increases capital accumulation, it reduces the relative share of expenditures not spent because of the mortality differential[16]. The net impact on the tax rate is thus ambiguous, but we can reasonably assume that the direct impact is higher than the one going through capital accumulation. It implies that a more redistributive pension system reduces the tax rate, which has a positive impact on the wealth level.

Element C has an ambiguous sign. Indeed, a more redistributive pension system increases the wage level of every agent, which has a positive impact on pensions. However, λ has a direct impact on pensions through the indexation of pensions on wages. For unskilled agents, a more redistributive pension system increases their pensions, and thus C is positive since every effect has the same sign. But for skilled agents, a more redistributive pension system decreases their pension for a given level of capital per worker. Consequently, the net impact on C is ambiguous for skilled agents.

Finally, element D is positive because of the negative impact of a more redistributive pension system on the interest factor.

We can reasonably conclude from this analysis that it is almost sure that unskilled agents highly benefit from a more redistributive pension system. However, the final impact is ambiguous for skilled agents. The indirect impact on capital accumulation has to be large for educated agents to benefit from this policy. It implies that life expectancy

[16] See the discussion about equation (13).

inequalities have to be high[17].

Let us now consider the impact of this policy on the welfare level of agents.

Using equations (1), (2), (3) and (4) we obtain:

$$U^i(\lambda) = \frac{(c^i(\lambda))^{1-\eta}}{1-\eta} + \beta(T^i)^\eta \frac{(d^i(\lambda))^{1-\eta}}{1-\eta} \tag{20}$$

with:

$$c^i(\lambda) = \frac{W^i(\lambda)}{1+\beta^{1/\eta}(R(k(\lambda)))^{(1-\eta)/\eta}T^i} \tag{21}$$

and,

$$d^i(\lambda) = \frac{W^i(\lambda)\beta^{1/\eta}T^i}{(R(k(\lambda)))^{-1/\eta}+\beta^{1/\eta}T^i(R(k(\lambda)))^{-1}} \tag{22}$$

The impact of a decrease in λ on consumption flows is *a priori* ambiguous. Indeed, a more redistributive pension system has an impact on the wealth level of agents. However, as it also has an impact on the interest factor, it influences the price of the second period consumption (d)[18]. Let us first consider that a decrease in λ has a positive impact on the wealth level of agents (W^i). Then, two cases have to be considered. The first one is such that $\eta < 1$. It implies that the intertemporal elasticity of substitution is high. In that case, a decrease in λ has a positive impact on the first period consumption (c). Indeed, the wealth level and the price of the second period consumption increase. Consequently, agents prefer increasing their first period consumption. The net impact on the second

[17] In the calibration exercise, we emphasize this point.

[18] The price of the second period consumption is the inverse of the interest factor. A decrease in R increases the price of the second period consumption.

period consumption is ambiguous. It depends on the scale of the wealth effect and of the price effect (through the interest factor).

The second case which has to be considered, is the case in which $\eta > 1$, i.e. the case of a small intertemporal elasticity of substitution. In that case, the impact of a more redistributive pension system on consumption levels is ambiguous whatever the period considered.

Finally, if a decrease in λ has a negative impact on the wealth level, then if $\eta > 1$, consumption levels of every period decrease. If $\eta < 1$, then the second period consumption decreases whereas the impact on the first period consumption is ambiguous. However, we can reasonably assume that the consumption level of every period will decrease.

Since we cannot determine *a priori* the impact of a more redistributive pension system on wealth, on welfare, and on wealth inequalities, we calibrate and we solve our model numerically.

4 Calibration and Results

Firstly, we specify our production function and we detail our calibration choices. Then, we give the results of the numerical resolution of our model.

4.1 Calibration

In this section, we specify the functional form of the production function and we calibrate our model to study the impact of the redistribtutivity of pension systems on macroeconomic variables.

Firstly, we assume that the production function has the following form:

$$F(L_t^u, X_t) = A\left[\alpha(L_t^u)^\upsilon + (1-\alpha)X_t^\upsilon\right]^{1/\upsilon} \tag{23}$$

and,

$$X_t \equiv G(K_t, L_t^s) = [bK_t^\gamma + (1-b)(L_t^s)^\gamma]^{1/\gamma} \tag{24}$$

with $A > 0$ the level of the technology, $\upsilon, \gamma < 1$ and $b, \alpha \in (0,1)$. Using Sato's (1967) results and the study of Duffy *et al.* (2004), we show that there exists a capital-skill complementarity if and only if $\upsilon > \gamma$. This condition is necessary and sufficient for capital and skilled labor to be more complementary than capital and unskilled labor[19].

It can be shown that the wage inequality ratio: $I_w = w^s(k_t)/w^u(k_t)$ is an increasing function of k_t if and only if $\upsilon > \gamma$.

We calibrate the parameters υ and γ of the production function to match the findings of Fallon and Layard (1975). They find that, for a restricted set of rich countries, the elasticity of substitution between capital and unskilled labor is 1.85, whereas the one between capital and skilled labor is 0.55. Given our technology, it implies that $\upsilon = 0.46$ and $\gamma = -0.81$.

Secondly, we calibrate the basic parameters of the model. The length of each period is 40 years. The growth rate of the population is $n = 0.3$. It corresponds to an annual growth rate of the population of 0.65% (Charpin, 1999). The pure time preference factor is $\beta = 0.6$ (d'Autume 2003), i.e. an annual psychological discount rate of 1.3%. Moreover, we assume that $\eta \geq 1$. It means that the intertemporal elasticity of substitution is low. Agents prefer smoothing their consumption. This assumption is in accordance with Attanasio *et al.*

[19] See Duffy *et al.* (2004). This condition is necessary and sufficient using either the Allen-Uzawa partial elasticity of substitution, or the Hicks-Allen direct partial elasticity of substitution.

(1999) and with Cooley and Prescott (1995). It is the same as the one used in Casamatta *et al.* (2000). More specifically, we assume that $\eta = 1.5$ as in Sommacal (2006).

Moreover, we first assume that $\varphi = 0.55$, $\sigma = 0.45$ and thus that $\epsilon = 0.1$. It means that the length of life of an educated agent is 82 years[20], and the one of unskilled agents is 78 years. The life expectancy differential is 4 years. The life expectancy gap is smaller than the one found by Mesrine (1999) between the highest and the lowest socio-professional group. Since the indirect effects depend on the value taken by ϵ, we discuss the impact of a redistributive policy for different values taken by this parameter.

The share of the educated population is 0.4 ($e = 0.4$) as in Sommacal (2006) and in Acemoglu (2002). As in their studies, our model will have to match the wage gap I_w found by Acemoglu (2002).

Thirdly, we calibrate our model to match some empirical facts. The average replacement rate is fixed for the tax rate of the pension system to be around 20% as reported in Nyce and Schieber (2005). We obtain $\nu = 0.55$. This value is in accordance with the empirical findings of Nyce and Schieber (2005), and this value is the same as the parametrization used in d'Autume (2003).

There only remains to fix α, b and A. We fix them for our model to reproduce three facts. Firstly, the capital share in total output has to be near 0.33. Secondly, the annual interest rate has to be in the reasonable interval $[0.03, 0.05]$. And finally, following Sommacal (2006) and Acemoglu (2002), the wage premium w^s/w^u has to be near 1.7. If $(A, b, \alpha) = (3, 0.12, 0.23)$, then these three facts are observed at the steady state of our economy. With this calibration, we find a tax rate τ for the pension system around 0.21. Table 1 summarizes our calibration.

[20] In our model we do not include the very first period of life during which an agent is young. We assume that the length of this period is 20 years. Thus the life expectancy of skilled agents is obtained by: $20 + 40 + \varphi \times 40$.

Parameter	Value	Parameter	Value
n	0.3	η	1.5
β	0.6	A	3
φ	0.55	b	0.12
σ	0.45	α	0.23
ν	0.55	υ	0.46
e	0.4	γ	-0.81

Table 1: Calibration of the model

4.2 Steady State Effects

In this section we analyze the impact of a decrease in λ on the economic variables at steady state (see Figures 2-10).

We observe that the capital level per capita is a decreasing function of λ as in our proposition 1. It implies that wage levels and wage inequalities increase with the redistributivity of pension systems. Another important point is that the decrease in λ has a negative impact on the pensions of educated agents. It means that the direct decrease in the indexation of pensions on activity wages overcompensates the increase in wages implied by a more redistributive pension system. Obviously, the pensions of unskilled agents decrease with λ because all effects go in the same way.

The total impact of a decrease in λ on the wealth of every agent is positive. It means that the decrease in the pensions of skilled agents is overcompensated by the increase in their wages. For unskilled agents the positive impact is more trivial as wages and pensions increase.

We also find that wealth inequalities decrease with the level of redistribution of pension systems. It implies that the direct redistribution of pension systems overcompensates the increase in wage inequalities.

The tax rate is a decreasing function of the beveridgian part of pension systems as can be observed empirically. Finally, we find that every agent of the economy benefits from a

Variables	$\epsilon = 0.02$	$\epsilon = 0.1$	$\epsilon = 0.15$
k	+	+	+
I_w	+	+	+
p^s	-	-	-
p^u	+	+	+
W^s	-	+	+
W^u	+	+	+
I_W	-	-	+
U^s	-	+	+
U^u	+	+	+
τ	-	-	-

Table 2: Impact of a decrease in λ on macroeconomic variables

more redistributive pension system because his utility decreases with λ.

Let us now consider different values which can be taken by ϵ, denoting the mortality differential. It can be numerically shown that for $\epsilon \in (0, 0.15)$, A, b and α can keep the same value without altering the matching properties of our model too much. Let us recall that the larger ϵ is, the higher inequalities of length of life are. In table 2 we test the robustness of our results for different values of ϵ, keeping φ at its initial value ($\varphi = 0.55$). The sign $+$ $(-)$ means that the redistributivity of pension systems has a monotonous positive (negative) impact on the variable.

Firstly, comparing the first column to the second one, we observe that if life expectancy inequalities are not sufficiently high, the indirect effects of a more redistributive pension system are small, thus skilled agents do not benefit from such a policy. Their utility level decreases because the decrease in their pension level is higher than the increase in their wage level.

Secondly, comparing the case in which $\epsilon = 0.1$, to the case in which $\epsilon = 0.15$, we observe that each qualitative result remains unchanged for different values of σ, except for the wealth inequality ratio. Indeed, we observe that an increase in the redistributivity of pension systems decreases wealth inequalities as long as ϵ is not too large. However, for ϵ sufficiently high, an increase in the Beveridgian part of pension systems increases wealth inequalities. The main explanation is that if ϵ is sufficiently high, unskilled agents do not benefit from the redistributive properties of pension systems for a long time. Then, the increase in wage inequalities overcompensates the decrease in pension inequalities.

4.3 The Transitional Dynamic

Let us now consider the transitional dynamic of our macroeconomic variables if the pension system becomes more redistributive. To study the transitional dynamic, we assume that $\epsilon = 0.1$. Thus, we consider the case in which, at steady state, the welfare level of every agent increases. We try to know if there is a transitional cost for agents.

We assume that an economy is initially (at period 1) at steady state. This steady state is characterized by a given value of the parameter λ. The government changes the value of this parameter from period 3 on, and every agent expects this change[21]. We study the dynamic of our model if the pension is initially Bismarkian ($\lambda = 0.885$, as in Hairault and Langot (2008) on the French case, and as in Hachon (2008)), and if it becomes more Beveridgian ($\lambda = 0.685$).

Concerning capital accumulation, we have $k_1 = k^i$, with k^i the steady state value of k with the initial value of λ. Moreover, as equation (17) determines k_2 with λ in the RHS and in the LHS, we also have: $k_2 = k^i$. However, as k_3 depends on the level of

[21] Hachon (2008) makes the same exercise with an unexpected change in λ. He obtains the same qualitative results.

pensions of period 3 received by agents born in period 2, then k_3 differs from k^i. Then, k_t converges towards its new steady state value. Figure 11 illustrates this convergent dynamic.

Figure 13 shows that the welfare level of unskilled agents continuously increases along the transitional path. However, figure 12 shows that the welfare level of skilled agents of generations 2 and 3 decreases compared to the welfare level of skilled agents of period 1, whereas in the long-term skilled agents benefit from this redistributive policy.

Finally, figure 14 shows that, at first, the wealth inequality ratio highly decreases because of the direct redistribution of public resources in favour of unskilled agents. However, from period 3 on, this ratio increases and converges towards its new value because of the increase in the wage inequality ratio (linked to capital accumulation).

5 Conclusion

In this paper we show that an increasing redistribution of pension systems increases capital accumulation. Given our capital-skill complementarity technology, it implies that a more redistributive pension system increases wage inequalities. However, in a life-cycle perspective, this policy redistributes wealth among the population if the mortality differential is not too large. Moreover, it is possible even for rich agents to benefit from this structure because of the increase in capital accumulation.

A future work will introduce an endogenous labor supply and study the distorsive impact of a more redistributive policy. Intuitively, the tax rate should decrease and the capital accumulation should increase. It would imply that labor supply would be an increasing function of the degree of redistribution of pension systems. It would dramatically contrast with the results of Sommacal (2006).

APPENDIX

Appendix 1: Proof of Proposition 1.

Given assumption 3, equation (17) can be rewritten as:

$$LHS(\Psi(k_t), \lambda) - RHS(\Psi(k_t), k_t, \lambda) = 0$$

Differentiating this equation with respect to k_t gives:

$$LHS_1()\Psi_1() - RHS_1()\Psi_1() - RHS_2() = 0$$

with $f_i()$ the derivative of $f()$ with respect to its ith argument. It implies:

$$\Psi_1() = \frac{RHS_2()}{LHS_1() - RHS_1()}$$

Under assumptions 3 and 4 there exists at least one stable steady state k^s_{SS}. It implies that: $LHS_1() - RHS_1() - RHS_2() > 0$.

Let us now consider the net impact of an increase in λ on k^s_{SS}. We differentiate equation (17) with respect to k^s_{SS} and λ. We obtain:

$$\left[LHS_1() - RHS_1() - RHS_2()\right] dk^s_{SS} = \left[RHS_3() - LHS_2()\right] d\lambda$$

The factor before dk^s_{SS} is strictly positive under the assumption of stability of the equilibrium. To determine the sign of $dk^s_{SS}/d\lambda$ it is sufficient to know the sign of $(RHS_3() - LHS_2())$. We show that it is negative. Indeed as long as $w^s(k) > w^u(k)$, $RHS_3()$ is a decreasing function of λ because of the positive impact of λ on τ. There only remains to know the sign of $LHS_2()$. If it is positive then we prove the proposition.

$LHS()$ can be rewritten as:

$$LHS(k,\lambda) = (1+n)k + \frac{\nu\varphi\overline{w}(k)}{\varphi(\beta R(k))^{1/\eta} + R(k)} + (1-e)\nu(\lambda w^u(k) + (1-\lambda)\overline{w}(k))\left[\frac{-\epsilon R(k)}{\left[(\varphi-\epsilon)(\beta R(k))^{1/\eta} + R(k)\right]\left[\varphi(\beta R(k))^{1/\eta} + R(k)\right]}\right]$$

It implies that as long as $w^s(k) > w^u(k)$, then $LHS_2() > 0$.□

Appendix 2[22]:

Firstly, let us give a preliminary result. We consider the case of a perfect competition on the final good market and on inputs markets.

Lemma 1: If there exists a production function $Y = F(K,L)$ satisfying the Inada conditions, and which is homogenous of degree 1 then we have:

$$\frac{dw}{w} = \frac{\alpha_K}{\sigma_{K,L}}\frac{dk}{k} = \frac{\alpha_K}{\sigma_{K,L}}\left[\frac{dK}{K} - \frac{dL}{L}\right] \tag{25}$$

and

$$\frac{dr}{r} = -\frac{1-\alpha}{\alpha}\frac{dk}{k} = -\frac{1-\alpha}{\alpha}\left[\frac{dK}{K} - \frac{dL}{L}\right] \tag{26}$$

with $k = K/L$, $\alpha_K = \frac{KF_K()}{Y}$ and $\sigma_{K,L}$ the elasticity of substitution between K and L defined by:

$$\sigma_{K,L} = \frac{dk/k}{d(\frac{w(k)}{r(k)})/(\frac{w(k)}{r(k)})} \tag{27}$$

Proof: We start with the result: $w = f(k) - kf'(k)$. Then it is sufficient to use equation 27 knowing that $dr/dk = f''(k) = -dw/dk \times 1/k$.□

[22] I thank A.d'Autume for the proof presented here.

There only remains to apply to apply this result to the production function (5). Then we have:

$$\frac{dw^u}{w^u} = \frac{\alpha_F}{\sigma_F}\left(\frac{dG}{G} - \frac{dL^u}{L^u}\right) \tag{28}$$

or

$$\frac{dw^u}{w^u} = \frac{\alpha_F}{\sigma_F}\left(\alpha_G\frac{dK}{K} + (1-\alpha_G)\frac{dL^s}{L^s} - \frac{dL^u}{L^u}\right) \tag{29}$$

α_F denotes the share of $G()$ in function $F()$, α_G denotes the share of K in $G()$, and σ_F denotes the elasticity of substitution between the two arguments of $F()$. In the same way, using the fact that at equilibrium we have:

$$w^s = F'_G.G'_{L^s} \tag{30}$$

we have:

$$\frac{dw^s}{w^s} = -\frac{1-\alpha_F}{\sigma_F}\left(\frac{dG}{G} - \frac{dL^u}{L^u}\right) + \frac{\alpha_G}{\sigma_G}\left(\frac{dK}{K} - \frac{dL^s}{L^s}\right) \tag{31}$$

with σ_G the elasticity of substitution between the two inputs of function $G()$. The previous equation implies:

$$\frac{dw^s}{w^s} = \left[-\frac{1-\alpha_F}{\sigma_F}\alpha_G + \frac{\alpha_G}{\sigma_G}\right]\frac{dK}{K} + \left[-\frac{1-\alpha_F}{\sigma_F}(1-\alpha_G) - \frac{\alpha_G}{\sigma_G}\right]\frac{dL^s}{L^s} + \frac{1-\alpha_F}{\sigma_F}\frac{dL^u}{L^u} \tag{32}$$

Assumption A1 implies that for $dL^s/L^s = dL^u/L^u = 0$ and if $dK/K > 0$ then:

$$\frac{dw^s}{w^s} > \frac{dw^u}{w^u} \;\Leftrightarrow\; -\frac{1-\alpha_F}{\sigma_F}\alpha_G + \frac{\alpha_G}{\sigma_G} > \frac{\alpha_F}{\sigma_F}\alpha_G \;\Leftrightarrow\; \frac{1}{\sigma_G} > \frac{1}{\sigma_F} \tag{33}$$

It means that the wage inequality differential increases with capital accumulation iff the elasticity of substitution between the arguments of function $F()$ is higher than the one between the arguments of function $G()$.□

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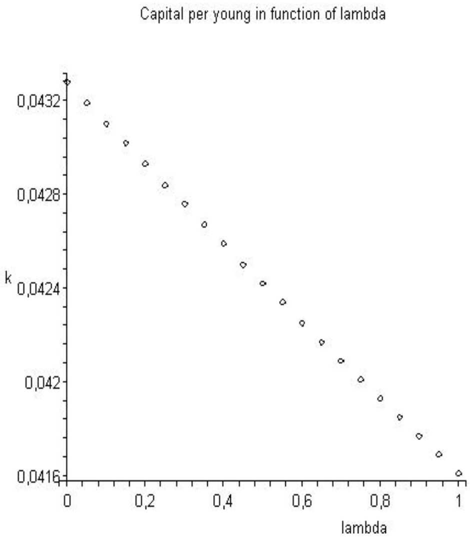

Figure 2: Capital per young (k^s) in function of λ

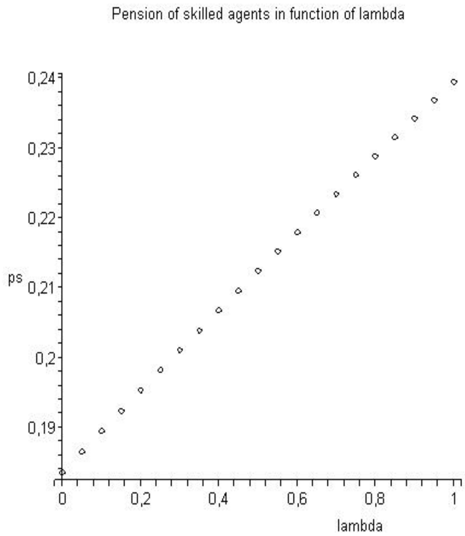

Figure 3: $p^s(k)$ in function of λ

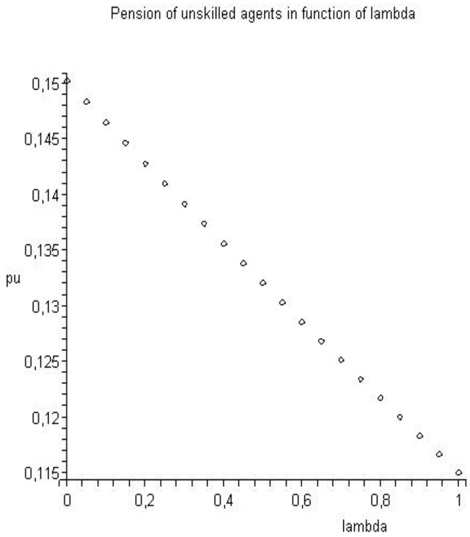

Figure 4: $p^u(k)$ in function of λ

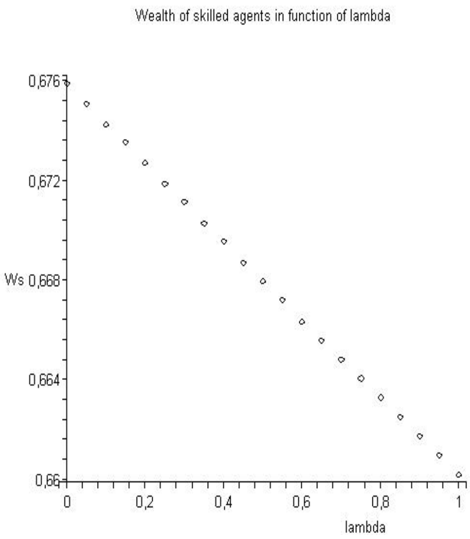

Figure 5: $W^s(k)$ in function of λ

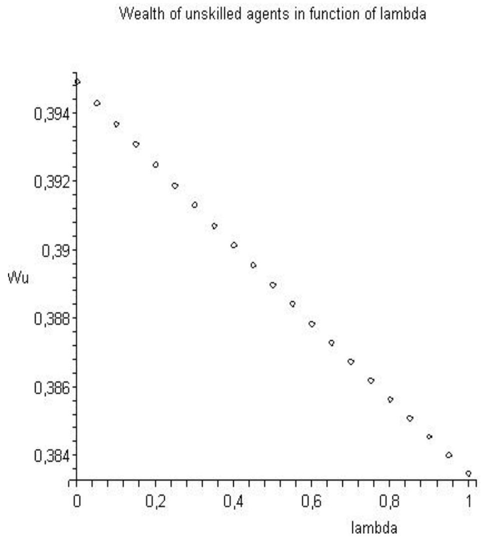

Figure 6: $W^u(k)$ in function of λ

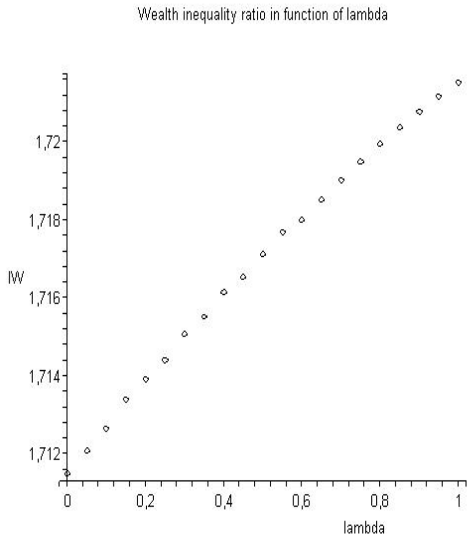

Figure 7: $I_W(k)$ in function of λ

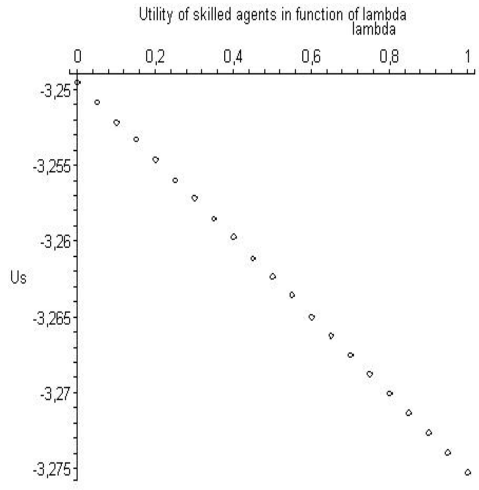

Figure 8: $U^s(k)$ in function of λ

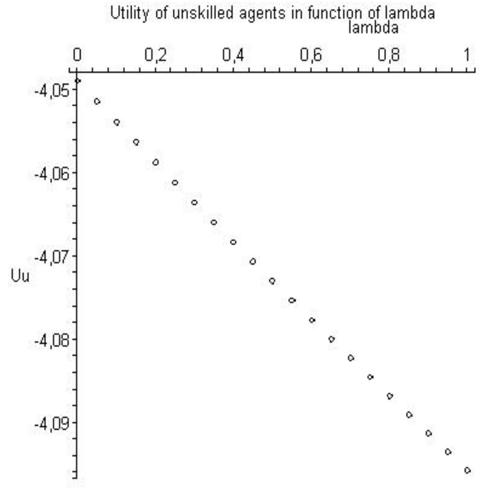

Figure 9: $U^u(k)$ in function of λ

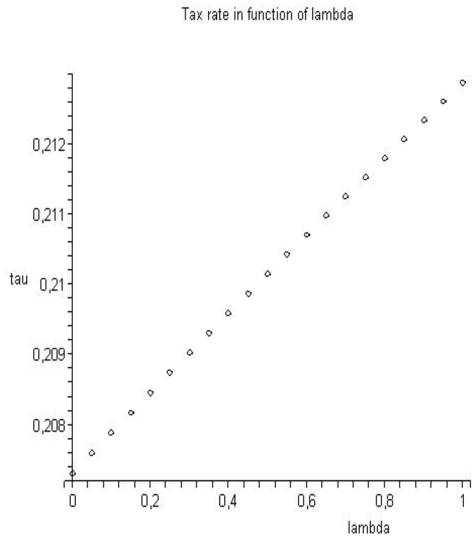

Figure 10: $\tau(k, \lambda)$ in function of λ

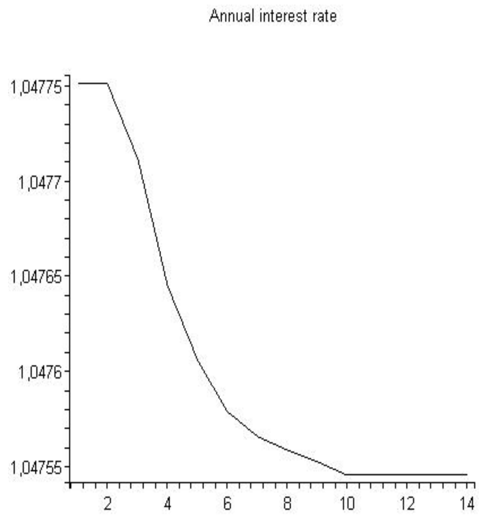

Figure 11: Transitional dynamic of the Annual Interest Rate

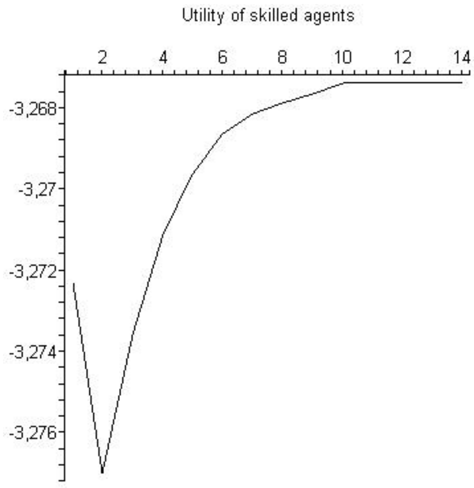

Figure 12: Transitional dynamic of U^s

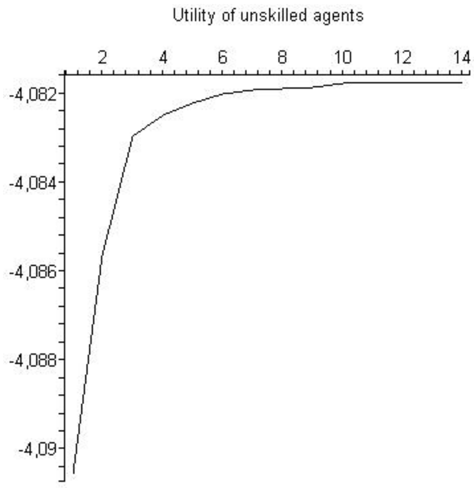

Figure 13: Transitional dynamic of U^u

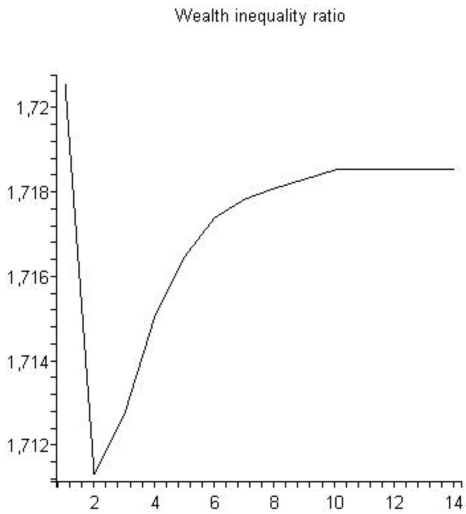

Figure 14: Transitional dynamic of I_W