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Who Really Benefits from Pension Systems? When Life Expectancy Matters*

Christophe Hachon†

Abstract

A growing literature shows that life expectancy depends on the wage level. Using an overlapping generations model with a small open economy, we explain why this result can change the redistributive properties of pension systems. Net contributions are now progressive for beveridgian pension systems and regressive for bismarkian ones. For mixed pension systems it is possible that collected resources are redistributed in favour of the ends of the distribution of wages.

Keywords: Pension system, inequality, length of life, net contribution.

JEL Classification: H55.

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†CES, University Paris 1 Pantheon-Sorbonne, 106-112 Boulevard de l'hôpital 75 013 Paris. E-mail: Christophe.Hachon@malix.univ-paris1.fr

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1 Introduction

A wide literature studies the macroeconomic impact of the ageing of the population, notably on the sustainability of pension systems[1]. This ageing process implies that reforms have to be adopted by developed countries in order to limit the fiscal burden of pension systems. Another dimension of the length of life has been explored recently. Indeed, agents differ by their life expectancy. More particulary, these differences between socio-professional groups are wide. Mesrine (1999) studies the inequalities of length of life according to socio-professional groups in France[2]. The most striking feature of his paper is that a worker has a probability to die between 35 and 65 years old almost twice higher than that of an executive manager. Furthermore, their life expectancy at 35 is 38 and 44 respectively. The same qualitative results are observed in the United-States (Panis and Lillard (1995), Deaton and Paxson (2000)). Finally, Robert-Bobbée and Cadot (2007) show that this inequality is also observed for elderly people. For agents who are 86, the ones with highest education level can expect to live 20% longer than the ones with lowest education level.

As socio-professional groups are linked to earnings, we can conclude from the previous results that earnings have an impact on the length of life. Some empirical studies deal with this link[3]. These life expectancy inequalities can have strong implications for the redistributive properties of pension systems. Indeed, as rich agents live and benefit from a pension for a longer time, pension systems are not as redistributive as they seem. Consequently, to study these properties, two dimensions have to be analyzed: (1) the distinction between Beveridgian and Bismarkian systems, and (2) the relationship between wages and life expectancy. Some authors studied the empirical implications of this distinction. Coronado *et al.* (2000) and Liebman (2001) show that the pension system of the United-States is far less progressive than is usually mentioned. Bommier *et al.* (2003) find the same qualitative results on french data.

However, theoretical implications of these new results have not been clearly studied. Mitchell and Zeldes (1996) explain that:

[1]See d'Autume (2003), Charpin (1999), Cremer and Pestieau (2000) or Galasso and Profeta (2004) among others. Here and in the rest of this paper we only consider Pay-As-You-Go pension systems.

[2]These inequalities also depend on other factors like sex or the geographical localization. For example, in France the life expectancy of women is 84.1, whereas that of men is only 77.2 (INSEE, 2006). Moreover, Rican and Salem (1999) show that there are strong disparities according to the localization of people in France.

[3]See Attanasio and Emmerson (2001), Bommier *et al.* (2003) or Adams *et al.* (2003) for a survey.

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> *"Despite its intent, the system [the pension system] is less progressive than it might seem, because there is a positive correlation between lifetime earnings and length of life."*

They emphasize the main role played by the inequalities of length of life but this sentence raises a question: can a pension system be regressive?

Borck (2007) has exploited this idea. He shows that the size of a pension system can be determined by a coalition of elderly, very poor and very rich agents. Poor agents benefit from the Beveridgian part of the pension system, whereas rich agents benefit the longest time from the pension system. This paper of Borck can be seen as a study on the consequences of the redistributive properties of pension systems when inequalities of length of life are taken into account. Nevertheless, it is not an analysis of the redistributivity itself. Consequently, our paper is the next step in order to analytically clarify this last point.

A pension system is purely Beveridgian if every agent receives the same pension. Conversely, a pension system is purely Bismarkian if pensions completely depend on the wages of agents. A pension system is mixed if it has a Beveridgian and a Bismarkian component. The more a pension system is Beveridgian, the higher intra-generational transfers are. Countries highly differ by this intra-generational component. France, Germany and Italy have a Bismarkian structure. Canada, the Netherlands and New-Zeland are essentially Beveridgian. Finally, Japan, the United-Kingdom and the United States have mixed pension systems (Sommacal (2006), Casamatta *et al.* (2000)).

In this paper, we use an overlapping generations model in which agents differ by their wage and by their length of life. The pension system can be Beveridgian, Bismarkian or a mix of the two as in Casamatta *et al.* (2000). To study the redistributivity of pension systems we use the concept of "net contributions". The net contribution of an agent with a given wage, is the actualized difference (at the growth rate of the population) between the tax paid and the benefit from the pension system (Drouhin, 2001). A positive net contribution implies that agents pay more than they receive from the pension system.

In order to well understand the qualitative changes induced by inequalities of length of life we first consider that every agent has the same length of life. In this specific case, it is possible to show that if pension systems are at least partially Beveridgian then it is also progressive. But if the pension system is Bismarkian then the net contribution for each agent is nil. Afterwards, we introduce inequalities of length of life and we consider the case of a Beveridgian, a Bismarkian and of a mixed pension system successively. If the pension system is Beveridgian then it is progressive, but the poorest do not necessarily benefit the

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most from the pension system. Furthermore, the share of the population who benefits from a negative net contribution changes. If the pension system is Bismarkian, then we show that it is regressive because the poorest have the most positive net contribution and the richest the most negative one. If we have a mixed pension system our analytical results in terms of net contributions can be generalized only if pension systems tend towards either a Beveridgian or a Bismarkian structure. For intermediate cases, a numerical resolution calibrated on French data is used. It does not describe the exact structure of the French pension system but it emphasizes its important qualitative properties. We use different calibrations for the function which links the length of life with the wage level. We show that it is possible that the ends of the distribution of wages benefit from a negative net contribution.

Our paper is organized as follows. In section 2, we present our model. In section 3, the study of the redistributive properties of pension systems without inequalities of length of life is detailed. In the following sections we assume that the length of life is linked to the wage level. In section 4 and 5, we study a Beveridgian and a Bismarkian pension system respectively. In section 6, we emphasize the main properties of mixed pension systems. In section 7 we calibrate our model on french data. Section 8 provides some concluding remarks.

2 The Model

We consider a small open economy in which agents live two periods[4]. At each period t, the number of young agents is N_t. Population is assumed to grow at a constant rate n, such that $N_t = (1+n)N_{t-1}$. These agents are heterogenous since each of them has a wage w which belongs to the interval $\Omega_w = [w_-, w_+]$, with $w_- > 0$. Wages are distributed randomly among the population. $f(w)$ denotes the density function of the random variable w. Consequently, it is also the fraction of the population having a wage level w[5]. The average wage of this economy can be written:

$$\bar{w} = \int_{\Omega_w} w f(w) dw \tag{1}$$

Furthermore, we assume that agents live only a fraction T of their second period of life[6]. This length of life is supposed to be linked to the wage level of agents: $T = T(w)$, and

[4]The length of each period is normalized to 1.

[5]We assume that the size of each generation is sufficiently large for the law of large numbers applies.

[6]There is no uncertainty in our model. Consequently agents are sure to live until the end of the fraction T of their second period of life. However, our model can also be interpreted as a model in which $(1-T)$

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more specifically we assume that $T'(w) > 0$. It represents the inequalities of length of life in function of socio-professional groups. The average length of life is:

$$\bar{T} = \int_{\Omega_w} T(w)f(w)dw \tag{2}$$

The linkage between the wage level and the length of life is measured by the covariance:

$$COV_{T,w} = \int_{\Omega_w} T(w)wf(w)dw - \bar{T}\bar{w} \tag{3}$$

As we assume that $T'(w) > 0$, then we have $COV_{T,w} > 0$[7].

Moreover, we make the following assumption about the function $T(w)$:

Assumption 1: $T'(w) > 0$, $T''(w) < 0$, $E_{T/w} = T'(w)\frac{w}{T(w)} < 1$, and $-T''(w)\frac{w}{T'(w)} < 2$.

The first part of this assumption is standard and represents the decreasing marginal impact of wages on the length of life. The assumption on the elasticity implies that an increase in wages of $x\%$ implies an increase in the length of life of less than $x\%$. The last part of this assumption is only a technical hypothesis which will be used later in the paper[8].

Each agent works when he is young and retires at the end of his first period of life[9]. It is the same assumption than this used in Casamatta *et al.* (2000) or in Borck (2007). As long as an agent works, he pays a payroll tax τ. This tax is used to finance a PAYG pension system. When old, an agent receives a pension $p(w)$. Pensions are paid as long as agents are still alive, i.e. during a fraction $T(w)$ of their second period of life (d'Autume, 2003). Furthermore, pensions per unit of time are partly indexed on the wage of the first period of the agent and on the average wage of the economy $(\alpha w + (1-\alpha)\bar{w})$. α measures the size of

would be the probability to die at the end of the first period of life as in Drouhin (2001).

[7]See Appendix A.

[8]Some main functions respect this property. For example: $T(w) = aw+b$, or $T(w) = \gamma w^{\xi}$ with $a,b,\gamma > 0$ and $1 > \xi > 0$.

[9]In this paper we do not consider the length of education and the retirement age whereas it has an impact on the redistributive properties of pension systems. Indeed, the length of education and the retirement age are positively correlated with the wage level. Then, a strong link between education and wages decreases the progressivity of pension systems, whereas the positive correlation between the retirement age and the wage level increases the progressivity of pension systems. However, in this paper we only analyse the redistributive properties of unfunded pension systems when life expectancy inequalities are taken into account. In this way we emphasize the main role played by the mortality differential neutralizing other channels.

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the Bismarkian part of the pension system, whereas $(1-\alpha)$ measures the Beveridgian part of the pension system. When $\alpha = 1$ then the pension system is completely Bismarkian because pensions are only indexed on activity wages of each agent. Conversely, when $\alpha = 0$ every agent receives the same pension. In that case, the pension system is Beveridgian.

Finally, agents receive only a fraction (λ) of this weighted average $(\alpha w + (1-\alpha)\bar{w})$ per unit of time. λ denotes the average replacement rate of the pension system. Consequently, the pension $p(w)$ which an agent receives during his second period of life is:

$$p(w) = \lambda(\alpha w + (1-\alpha)\bar{w})T(w) \tag{4}$$

We assume that there is no debt in this economy. It implies that all pensions have to be financed by a tax on wages. The budget constraint of the government can be written:

$$N_t \int_{\Omega_w} \tau w f(w) dw = N_{t-1} \int_{\Omega_w} \lambda(\alpha w + (1-\alpha)\bar{w})T(w)f(w)dw$$

Some straightforward calculations imply that:

$$\tau = \lambda \frac{\alpha COV_{T,w} + \bar{w}\bar{T}}{(1+n)\bar{w}} \tag{5}$$

Let us first study the case where every agent has the same length of life, that is: $T(w) = \bar{T}$, $\forall w$. In that case we have $COV_{T,w} = 0$, and the tax rate becomes (as in d'Autume (2003)):

$$\tau = \lambda \frac{\bar{T}}{(1+n)}$$

The higher the replacement rate is, the higher the tax rate necessary to finance the pension system is. Indeed, an increase in the generosity has to be financed with a higher tax on young agents. Furthermore, the higher the old age dependency ratio $\left(\frac{\bar{T}}{(1+n)}\right)$ is, the higher the tax rate has to be. If for each worker there are more old agents, and for a given generosity of the pension system, then the tax rate has to increase to finance these additional pensions.

Let us now consider the case where $COV_{T,w} > 0$, i.e. that there are inequalities of length of life. Everything else equal, the introduction of inequalities of length of life increases the tax rate of the pension system. Indeed, the richer agents are, the longer they live and then the longer the period during which they receive a pension is. Conversely, the poorer agents are, the shorter the period during which they receive a pension is. If pensions are even partially indexed on the activity wage of agents (by a coefficient α) then the highest pensions are paid to people who live for the longest period. The tax rate has to increase to

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finance this additional spending. Now, if the pension system has a Beveridgian structure ($\alpha = 0$), then even if the covariance is large, the decrease in spending for agents with a length of life smaller than $\bar{T}$ exactly compensates the increase in spending for the others[10].

At each period t, a group of agents with a wage w pays: $\tau w f(w) N_t$, whereas at the same time agents with the same productivity receive $p(w)f(w)N_{t-1}$. Our main objective is to know if at each period a group of agents with a wage w receives more from the pension system than he pays for it, i.e. if $p(w)f(w)N_{t-1}$ is larger than $\tau w f(w) N_t$. But the model also permits to determine the wage of the group who benefits the most from the pension system, i.e. who receives the most given the amount he pays.

At each period t, the net contribution of a group with a wage w is:

$$CN_{w,t} = \tau w f(w) N_t - p(w) f(w) N_{t-1}$$

or,

$$CN_{w,t} = N_t f(w) \left[\tau w - \frac{p(w)}{1+n} \right]$$

The member between brackets represents the net contribution of an agent with a wage w if he uses the growth rate of the population as actualisation rate (Drouhin, 2001). A positive net contribution means that a group pays more for the pension system than he receives from it. Using equations (4) and (5) we obtain:

$$CN_{w,t} = \lambda f(w) N_{t-1} \left[\frac{\alpha COV_{T,w} + \bar{w}\bar{T}}{\bar{w}} w - (\alpha w + (1-\alpha)\bar{w}) T(w) \right] \tag{6}$$

Integrating this function over the interval Ω_w, it is straightforward to show that $\int_{\Omega_w} CN_{w,t} dw = 0$.

But this amount is biased by the size of each group. Indeed, a high net contribution can reflect a large size of a group. So it is better to use the individual net contribution for each group. It can be written:

$$CN^i_{w,t} = \lambda \left[\frac{\alpha COV_{T,w} + \bar{w}\bar{T}}{\bar{w}} w - (\alpha w + (1-\alpha)\bar{w}) T(w) \right] \equiv \lambda A(w) \tag{7}$$

Note that the size of the pension system (λ) has only a quantitative effect because it does not influence the sign of $A(w)$. λ amplifies the net contribution of each agent.

[10] Because of the linearity of pensions in (4).

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The form and the sign of the function $A(w)$ are not trivial because of the function $T(w)$. To well understand the main implications of such a function we first study the case where there are no inequalities of length of life. Then, we introduce inequalities of length of life and we study the case where $\alpha = 0$ and the case where $\alpha = 1$ successively. Finally, we give the main properties of a mixed pension system, i.e. a system with a Beveridgian and a Bismarkian part.

3 The Benchmark Case

This section details the results for the case where every agent in our economy has the same length of life: $T(w) = \bar{T}$, $\forall w$. It is an usual assumption. Agents only differ by their wages. This uni-dimensionality of the heterogeneity often simplifies the analysis but masks a very different reality. Let us first study the conclusions that would be obtained if we had only considered wage inequalities.

If $T(w) = \bar{T}$, $\forall w$, then $COV_{T,w} = 0$ and finally:

$$A_1(w) = (1 - \alpha)\bar{T}(w - \bar{w}) \tag{8}$$

$A(w)$ is a strictly increasing function of w as long as $\alpha \in [0, 1)$ (see figure (1)). Furthermore, the net contribution is negative for agents having wages below the average. It implies that poor agents receive more from the pension system than they pay for it. Conversely, the net contribution is positive for agents having wages above the average wage.

Proposition 1: *(i) Every agent with a wage below (above) the average wage has a negative (positive) net contribution. (ii) The poorest benefit the most from the redistributive properties of the pension system. This net benefit is a decreasing function of w.*

Proof: (i) $A(w)$ is negative for $w < \bar{w}$ and positive for $w > \bar{w}$. (ii) $A(w)$ is a strictly increasing function of w.□

The second part of this proposition implies that the poorest have the highest negative net contribution. This result depends on the assumption that the pension system is at least partly Beveridgian ($\alpha \in [0, 1)$). If $\alpha = 1$ then the pension system is completely Bismarkian because the pension is only indexed on the activity wage of agents.

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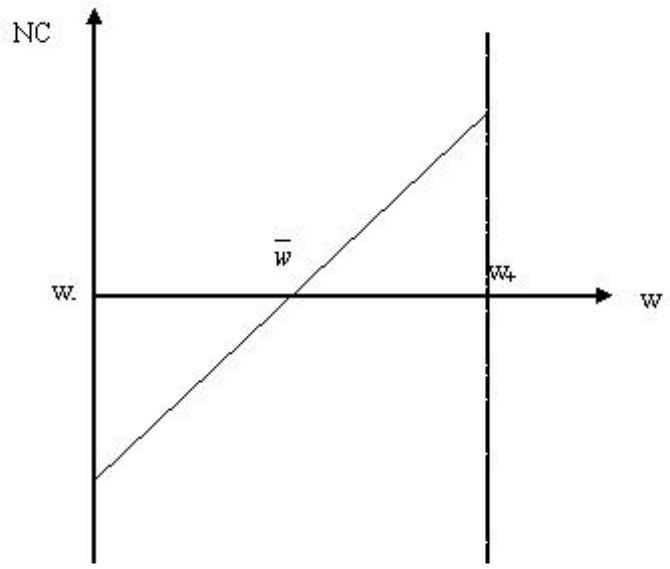

Figure 1: Net Contributions for $T(w) = \bar{T}\ \forall w$

Proposition 2: *If $\alpha = 1$ then the net contribution of each group is nil. The pension system is not redistributive.*

Proof: See equation (8).□

This result ensures that a Bismarkian pension system in an economy where agents only differ by their wages, is neutral in terms of net contribution. Pensions exactly compensate contributions to the pension system.

These two results are frequently encountered in the literature. The following sections show that these results depend on the assumption that every agent has the same length of life. If it is not the case, then the higher the wage is, the longer agents live, and the more they benefit from the pension system. Consequently, the intuition is that the redistributive properties mentioned above change. Firstly because the poorest do not necessarily benefit the most from a Beveridgian pension system. Secondly because the Bismarkian pension systems is not neutral.

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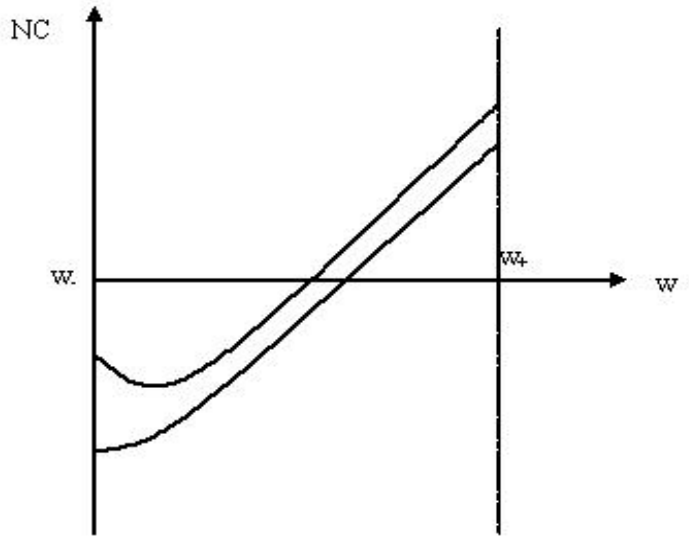

Figure 2: Net Contributions for $\alpha = 0$.

4 Pure Beveridgian Pension Systems

For pure Beveridgian pension systems we have $\alpha = 0$, i.e. that every agent receives the same pension. This pension is indexed on the average wage of the economy. From this section we assume that agents also differ by their length of life: $T(w)$, with $T'(w) > 0$. The expression for $A(w)$ becomes:

$$A_2(w) = w\bar{T} - \bar{w}T(w) \tag{9}$$

The properties of this function are such that: $A_2'(w) = \bar{T} - \bar{w}T'(w)$ and $A_2''(w) = -\bar{w}T''(w)$. The sign of $A_2'(w)$ is indeterminate, but $A_2''(w)$ is clearly positive under the assumption 1 (see figure (2)).

Proposition 3: *There exists a threshold $\hat{w}$ such that the net contribution is negative (positive) for $w < \hat{w}$ ($w > \hat{w}$). Furthermore $\hat{w} > \bar{w}$ if and only if $\bar{T} < T(\bar{w})$.*

Proof: Under assumption 1, we know that $T(w)/w$ is a decreasing function of w. Furthermore, $A_2(w) > 0$ if and only if $T(w)/w < \bar{T}/\bar{w}$. $A_2(w)$ can not be positive for every w as the sum of net contributions is equal to 0. Finally, as $T(w)/w$ is a decreasing function

of w, we conclude that there exists a threshold value $\hat{w}$ such that $A_2(w) < 0$ for $w < \hat{w}$, and $A_2(w) > 0$ for $w > \hat{w}$. $\hat{w} > \bar{w}$ if and only if $A_2(\bar{w}) < 0$.□

This result ensures that poor agents (with a wage below $\hat{w}$) have a negative net contribution. Moreover, if the average length of life is smaller than the length of life of the average wage then a larger group benefits from the redistributive effect of the pension system. The more $\bar{T}$ is different from $T(\bar{w})$, the more $\hat{w}$ removes away from $\bar{w}$. This result is intuitive. Indeed, a high average length of life implies that the taxe rate of the pension system is higher. Then the share of the population who benefits from a negative net contribution decreases ($\hat{w} < \bar{w}$).

But we also have to compare the net contribution $A_2(w)$ with $A_1(w)$. It is easy to show that $A_2(w) > A_1(w)$ ($<$) as long as $T(w) < \bar{T}$ ($>$). It implies that poor workers benefit less from the pension system and that rich workers pay less for the pension system.

Because of the convexity of the function $A_2(w)$, if there exists a value w^M such that $A_2'(w^M) = 0$, then it is a minimum. It has two implications for our analysis. (i) The richest contribute the most to the pension system because of the convexity of $A_2(w)$. They have the highest positive net contribution. (ii) The poorest do not necessarily benefit the most from the redistributivity of the pension system.

Lemma 1: *$\hat{w} > w^M > w_-$ if and only if $A_2'(w_-) < 0$. $w^M = w_-$ if and only if $A_2'(w_-) \geq 0$.*

Considering $A_2'(w_-)$, this expression is negative if $T'(w_-)$ is sufficiently large. It means that the marginal impact of wages on the length of life is large for low values of wages. Agents with a wage slightly higher than w_- can expect to live a lot of more time than agents with a wage w_-. They receive a pension during this additional time. Consequently, it is the agents from the group w^M who benefit the most from the pension system and w^M can be different from w_-.

The two main implications are: (i) The share of the population who benefits from the pension system can differ from the interval $[w_-, \bar{w}]$. (ii) The agents who benefit the most from the pension system are not necessarily the poorest. It depends on the properties of the function $T(w)$.

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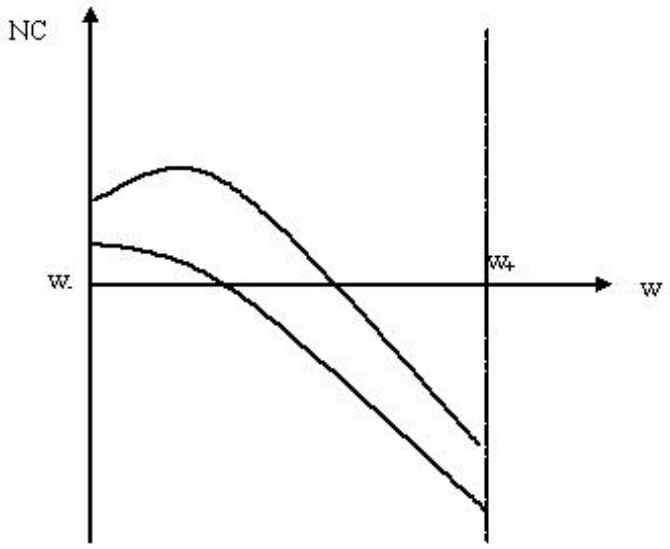

Figure 3: Net Contributions for $\alpha = 1$.

5 Pure Bismarkian Pension Systems

For pure Bismarkian pension systems, pensions are only indexed on activity wages ($\alpha = 1$). In section 3, we obtained that the net contributions are nil for every group if there are no inequalities of length of life. But the introduction of these inequalities considerably changes the qualitative results.

For $\alpha = 1$, $A(w)$ can be written:

$$A_3(w) = w\frac{COV_{T,w} + \bar{w}\bar{T}}{\bar{w}} - wT(w) \tag{10}$$

The sign of $A_3'(w) = \frac{COV_{T,w}+\bar{w}\bar{T}}{\bar{w}} - T(w) - wT'(w)$ is indeterminate, but $A_3''(w) = -2T'(w) - T''(w)w$ is clearly negative under assumption 1. Then $A_3(w)$ is a concave function of w (see figure (3)).

Proposition 4: *There exists a threshold $\breve{w}$ such that the net contribution is positive (negative) for $w < \breve{w}$ ($w > \breve{w}$). $\breve{w}$ is determined by the following equation:*

$$T(\breve{w}) = \frac{COV_{T,w}}{\bar{w}} + \bar{T}$$

Proof: $A_3(w) > 0$ if and only if $T(w) < \frac{COV_{T,w}+\bar{w}\bar{T}}{\bar{w}}$. As $T(w)$ is a strictly increasing

function of w and as the sum of the net contributions is equal to 0, then as long as $w < \breve{w}$ we have $T(w) < T(\breve{w})$ and the net contribution is positive. But for $w > \breve{w}$ the net contribution is negative.□

This result is completely different from this of proposition 2. Rich agents benefit from a negative net contribution whereas poor agents have a positive net contribution. But the result of this section is even more surprising. Indeed, as $A_3(w)$ is a concave function of w then if there exists a value w^{max} such that $A_3'(w) = 0$, it is a maximum. The highest positive value of $A_3(w)$ is obtained for $w = w_+$. Given the result of proposition 4 we can write that $w^{max} \in [w_-, \breve{w})$, and more precisely:

Lemma 2: *$w^{max} = w_-$ as long as $A_3'(w_-) = \frac{COV_{T,w} + \bar{w}\bar{T}}{\bar{w}} - T(w_-) - w_- T'(w_-) \leq 0$. $w^{max} \in (w_-, \breve{w})$ if and only if $A_3'(w_-) > 0$.*

$A_3'(w_-)$ is positive if $T'(w_-)$ is not too large, i.e. if agents with a wage slightly higher than w_- can expect to have a life expectancy only just higher than this of agents with a wage w_-.

Two main conclusions can be obtained from this analysis. (i) Pure Bismarkian pension systems are regressive because poor agents have a positive net contribution and rich agents have a negative one. (ii) If $T'(w_-)$ is sufficiently large, then the poorest have the highest positive net contribution. The concavity of $A_3(w)$ also implies that the richest have the highest negative net contribution, what reinforces our previous conclusion.

6 Mixed Pension Systems

Let us assume from now that $\alpha \in (0, 1)$. If α tends towards 1 then the pension system becomes more Bismarkian. Conversely, if α tends toward 0 the pension system becomes more Beveridgian as the pension depends less on the wage of agents. The function $A(w)$ can be written as:

$$A_4(w, \alpha) = w\frac{\alpha COV_{T,w} + \bar{w}\bar{T}}{\bar{w}} - (\alpha w + (1 - \alpha)\bar{w})T(w) \tag{11}$$

But this function has indeterminate properties. Indeed $\partial A_4(w, \alpha)/\partial w$ can be positive, negative or null. And $\partial^2 A_4(w, \alpha)/\partial w^2$ can also be positive, negative or null.

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In order to obtain clear analytical results we specify the function $T(w)$. We assume it has the following form:

$$T(w) = \gamma w^{\xi} \tag{12}$$

with $\gamma > 0$ and $1 > \xi > 0$. This function respects every property of assumption 1. Let us first study the concavity and the convexity of the function $A_4(w)$. Note that:

$$\frac{\partial A_4(w,\alpha)}{\partial w} = \frac{\alpha COV_{T,w} + \bar{w}\bar{T}}{\bar{w}} - T'(w)(\alpha w + (1-\alpha)\bar{w}) - \alpha T(w) \tag{13}$$

and that:

$$\frac{\partial^2 A_4(w,\alpha)}{\partial w^2} = -T''(w)(\alpha w + (1-\alpha)\bar{w}) - 2\alpha T'(w) \tag{14}$$

Lemma 3: *There exists a threshold value $w_p(\alpha)$ such that if $w < w_p(\alpha)$ ($> w_p(\alpha)$) then $\frac{\partial^2 A_4(w,\alpha)}{\partial w^2} > 0$ (< 0). It is such that $\frac{\partial w_p(\alpha)}{\partial \alpha} < 0$. Furthermore, there exists an interval $(0, \alpha_+)$ such that for $\alpha \in (0, \alpha_+)$ we have $w_p(\alpha) > w_+$; and a second interval $(\alpha_-, 1)$ such that for $\alpha \in (\alpha_-, 1)$ we have $w_p(\alpha) < w_-$, with $\alpha_+ < \alpha_-$.*

Proof: Using equation (12) we obtain that $T''(w)/T'(w) = (\xi - 1)/w$. Then using equation (14) it is straightforward to show that $\frac{\partial^2 A_4(w,\alpha)}{\partial w^2} > 0$ if and only if $w < \bar{w}\frac{(1-\xi)(1-\alpha)}{\alpha(1+\xi)} \equiv w_p(\alpha)$, with $w_p'(\alpha) < 0$. $w_p(0) = +\infty$ and $w_p(1) = 0$.□

This result gives the properties in terms of convexity and of concavity of the function $A_4(w)$. It implies that for a value of α sufficiently small (inferior to α_+) $A_4(w)$ is convex. Conversely, for α sufficiently high (superior to α_-) $A_4(w)$ is concave.

Proposition 5: *If $A(w_-, \alpha_-) > 0$ and if $A(w_-, \alpha_+) < 0$ then for $\alpha \in (0, \alpha_+)$ net contributions behave qualitatively as in the pure Beveridgian case. And for $\alpha \in (\alpha_-, 1)$ net contributions behave qualitatively as in the pure Bismarkian case.*

Proof: The first two conditions ensure that the poorest have a positive (negative) net contribution in the interval $(\alpha_-, 1)$ ($(0, \alpha_+)$).□

This result extends the qualitative properties of Bismarkian and Beveridgian pension systems to intervals for the parameter α and not only to extreme values 0 and 1. Every

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pension system with an α in the interval $(\alpha_-, 1)$ is regressive. Conversely, every pension system with an α inferior to α_+ is redistributive, whereas the poorest do not necessarily benefit the most from this pension system.

For $\alpha \in (\alpha_+, \alpha_-)$, we have to calibrate our model to know the exact form of net contributions.

7 Calibration on French Data

In order to calibrate our model we first have to specify wage inequalities. We assume that wages belong to the interval: $\Omega_w = [0.2, 9]$. This interval is sufficiently large for the wage w_+ is 45 times higher than w_-. Piketty (2002), studying the distribution of wages in France, finds a ratio of 5 between the wages of the first and of the last decile. The gap between this empirical fact and our calibration can be explained by the fact that we use the two extreme values of a *continuum* and as a consequence wage inequalities are greater. We choose this interval for Ω_w because once it is combined with the density function of w, our model matches the Gini coefficient of the wage distribution calculated by Hairault and Langot (2008) on French data.

The density function of the distribution of wages among the population has to respect the essential property: mode<median<mean (Lambert, 2001, p.23). This property is a common feature of most industrialized countries. It implies that the wage distribution among the population is asymetric. The most common income level is less than the median wage. And because of strong wage inequalities the median wage is less than the average wage of the economy. Furthermore, the Gini index of wages has to tend towards 0.32 (Hairault and Langot, 2008). Lambert (2001) shows that the Gini index can be calculated as:

$$G = -1 + 2\int_{w_-}^{w_+} \frac{wF(w)f(w)}{\bar{w}} dw$$

A useful density function is the density function of Weibull. It is asymmetric and it has the following form:

$$f(w) = \frac{c}{b}\left(\frac{w-a}{b}\right)^{c-1} e^{-\left(\frac{w-a}{b}\right)^c} \tag{15}$$

if $w > a$, and $f(w) = 0$ for $w \leq a$, with b and $c > 0$. The only problem with the use of this function is that the $\int_{\Omega_w} f(w)dw$ is not exactly 1. But the following calibration is such that this integral is approximately 1: $a = 0.2$, $b = 2$ and $c = 1.55$. Furthermore, it implies

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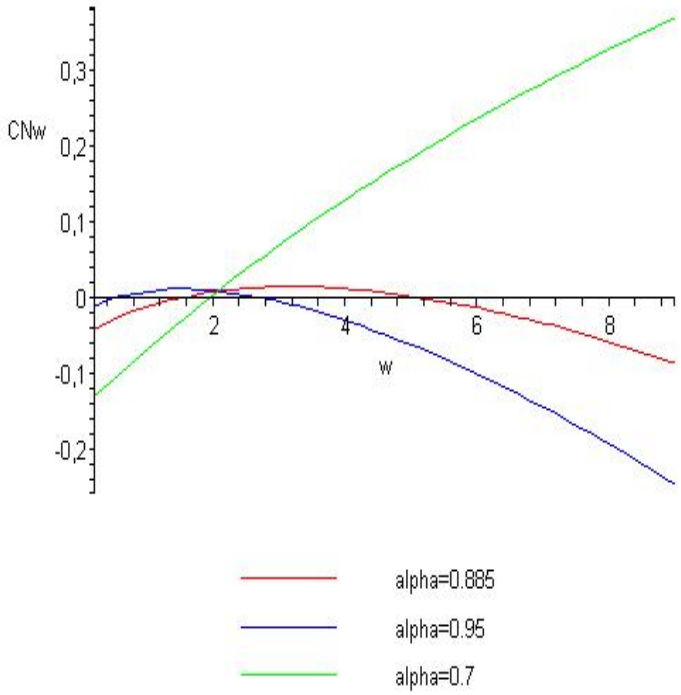

Figure 4: Net Contributions for different values of α and for $\beta = 0.09$

that $F(\bar{w}) > 0.5$, with $F()$ the cumulative distribution function, $\bar{w} = 2$, $w_{median} = 1.78$ and $G = 0.3245$.

We now have to specify the relationship between wages and the length of life. We assume that it has the following form:

$$T(w) = \mu w^{\beta} \tag{16}$$

with β the elasticity of the length of life with respect to wages. μ is a scale parameter. We assume that $\mu = 0.2$. In the following analysis we study the impact of a change in the parameter β, knowing that this value is clearly less than 1 (Bommier *et al.*, 2006).

Finally, we assume that $\alpha = 0.885$ (Hairault and Langot, 2008), but we study what happens if α varies around this benchmark value.

Firstly, let us consider that $\beta = 0.09$, i.e. that an increase in wages of 1% implies an increase in the length of life of 0.09%. Figure (4) illustrates this case.

For the value of α calculated by Hairault and Langot (2008), then it is clear that agents

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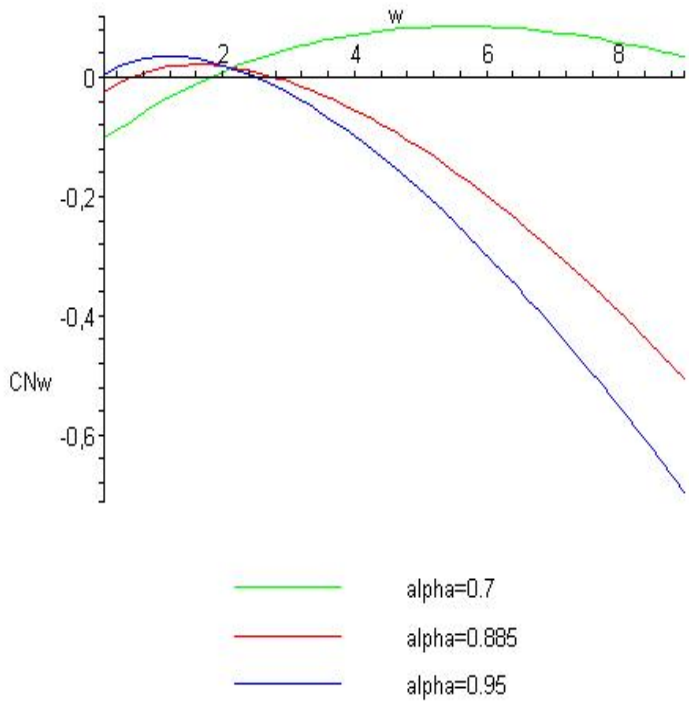

Figure 5: Net Contributions for different values of α and for $\beta = 0.18$

with a wage below 1.6 and above 5.2 benefit from the pension system. The richest have a net contribution more negative than the net contribution of the poorest. Agents with a wage in the interval [1.6,5.2] have a positive net contribution and finance the negative net contributions of the two extremes. If α increases, i.e. that the pension system becomes more Bismarkian, then less poor agents benefit from a negative net contribution whereas the net contributions becomes negative for wages above 2.8. It is almost a regressive pension system. Finally, if $\alpha = 0.7$, then the redistributive properties of the pension system endure. The poorest benefit the most from the pension system and the richest have the highest positive net contribution.

Let us now assume that $\beta = 0.18$. The elasticity of the length of life is twice as high as before. Then the previous qualitative results endure (see figure (5)), but pension system is clearly regressive because only very small wages benefit from a negative net contribution. Agents with a wage above 2.4 have a negative net contribution and the richest benefit the most from the pension system. This result is also more true for $\alpha = 0.95$. Finally, even for $\alpha = 0.7$ the redistributive properties of the pension system are less clear.

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8 Conclusion

Our contribution permits to clarify theoretically the debate on the redistributive properties of pension systems when there exists inequalities of length of life and notably the sentence from Mitchell and Zeldes (1996) cited above. It is shown that Beveridgian pension systems are less progressive than they seem and that Bismarkian pension systems are regressive. Moreover, it permits to show that for mixed pension systems, there can be a redistribution of resources from the middle to the ends of the distribution of wages. This last point would have to be taken into account for empirical analysis which study the progressivity of pension systems.

Our theoretical analysis is a first attempt which clarifies the debate on the progressivity of pension systems once life expectancy inequalities are taken into account. The next step on our research agenda would be to use a micro-simulation model, as in Liebman (2001), on French data. It would permit to quantify the impact of life expectancy inequalities on the redistributive properties of pension systems.

APPENDIX A

Indeed, the covariance can also be written as: $\int_{\Omega_w}(w-\bar{w})(T(w)-\bar{T})f(w)dw$. But as $\int_{\Omega_w}(w-\bar{w})f(w)dw=0$, we can write that: $\int_{\Omega_w}(w-\bar{w})(T(w)-\bar{T})f(w)dw=\int_{\Omega_w}(w-\bar{w})(T(w)-X)f(w)da$, with X a constant, whatever the value of X. So it is particulary true for $X=T(\bar{w})$. Then we can write that: $\int_{\Omega_w}(w-\bar{w})(T(w)-\bar{T})f(w)dw=\int_{\Omega_w}(w-\bar{w})(T(w)-T(\bar{w}))f(w)dw$. The right-hand-side is positive as it is an integral on a product of terms which have the same sign because $T'(w)>0$.□

REFERENCES

Adams, P., Hurd, M.D., McFadden, D., Merrill, A., Ribeiro, T., 2003. "Healthy, Wealthy and Wise ? Tests for Direct Causal Paths between Health and Socio-Economic Status". *Journal of Econometrics*, 112, 3-56.

Attanasio, O., Emmerson, C., 2001. "Differential Mortality in the UK". *IFS Working Paper* 01/16.

d'Autume, A., 2003. "Vieillissement et choix de l'âge de la retraite. Que peut nous dire le modèle à générations ?". *Revue Economique*, 54 (3), 561-571.

halshs-00279651, version 1 - 14 May 2008

Bommier, A., Magnac, T., Rapoport, B., Roger, M., 2006. "Droit à la Retraite et Mortalité Différentielle". *Economie et Prévision*, 168, pp.1-16.

Borck, R., 2007. "On the Choice of Public Pensions when Income and Life Expectancy are Correlated". *Journal of Public Economic Theory*, 9, 711-725.

Casamatta, G., Cremer, H., Pestieau, P., 2000. "The political economy of social security". *Scandinavian Journal of Economics*, 102, pp.503-522.

Charpin, J.M., 1999. "L'avenir de nos retraites". *La documentation française.*

Coronado, J.L., Fullerton, D., Glass, T., 2000. "The Progressivity of Social Security". *NBER working papers* 7520.

Cremer, H., Pestieau, P., 2000. "Reforming our pension system: Is it a demographic, financial or political problem?". *European Economic Review*, 44(4-6), pp.974-983.

Drouhin, N., 2001. "Inégalités face à la mort et systèmes de retraite". *Revue d'Economie Politique*, Hors-Serie: Epargne et Retraite.

Galasso, V., Profeta, P., 2004. "Lessons for an ageing society: the political sustainability of social security systems". *Economic Policy*, 19(38), pp.63-115.

Hairault, J.O. and Langot, F., 2008. "Inequality and Social Security Reforms". *Journal of Economic Dynamics and Control*, 32 (2), 386-410.

Lambert, P.J., 2001. "The Distribution and Redistribution of Income". *Manchester University Press.*

Liebman, J.B., 2001. "Redistribution in the Current U.S. Social Security System". *NBER working paper 8625.*

Mesrine, A., 1999. "Les différences de mortalité par milieu social restent fortes". *Données Sociales*, INSEE, pp.228-235.

Mitchell, O.S., Zeldes, S.P., 1996. "Social Security Privatization: A Structure for Analysis". *American Economic Review*, 88 (2), pp.363-367.

Monteil, C., Robert-Bobée, I., 2005. "Les différences sociales de mortalité : en augmentation chez les hommes, stables chez les femmes". *INSEE première*, No.1025.

Piketty, T., 2002. "L'économie des Inégalités". *Repères: La découverte.*

Robert-Bobée, I., Cadot, O., 2007. "Mortalité aux Grands Ages : encore des écarts selon le diplôme et la catégorie sociale". *INSEE première*, No.1122.