

FUZZY EVALUATION OF RESIDUALS IN FDI METHODS

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Abstract: This paper presents a Fault Detection and Isolation (FDI) method based on a fuzzy evaluation of residuals. The advantages of this non-Boolean evaluation are described. The ability of this approach to integrate a persistence index of the residual deviations and the sensitivity of the residuals with regard to faults is particularly studied. Some results issued from the implementation of the proposed approach on an urban water supply network illustrate its pertinence. *Copyright © 2000 IFAC*

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1. PROBLEM POSITION

Chow and Willsky (1984) have described the organization of a model-based Fault Detection and Isolation (FDI) procedure. The first stage of the proposed procedure is the generation of residuals that are revealing of the concordance between the model corresponding to the normal functioning state and the measurements. The second stage concerns the residual evaluation in order to detect and locate the eventual faults occurring on the system. The residuals are subjected to statistical testing, to eliminate, or at least alleviate, the noise effects. The test thresholds are determined on the basis of the nominal noise variances. Each individual test yields a Boolean decision, the full set of parallel tests resulting in a Boolean vector that is called the failure signature. This experimental signature is then compared with the columns of the theoretical signature matrix that indicates the occurrence of the variables (or faults) in the different equations (or residuals).

In practice, a fault of intermediate size may cause some of the tests fire while others not. So the resulting signature is a degraded version of the theoretical ones. If there is a failure that corresponds to this degraded signature, this partial firing leads to misisolation of the fault. Gertler and co-workers (Gertler, 1991, Gertler and Anderson, 1992) have extensively studied this problem and have proposed some solutions to enhance the isolation of faults. Despite these enhancements this type of approaches is still difficult to implement because the obtained decision is "absolute"; no means are given to appreciate the quality of the result of the decision process.

Alternate approaches that recognize the role of uncertainty are required. In the proposed method, incremental changes in the plant state results in incremental changes in degrees of belief of fault hypotheses. This eliminates the noise effects that become dominant when residual values are near threshold values. Also, a ranking of probable faults is obtained (Kramer, 1987).

The proposed paper will be organized as follows. In a second section, a short introduction recalls the basic principles of model-based fault detection and isolation methods. In particular, the isolation of faults based on a comparison of an experimental fault signature to theoretical signatures is explained. The drawbacks of this type of methods are pointed out. The third section concerns the fuzzy based fault isolation. A new conjunction operator for evaluating the truth degree of the rules is presented. Its structure is justified with regard to the problem of fault detection and isolation.

One of the drawbacks of the classical methods is the instability of the decision with regard to time when the residual values are near the thresholds. That leads to an unceasing switch between the normal (no fault) and fault situations. In order to remedy to this situation a persistence index is introduced in the calculus of the membership grade of the residuals to the different modalities. It will be presented in the fourth part of the proposed paper. Moreover, it will be shown that the flexibility of the method also allows the sensitivity of the residuals to be taken into account in the fuzzy evaluation. Some results issued from the implementation of the proposed method on an urban water supply network will be provided.

2. FDI BASED ON A BOOLEAN SIGNATURE ANALYSIS

As previously mentioned, model-based FDI encompasses two main steps: generation of residuals and decision. This latter itself may be split up in a stage of fault detection and a stage concerning fault isolation (or localization).

Let us consider the following set of four residuals r_i that link three measurements x_j :

$$\begin{cases} r_1 = f_1(x_1, x_2) \\ r_2 = f_2(x_1, x_3) \\ r_3 = f_3(x_1, x_2, x_3) \\ r_4 = f_4(x_2, x_3) \end{cases} \quad (1)$$

The theoretical matrix of fault signature, Σ , is defined by coding with binary variables, the occurrence of variables in the different residuals. For the considered example, we have:

$$\Sigma = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \quad (2)$$

This matrix expresses the theoretical influence of the faults onto the residuals. Indeed, to the j th column (signature) of Σ that will be denoted $\Sigma_{\cdot j}$, may be associated the fault δ_j . Then, the corresponding signature indicates how the fault affects the different residuals. A signature corresponding to a normal functioning may complete this matrix; this one is only constituted of null elements.

Statistical methods of abrupt change detection permit, at each sample time k , to code in a binary form also, a set of experimental residuals $r_i(k)$, $i = 1, \dots, N$:

$s_i(k) = 0$, if the taking into account of the value $r_i(k)$ in the analysis allows an abrupt change in r_i to be detected

$s_i(k) = 1$, if this value does not allow an abrupt change in the residual r_i to be detected.

By this way, one builds the experimental signature $S(k) = (s_1(k) \ \dots \ s_N(k))^T$. The fault localization is then obtained by searching, in the theoretical signature matrix, that corresponding to the observed experimental signature.

In practice, the experimental signatures are often degraded. The most frequent situation is the transformation of a "1" into a "0" corresponding to a non-detection. This situation has been extensively studied by Gertler (Gertler, 1988), (Gertler and Singer, 1990), (Gertler and Anderson, 1992). Indeed, according to the residual sensitivity to certain faults (this sensitivity does not intervene in the proposed coding), certain small magnitude faults cannot be detected by the proposed statistical tests. The

localization must therefore be done with the help of a distance calculus between the experimental and theoretical signatures. The most likelihood fault is that corresponding to the theoretical signature closer to the experimental one. The most frequently employed signatures are the Euclidian one, defined by:

$$d_j(k) = \left(\sum_{i=1}^N (\Sigma_{ij} - s_i(k))^2 \right)^{1/2} \quad (3)$$

$$d_j(k) = \left((\Sigma_{\cdot j} - S(k))^T (\Sigma_{\cdot j} - S(k)) \right)^{1/2}$$

and the Hamming distance:

$$d_j(k) = \sum_{i=1}^N (\Sigma_{ij} \oplus s_i(k)) \quad (4)$$

where the symbol \oplus denotes the logic exclusive or operator.

The performance of this recognition step depends on the dissimilarity of the theoretical signatures. In the vector space $\{0, 1\}^N$, the points representing the binary vectors of signatures must be as distant as possible. The goal of the structuration of residuals is precisely to increase the distances between the different theoretical signatures. Indeed, facing with k decision errors when analyzing the experimental residuals, it needs a Hamming distance equal, at least, to $2k + 1$ for guaranteeing a correct localization of a fault.

More information may also be kept by using a ternary coding of the residuals that take into account the sign of the detected abrupt change. In this case, we have:

$s_i(k) = 1$, if the detected abrupt change in the residual r_i is positive,

$s_i(k) = -1$, if the detected abrupt change in the residual r_i is negative

A similar analysis to those previously presented allows the fault to be detected.

3. NON-BOOLEAN EVALUATION OF RESIDUALS

The method presented in this paragraph uses a non-Boolean inference technique and is based on elementary notions related to fuzzy set theory. A fuzzy set is characterized by a membership function expressing the progressive character of the transition between "belonging" and "not belonging". It is then defined by:

$$A = \{ \{x, \mu_A(x)\} \mid x \in X \} \quad (5)$$

where $\mu_A(x)$ represents the membership function of the x element to the fuzzy set A defined on the definition support X and taking values in the interval $[0, 1]$.

Fuzzy sets can represent "linguistic variables" which express a qualitative or imprecise knowledge such that "the temperature is low" or "the magnitude of the residual is great".

For the residual analysis, three fuzzy sets or modalities have been defined. The modality r_i^+ (respectively r_i^- and r_i^0) is related to a "positive" residual (respectively "negative" and "null") in a linguistic sense. For a residual $r_i(k)$, the membership functions to these modalities are defined on the following manner:

$$\mu_{r_i^0}(k) = 1 - \frac{(r_i(k)/\tau_i)^\rho}{1 + (r_i(k)/\tau_i)^\rho} \quad (6a)$$

$$\mu_{r_i^+}(k) = \begin{cases} 0 & \text{si } r_i(k) \leq 0 \\ 1 - \mu_{r_i^0}(k) & \text{si } r_i(k) > 0 \end{cases} \quad (6b)$$

$$\mu_{r_i^-}(k) = \begin{cases} 1 - \mu_{r_i^0}(k) & \text{si } r_i(k) \leq 0 \\ 0 & \text{si } r_i(k) > 0 \end{cases} \quad (6c)$$

where ρ even real and τ_i are shape parameters chosen by the user. Notice that $(\mu_{r_i^0}(k))$ is a simplified notation that stands for $\mu_{r_i^0}(r_i(k))$.

The figure 1 shows the shape of these membership functions.

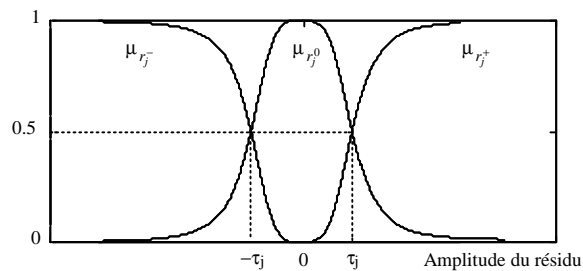


Figure 1. Aspect of the membership functions

The fault localization is based on a conjoined analysis of the residuals and the theoretical signature matrix. Let us consider the following signature matrix:

$$\Sigma = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}$$

related to three residuals and two potential faults (the third column corresponds to the absence of fault)

This matrix express, for example, the fact that a positive fault f_1^+ (first column of the matrix) induces a positive deviation of the first residual, a negative one of the second and does not influence the third residual.

Fault isolation corresponds to solve the reverse problem, i.e. deduce, from the different residuals

values, the variable that is affected by the fault(s). For that purpose, one constitutes, from the signature matrix, a rule base (fuzzy inference rules) linking symptoms (residual deviations) to causes (occurrence of fault(s)). Each rule is made up of a premise part related to the different residuals and a consequence part related to a fault. For example, from the previous signature matrix and for the fault f_1^+ , one can write:

$$\text{if } r_1(k) \text{ is } r_1^+ \text{ and } r_2(k) \text{ is } r_2^- \text{ and } r_3(k) \text{ is } r_3^0 \text{ then } f_1^+ \quad (7)$$

The quantifying of the different propositions is done by the means of membership grades of the residuals to the different modalities. For example, the premise proposition " $r_1(k)$ is r_1^+ " is quantified by the value $\mu_{r_1^+}$ of the membership grade of $r_1(k)$ to r_1^+ .

At each instant and for each rule of the base, the truth degree of the consequences (or the truth degree of the rules) must be evaluated with regard the validity of the premise. The "and" operator in the rule (7) is an aggregation or conjunction operator that describes the combination of the membership grades of the premise used for the elaboration of the truth degree of the corresponding rule. Various conjunction operators can be used as for example the "product", the "minimum" or the "mean" (Bouchon-Meunier, 1995). Considering the rule (7) related to fault f_1^+ , and using these operators, the truth degrees are given by the following expressions:

$$\mu_{f_1^+}(k) = \mu_{r_1^+}(k) \mu_{r_2^-}(k) \mu_{r_3^0}(k) \quad (8a)$$

$$\mu_{f_1^+}(k) = \min(\mu_{r_1^+}(k), \mu_{r_2^-}(k), \mu_{r_3^0}(k)) \quad (8b)$$

$$\mu_{f_1^+}(k) = \frac{1}{3} (\mu_{r_1^+}(k) + \mu_{r_2^-}(k) + \mu_{r_3^0}(k)) \quad (8c)$$

Nevertheless, these operators are not well adapted for the problem of fault isolation. Indeed, in the case where, among the set of residuals which intervene in the premises of a rule, one of them is weakly affected by a fault (due to a weak sensitivity of the residual or a small magnitude fault), the use of the minimum or product operators leads to a very small truth degree of this rule. The mean operator is less sensitive to this phenomenon; however, if the number of non-sensitive residuals to a particular fault is too much important (with regard to the number of residuals that are sensitive to the fault), it may produce relatively high value of truth degree of the rule even when the considered fault has not occur.

In order to alleviate these problems, Denis Mandel (Mandel, 1998) proposed to take into account separately, on one hand the modalities r_i^0 and, on the other hand, r_i^+ and r_i^- , by combining minimum and mean operators. The resulting aggregation operator is then defined as the minimum of the mean of the membership grades related to r_i^0 modalities and the mean of the membership grades related to r_i^+ and r_i^- . In the particular case of the rule (7), its expression is given by:

$$\mu_{f_1}(k) = \min \left(\mu_{r_3^0}(k), \frac{\mu_{r_1^+}(k) + \mu_{r_2^-}(k)}{2} \right) \quad (8d)$$

The isolation of the fault is based on the analysis of the truth degree of the rules. Indeed, the rule which truth degree is the highest designates the most likelihood fault.

4. ENHANCEMENT OF THE METHOD

Two enhancements of the described method are proposed. The first one concerns the taking into account of the persistence of a symptom. Indeed, in the previous approach, only the magnitude of the residuals is analyzed through the different membership grades. If the magnitude of the fault is continuously small, the associated truth degree of the corresponding rule remains small. By combining the magnitude of the residuals and the persistence of their deviations, it is possible to obtain a high truth degree of the rule associated to a certain fault even if this fault is very small but persistent.

For taking into account this notion of persistence, the membership grades of the residuals to the different modalities are modified following this scheme.

Let us considered an observation window comprising L samples of a residual r_i ($r_i(l)$, $l = k - L + 1, \dots, k$). Let S be a chosen threshold and $N(\mu)$ the number of values of the membership grade μ which go beyond this threshold S during this window. On this basis, three persistence indexes p_i^+ , p_i^- and p_i^0 corresponding to the persistence of the membership grades are evaluated. These indexes are equal to the ratio between the number $N(\mu)$ and the number of the window samples L . For example, for the modality r_i^- , the persistence index is given by:

$$p_i^-(k) = \frac{N(\mu_{r_i^-})}{L} \quad (9)$$

Therefore, new membership grades $\tilde{\mu}$ taking into account both the magnitude of the residuals and the persistence of their deviations are defined. Always for r_i^- modality, we have:

$$\tilde{\mu}_{r_i^-}(k) = \frac{p_i^-(k) + \mu_{r_i^-}(k)}{2} \quad (10)$$

The choice of this ordinary mean is arbitrary and a more sophisticated weighted mean can be employed according the relative importance of magnitude and persistence of residuals.

These new membership grades $\tilde{\mu}$ are then used, in replacement of the old ones (6), in the proposed approach.

The second enhancement that can be proposed concerns the integration of the sensitivity of the residuals with regard to faults. The presentation only deals with the situation where residuals are linear functions of measurements although the method can be extended, by using linearization techniques, to the nonlinear case.

In the linear case, the set of residuals may be expressed as $R = MX$, where X is the vector of measurements, R the vector of residuals and M the matrix of redundancy equations. In this situation, it is easy to evaluate the sensitivity of a particular residual to a given fault. Let us denote d_{ij} the magnitude of the fault on the j th variable that implies an overshoot of a given threshold of the i th residual (for example, this upper bound may be chosen equal to τ_i corresponding to a membership grade equal to 0.5). It is clear that the sensitivity of the i th residual to the j th fault evolves inversely with the magnitude d_{ij} . This remark helps to define weighting factors on the basis of these sensitivities as:

$$\delta_{ij} = \frac{1}{d_{ij}} \quad (11)$$

that corresponds to a kind of "normalized sensitivities". These coefficients δ_{ij} are then used as weighting factors of the membership grades $\mu_{r_i^+}$ and $\mu_{r_i^-}$ used in the j th rule. For example, expression (8d) related to rule (7) is transformed as follows:

$$\mu_{f_1^+}(k) = \min \left(\mu_{r_3^0}(k), \frac{\delta_{11}\mu_{r_1^+}(k) + \delta_{21}\mu_{r_2^-}(k)}{\delta_{11} + \delta_{21}} \right) \quad (12)$$

Of course the two proposed enhancements of the method may be implemented simultaneously.

5. APPLICATION TO A URBAN WATER SUPPLY NETWORK

The proposed approach has been applied on an urban water supply network. This process is depicted schematically in figure 2. The part of the water network considered in this study is the storage network. The function of this network consists, with help of pumping stations, in sharing out water reserves among the whole agglomeration, before its distribution to the consumers. More precisely, the storage network is composed of pumping stations that make up an arborescent network of about thirty tanks. On account of its importance for the water distribution reliability, the managers pay a great attention to the supervision of the storage network. Consequently, this part of the network has been supplied with numerous measuring devices. Indeed, to each tank corresponds a water level measurement and almost all the flowrate variables, roughly a hundred, are measured.

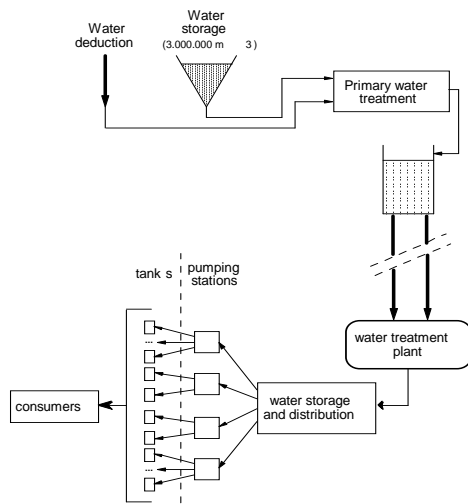


Figure 2. Water supply network

Due to lack of space, the results obtained by the suggested approach only concern one of the pumping stations. This latter is made up of four interconnected tanks. The measurements collected are the water level (h_i) of the tanks, different flowrates (q_i), binary variables indicating the state (on/off) of the pumps (w_i) and some pressure variables (p_i). Figure 3 shows some experimental signals (from top to bottom, a water level, a flowrate, an outflow of a pump, a binary state variable of a pump and a pressure). The sampling period of these measurements is equal to 1 minute.

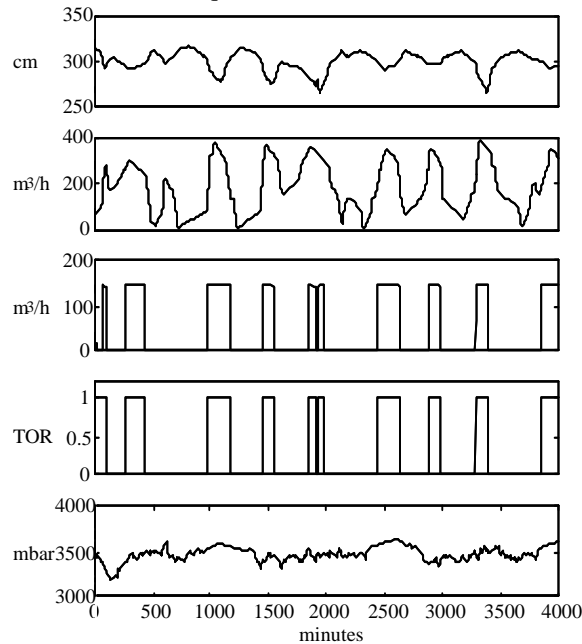


Figure 3. Experimental signals

Four types of redundancy equations has been established. The first one concerns dynamic material balance expressed for each tank while the second links nominal outflows of the pumps, binary state variables and measured outflows of the considered pumps. These two types of equations are considered as "exact". The two other redundancy equations, which are established by using a correlation analysis,

link on one-hand, flowrate and pressure measurements and, on the other hand, flowrates between themselves. Indeed, it is clear that water consumptions of some residential areas are very similar and present same cyclic shape. These last two types of redundancy equations are more "incertain" as their describing parameters are estimated on the basis of sets of measurements.

The detection and the localization of two different faults are now presented. The residuals presented figure 4 has been calculated from experimental data on a time period of about 10 000 minutes. During this time, residuals r_{13} and r_{32} are clearly perturbed by a fault. On the contrary, residuals r_{36} and r_{38} remain statistically null during this period. Expressions (13) give the structure of the redundancy equations that have been used for generating these residuals.

$$r_{13} = \mathcal{R}_1(-q_{39}, -q_{40}, -q_{41}, -q_{42}, +q_{44}) \quad (13a)$$

$$r_{32} = \mathcal{R}_2(+q_{39}, -q_{40}) \quad (13b)$$

$$r_{36} = \mathcal{R}_3(+q_{40}, -q_{41}) \quad (13c)$$

$$r_{38} = \mathcal{R}_4(+q_{41}, -q_{42}) \quad (13d)$$

One observes that q_{39} and q_{40} variables intervene both in the calculus of r_{13} and r_{32} residuals. However, the measurement of the flowrate q_{40} cannot be suspected to be faulty because it intervenes with a minus sign in the calculus of these two residuals whereas the signs of the two residual deviations are opposite. Moreover, the flowrate q_{40} also intervene in the calculus of the residual r_{36} that is not affected by this fault. Consequently, the measurement q_{40} is strongly suspected.

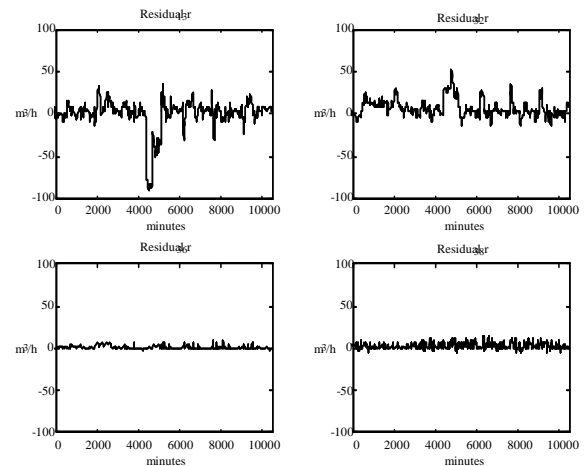


Figure 4. Experimental residuals

The fault hypothesis corresponding to negative faults (respectively positive) that affect q_{39} , q_{40} and q_{41} are denoted f_{39}^- (resp. f_{39}^+), f_{40}^- (resp. f_{40}^+) et f_{41}^- (resp. f_{41}^+). The truth degrees of these hypotheses (rules) are shown figure 5. The more likelihood hypothesis corresponds to a positive fault on q_{39} .

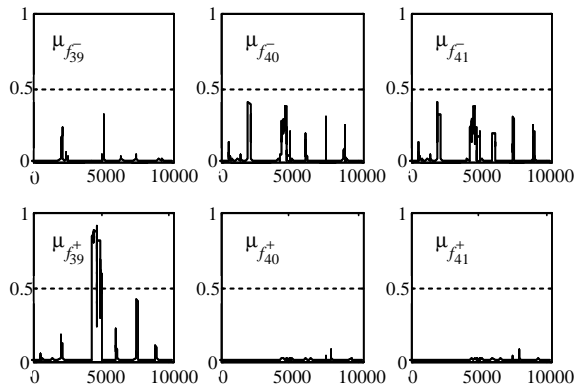


Figure 5. Truth degrees of fault hypotheses

The analysis of the evolution of q_{39} flowrate (figure 6) confirms the occurrence of a fault between samples 4 424 to 5 172. Although the proposed method has been mainly designed for the detection of sensor faults, a more precise analysis of this particular situation has shown that this fault corresponded to a leak (in fact a process fault) of about $30 \text{ m}^3/\text{h}$, during about 12 hours.

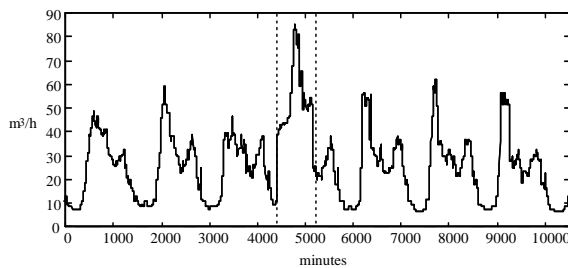


Figure 6. Temporal evolution of q_{39}

The second fault situation points out the importance of the integration of residual sensitivity to the different faults. Figure 7 shows the temporal evolution of the measurement of a pump outflow (q_{36}). From sample 5670, this measurement is affected by a bias. During a first period, this bias is of low magnitude, then, in a second period, this magnitude becomes very high. As in the second period, the detection of this fault is easy; it is not the case during the first one.

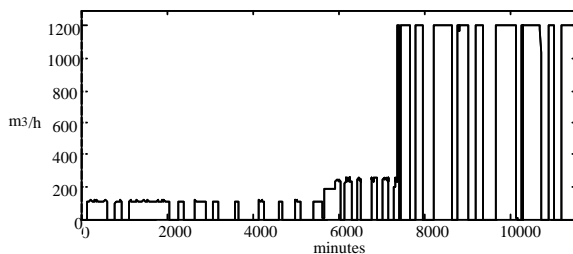


Figure 7. Temporal evolution of q_{36}

Remark that this bias only appears when the pump is functioning as measurement of the flowrate is automatically set up to zero by the programmable logic controller when the pump is stopped. Figure 8 presents the truth degrees of the hypothesis f_{36}^+

corresponding to the occurrence of a positive fault on the flowrate q_{36} with and without taking into account the sensitivity of the residuals to this fault as described in section 4.

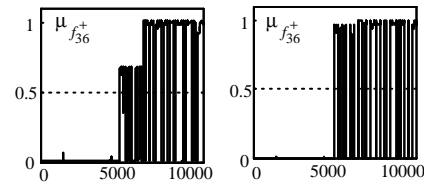


Figure 8. Truth degrees of fault hypotheses

In the first case, the truth degree of hypothesis f_{36}^+ stays around the value 0.5 during the first period due to the weak sensitivity of certain residuals to this fault. Taking into account these different sensitivities allows the contrast between the normal and fault situation to be enhanced as shown by the second figure where the truth degree of hypothesis f_{36}^+ is close to 1 as early as the fault occurs.

6. CONCLUSION

A Fault Detection and Isolation method based on fuzzy evaluation of residuals has been elaborated and implemented for the supervision of a water distribution network. If the fuzzy evaluation of the residuals has already been presented elsewhere, the main contribution of the study concerns the definition of a new conjunction operator well adapted to fault isolation and the integration of additional information such the persistence of the faults or the fault sensitivity of the residuals. For this application concerning a water distribution network, the fuzzy approach has demonstrated its superiority with regard to most classical approaches. Indeed, all the available data has been taken into account in order to produce the most accurate diagnosis of the process.

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