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STATIC OUTPUT FEEDBACK FOR TAKAGI-SUGENO SYSTEMS: AN LMI APPROACH

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Abstract

This paper studies the design of a static output feedback controller for nonlinear systems described by continuous-time Takagi-Sugeno (T-S) models. Motivated by stability results developed for parallel distributed compensation (PDC) controller, an Output PDC (OPDC) controller that corresponds to a nonlinear static output feedback control law is proposed. Both stabilisation and poles placement are addressed. An example is given to illustrate the approch.

1 Introduction

The issue of stability and the synthesis of controllers for nonlinear systems described by continuous-time Takagi-Sugeno models [14] has been considered actively. There has been also an increasing interest in the multiple model approach [17,13] which also uses the T-S systems to modelling.

During the last years, many works have been carried out to investigate the stability analysis and the design of state feedback controller of T-S systems. Using a quadratic Lyapunov function and PDC technique [6,8] sufficient conditions for the stability and stabilisability have been established [5,6,10,13,15,16]. The stability depends on the existence of a common positive definite matrix guarantying the stability of all local subsystems. The PDC control is a nonlinear state feedback controller. The gain of this controller can be expressed as the solution of a linear matrix inequality (LMIs) set [18]. Recently a number of control law have been derived from the PDC controller [4, 7, 19]. For example, a Dynamic PDC (DPDC) [7], which is a dynamic nonlinear control law, and a Proportional PDC (PPDC) controller [4], which allows to reduce the number of parameters in PDC technique, are used to stabilise T-S models. Also in [1] a dynamic output feedback controller is proposed for continuous T-S systems while in [11] a static output feedback control for switching discrete-time systems is studied. LMIs constraints have been also used for pole assignment in LMI regions [12] to achieve desired performances [2,9].

In this paper, the LMI approach is used to develop a static output feedback controller for nonlinear systems described by continuous-time T-S models. We propose an Output PDC (OPDC) controller which is useful when only the output of the system is available. Using the quadratic Lyapunov technique, sufficient conditions for the global asymptotic stability are derived in LMIs form for OPDC controller. An example is given to illustrate the result.

Notation: In this paper, we denote the symmetric definite positive matrix X by $X > 0$, the transpose of X by X^T and the Kronecker product by $\otimes$.

2 Continuous T-S model

A T-S model is based on the interpolation between several LTI local models as follows:

$$\dot{x}(t)=\sum_{i=1}^{n}\mu_i(z(t))(A_i x(t)+B_i u(t)) \tag{1}$$

where *n* is the number of submodels, $x(t)\in\mathbb{R}^p$ is the state vector, $u(t)\in\mathbb{R}^m$ is the input vector, $A_i\in\mathbb{R}^{pxp}, B_i\in\mathbb{R}^{pxm}$ and $z(t)\in\mathbb{R}^q$ is the decision variable vector.

The choice of the variable $z(t)$ leads to different class of systems. It can depend on the measurable state variables and possibly on the input; in this case, the system (1) describes a nonlinear system. It can also be an unknown constant value, system (1) then represents a linear differential inclusion (LDI). This variable can also be a function of the measurable outputs of the system, this case will be considered in the section 4.

The normalised activation function $\mu_i(z(t))$ in relation with the i^{th} submodel is such that:

$$\begin{cases}\sum_{i=1}^{n}\mu_i(z(t))=1\\ \mu_i(z(t))\geq 0 \quad \forall\, i\in\{1,..,n\}\end{cases} \tag{2}$$

The global output of T-S model is interpolated as follows:

$$y(t)=\sum_{i=1}^{n}\mu_i(z(t))C_i x(t) \tag{3}$$

where $y(t)\in\mathbb{R}^l$ is the output vector and $C_i\in\mathbb{R}^{lxp}$. More detail about this type of representation can be found in [14].

It should be point out that at a specific time, only a number s of local models are activated, depending on the structure of the activation functions $\mu_i(.)$.

3 Previous results

The PDC controller [6,8], which is nonlinear in general, is described by:

$$u(t)=\sum_{i=1}^{n}\mu_i(z(t))K_i x(t) \tag{4}$$

Substituting (4) in (1), we obtain the closed-loop continuous T-S model:

$$\dot{x}(t)=\sum_{i=1}^{n}\sum_{j=1}^{n}\mu_i(z(t))\mu_j(z(t))\tilde{A}_{ij}x(t) \tag{5}$$

where

$$\tilde{A}_{ij}=A_i+B_iK_j \tag{6}$$

For PDC controller design, it is supposed that the system (1) is locally controllable, i.e. the pairs $(A_i, B_i), \forall\, i\in\{1,..,n\}$ are controllable.

The stability conditions for system (5) are formulated by theorem 1 and for less of conservatism by theorem 2. In order to simplify the notation of the forthcoming equations, lets us denote:

$$L(\tilde{A}_{ij},P)=\left(\frac{\tilde{A}_{ij}+\tilde{A}_{ji}}{2}\right)^T P+P\left(\frac{\tilde{A}_{ij}+\tilde{A}_{ji}}{2}\right) \tag{7}$$

Theorem 1 [8]: The closed-loop continuous T-S model described by (5) is globally asymptotically stable if there exist a common symmetric positive definite P and semi-definite positive matrix R such that

$$L(\tilde{A}_{ii},P)+(s-1)R<0 \quad \forall\, i\in\{1,..,n\} \tag{8a}$$

$$L(\tilde{A}_{ij},P)-R\leq 0 \quad \forall\, i<j\in\{1,..,n\} \tag{8b}$$

and $\mu_i(z(t))\mu_j(z(t))\neq 0$ where s is the number of submodels simultaneously activated.

Proof: see [8] ■

Theorem 2 [5]: The closed-loop continuous T-S model described by (5) is globally exponentially stable if there exist symmetric positive definite matrix P and symmetric matrices R_{ij} such that:

$$L\left(\tilde{A}_{ii}, P\right) + R_{ii} < 0 \quad \forall\, i \in \{1,..,n\} \tag{9a}$$

$$L\left(\tilde{A}_{ij}, P\right) + R_{ij} \le 0 \quad \forall\, i < j \in \{1,..,n\} \tag{9b}$$

$$\tilde{R} = \begin{pmatrix} R_{11} & \cdots & R_{1n} \\ \vdots & \ddots & \vdots \\ R_{1n} & \cdots & R_{nn} \end{pmatrix} > 0 \tag{9c}$$

and $\mu_i(z(t))\mu_j(z(t)) \neq 0$.

Proof: see [5] ■

In the PDC technique, which is a state feedback law, the conditions (8) and (9) are easy to convert into an LMI problem [8,5].

4 Static output stabilisation

In the sequel, we assume that $C_i = C, \forall\, i \in \{1,..,n\}$ is full row rank and $z(t)$ is a function of the measurable outputs of the system, $z(t) = h(y(t))$.

The OPDC is a nonlinear static output feedback which shares the same activation functions as the T-S model (1):

$$u(t) = \sum_{i=1}^{n} \mu_i(z(t)) F_i y(t) \tag{10}$$

where $F_i \in \mathbb{R}^{m.l}$ is the local output feedback controller to determine. Taking into account the expression (10), the T-S system (1) becomes:

$$\dot{x}(t) = \sum_{i=1}^{n}\sum_{j=1}^{n} \mu_i(z(t))\mu_j(z(t))\bar{A}_{ij} x(t) \tag{11}$$

where

$$\bar{A}_{ij} = A_i + B_i F_j C,\ \forall\, i, j \in \{1,..,n\} \tag{12}$$

The synthesis of OPDC controller for T-S model can be done using the results of theorems 1, 2 by simply replacing $\tilde{A}_{ij}$ by $\bar{A}_{ij}$. We obtain respectively:

$$L\left(\bar{A}_{ii}, P\right) + (s-1)R < 0, \quad \forall\, i \in \{1,..,n\} \tag{13a}$$

$$L\left(\bar{A}_{ij}, P\right) - R \le 0 \quad \forall\, i < j \in \{1,..,n\} \tag{13b}$$

and

$$L\left(\bar{A}_{ii}, P\right) + R_{ii} < 0 \quad \forall\, i \in \{1,..,n\} \tag{14a}$$

$$L\left(\bar{A}_{ij}, P\right) + R_{ij} \le 0 \quad \forall\, i < j \in \{1,..,n\} \tag{14b}$$

$$\tilde{R} > 0 \tag{14c}$$

The obtained equations are also bilinear in P and $F_i, \forall\, i \in \{1,..,n\}$ but, in contrast to conditions (8) and (9), it is not easy to convert them into LMI problem. So, the solution is not guaranteed to belong to a convex domain and the classical tools for solving sets of matrix inequalities cannot be used. It constitutes the major difficulty of output feedback design.

In the following, sufficient conditions in LMIs form are given to ensure asymptotic stability of (11).

4.1 Stabilisation using OPDC

Theorem 3. Suppose that there exist matrices N_i, M, S and Q such that

$$Q > 0, S > 0$$

$$QA_i^T + A_i Q + C^T N_i^T B_i^T + B_i N_i C + (s-1)S < 0, \forall i \in \{1,\dots,n\} \tag{15a}$$

$$Q\left(A_i + A_j\right)^T + \left(A_i + A_j\right)Q + C^T\left(N_j^T B_i^T + N_i^T B_j^T\right) + \left(B_i N_j + B_j N_i\right)C - 2S \le 0 \quad \forall i < j \in \{1,\dots,n\} \tag{15b}$$

and

$$CQ = MC \tag{16}$$

with $\mu_i(z(t))\mu_j(z(t)) \neq 0$. Then the T-S model (11) is globally asymptotically stable with the OPDC controller $u(t) = \sum_{i=1}^{n} \mu_i(z(t)) F_i y(t)$ where

$$F_i = N_i M^{-1} \qquad \forall\, i \in \{1,..,n\} \tag{17}$$

Proof : The inequality (13a) is equivalent to

$$Q(A_i + B_i F_i C)^T + (A_i + B_i F_i C)Q + (s-1)S < 0 \quad (18)$$

where $P^{-1} = Q > 0$ and $S = P^{-1}RP$

From (16) and with the change of variables

$$F_i M = N_i \quad (19)$$

the inequality (18) becomes

$$QA_i^T + A_i Q + (B_i N_i C)^T + B_i N_i C + (s-1)S < 0 \quad (20)$$

The LMIs (15b) are obtained from (13b) by using the same changes of variables (19). Since the matrix C is assumed full row rank, we deduce from (19) that there exist a non-singular matrix $M = CQC^T\left(CC^T\right)^{-1}$ and then $F_i = N_i M^{-1}$ ■

Theorem 4. Suppose that there exist matrices N_i, M, S_{ij} and Q such that

$$Q > 0$$

$$QA_i^T + A_i Q + C^T N_i^T B_i^T + B_i N_i C + S_{ii} < 0,\ \forall i \in \{1,\ldots,n\} \quad (21a)$$

$$Q\left(A_i + A_j\right)^T + \left(A_i + A_j\right)Q + C^T\left(N_j^T B_i^T + N_i^T B_j^T\right) + \left(B_i N_j + B_j N_i\right)C + 2S_{ij} \le 0 \quad \forall i < j \in \{1,\ldots,n\} \quad (21b)$$

$$S = \begin{pmatrix} S_{11} & \cdots & S_{1n} \\ \vdots & \ddots & \vdots \\ S_{1n} & \cdots & S_{nn} \end{pmatrix} > 0 \quad (21c)$$

and

$$CQ = MC \quad (22)$$

with $\mu_i(z(t))\mu_j(z(t)) \neq 0$. Then the T-S model (11) is globally asymptotically stable with the OPDC controller $u(t) = \sum_{i=1}^{n} \mu_i(z(t)) F_i y(t)$ where

$$F_i = N_i M^{-1} \qquad \forall\, i \in \{1,..,n\} \quad (23)$$

Proof : It can be easily established by applying the same change of variables (19) to conditions (14).■

Remarks:

1. Since C is assumed full row rank, to respect constraint (22) it suffices to impose a particular structure to matrix Q depending on matrix C.
2. In the case where $F_i = F, \forall\, i \in \{1,..,n\}$ the synthesis of linear static output feedback $u(t) = Fy(t)$ can be reduced to find matrices N, M and Q such that:

$$\begin{pmatrix} QA_i^T + A_i Q + C^T N^T B_i^T + B_i NC & 0 \\ 0 & -Q \end{pmatrix} < 0, \forall i \in \{1,\ldots,n\} \quad (24a)$$

$$CQ = MC \quad (24b)$$

where

$$F = NM^{-1} \quad (25)$$

4.2 Stabilisation using modified OPDC

In particular case when the input matrices are linearly dependent (i.e. $\exists\, B \in \mathbb{R}^{p.m}$ and $\alpha_i > 0,\ i \in \{1,..,n\}$ such that $B_i = \alpha_i B$), an interesting control law inspired from the CDF (Compensation and Division for Fuzzy models) controller [3] should be considered instead of the OPDC controller (10). The modified OPDC proposed is of the form:

$$u(t) = \frac{\sum_{i=1}^{n} \mu_i(z(t)) \alpha_i F_i}{\sum_{i=1}^{n} \mu_i(z(t)) \alpha_i} y(t) \quad (26)$$

Consequently by substituting (26) into (1), the closed-loop T-S system (1) becomes

$$\dot{x}(t)=\sum_{i=1}^{n}\mu_i(z(t))\bar{A}_{ii}x(t) \quad (27)$$

where $\bar{A}_{ii}, i\in\{1,..,n\}$ is defined in (12). Notice that in this case, the T-S system (27) is written without the coupled terms (i.e. $i\neq j$). Indeed the T-S system (27) is globally asymptotically stable if there exist matrices $P>0$ and $F_i, i\in\{1,..,n\}$ such that

$$L(\bar{A}_{ii}, P)<0, \quad \forall\, i\in\{1,..,n\} \quad (28)$$

The following theorem gives sufficient conditions in LMIs form to ensure asymptotic stability of (27).

Theorem 5. Suppose that there exist matrices N_i, M and Q such that

$$Q>0$$

$$QA_i^T+A_iQ+C^TN_i^TB_i^T+B_iN_iC<0,\ \forall i\in\{1,\dots,n\} \quad (29)$$

and

$$CQ=MC \quad (30)$$

then the T-S model (27) is globally asymptotically stable with the modified OPDC controller (26) where

$$F_i=N_iM^{-1} \qquad \forall\, i\in\{1,..,n\} \quad (31)$$

Proof : It can be easily established by applying the same change of variables (19) to conditions (28).■

Notice that in this particular case the modified OPDC controller (26) leads to n constraints instead of $n\frac{(n+1)}{2}$ with the OPDC controller (10).

4.3 LMI formulation for pole-placement

In order to achieve some desired transient performance, a pole placement should be considered. For many problems, exact pole assignment may not be necessary, it suffices to locate the pole of the closed loop system in a sub-region of the complex left half plane. This section discusses a pole assignment in LMI regions.

Definition 1 [12]. A subset D of the complex plane is called an LMI region if there exist a symmetric matrix $\alpha=(\alpha_{ij})\in\mathbb{R}^{p.p}$ and a matrix $\beta=(\beta_{ij})\in\mathbb{R}^{p.p}$ such that $D=\{z\in\mathbb{C}: f_D(z)<0\}$

where $f_D(z)=(\alpha_{ij}+\beta_{ij}z+\beta_{ji}\bar{z}), \quad \forall\, i,j\in\{1,..,p\}$.

Theorem 6 [12]. A matrix A is D-stable if and only if there exists a symmetric positive definite matrix X such that

$$M_D(A,X)<0$$

where $M_D(A,X)=\alpha\otimes X+\beta\otimes(AX)+\beta^T\otimes(AX)^T$ ■

For example, a disk region D_d centred at $(-q,0)$ with radius $r>0$ can be obtained by taking the matrices α and β as follows :

$$\alpha=\begin{pmatrix}-r & q\\ q & -r\end{pmatrix} \text{ and } \beta=\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}$$

What makes it possible to obtain the expression of this region:

$$f_{D_d}(z)=\begin{pmatrix}-r & z+q\\ \bar{z}+q & -r\end{pmatrix} \quad (32)$$

As it is shown in figure 1, this region which include conic region, allows to fix a lower bound on both the exponential decay rate: $-q+r$ and the damping ratio: $\xi_{\min}=\sqrt{1-r^2/q^2}$ $(r<q)$ of the closed-loop response.

Since the prescribed LMI region (32) will be added as supplementary constraints to these of the theorem 4 and theorem 5, it should be noted that it only suffices to locate the poles of the dominant term in the prescribe LMI regions, i.e. the case of $i=j$. It follows that the system (11) (or its particular form (27)) is D_d-stable if there exists a matrix $Q>0$ such that

$$\begin{pmatrix} -rQ & qQ+\bar{A}_{ii}Q \\ qQ+Q\bar{A}_{ii}^T & -rQ \end{pmatrix} < 0 \quad (33)$$

With the same change of variables (19) and constraint (30), the equation (33) leads to the following LMI formulation:

$$\begin{pmatrix} -rQ & qQ+A_iQ+B_iN_iC \\ qQ+QA_i^T+(B_iN_iC)^T & -rQ \end{pmatrix} < 0 \quad (34)$$

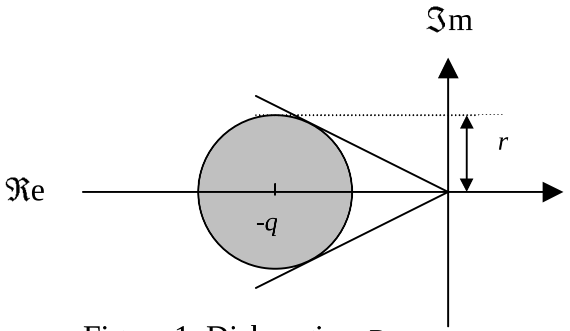

Figure 1. Disk region D_d

5 Numerical example

Consider the T-S model (1) where $s = n = 2$ and $z(t) = y(t)$:

$$\dot{x}(t) = \sum_{i=1}^{2} \mu_i(y(t))(A_i x(t) + B_i u(t))$$

$$y(t) = Cx(t) \quad (35a)$$

with

$$A_1 = \begin{pmatrix} 2 & -10 \\ 1 & 0 \end{pmatrix}, \quad B_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (35b)$$

$$A_2 = \begin{pmatrix} 1 & -10 \\ 1 & 0 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 10 \\ 0 \end{pmatrix} \quad (35c)$$

$$C = \begin{pmatrix} 1 & 0 \end{pmatrix} \quad (35d)$$

$$\mu_1(y(t)) = \frac{(1-\tanh(y(t)))}{2}, \quad \mu_2(y(t)) = \frac{(1+\tanh(y(t)))}{2} \quad (35e)$$

Since $B_2 = \alpha B_1, \alpha = 10$, the synthesis of an OPDC controller (26) for the continuous T-S (35) is reduced to the resolution of two LMIs $(i{:}1, 2)$ derived from (29) with the constraint (30). The following output feedback gains are obtained with a pole placement using conditions (34) such that $q = 10$ and $r = 9$:

$$F_1 = \text{-6.9638}, \; F_2 = \text{-0.6094} \quad (36)$$

We can verify that the eigenvalues of $A_i + B_i F_i C, \; i{:}1, 2$ are respectively $-2.5472 \pm 1.8740i$ and $-2.4819 \pm 1.9597i$.

The simulation of the closed loop T-S model (35) with the modified OPDC control law $u(t) = \frac{(\mu_1(y(t))F_1 + \mu_2(y(t))\alpha F_2)}{(\mu_1(y(t)) + \mu_2(y(t))\alpha)} y(t)$ is presented in figure 3.

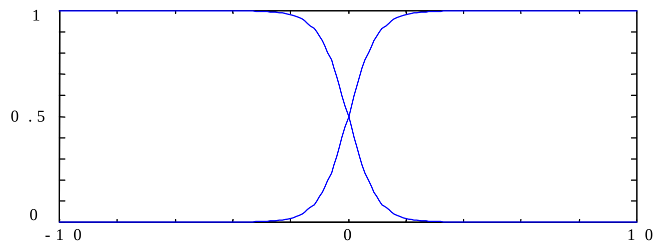

Figure 2. Activation functions (35e)

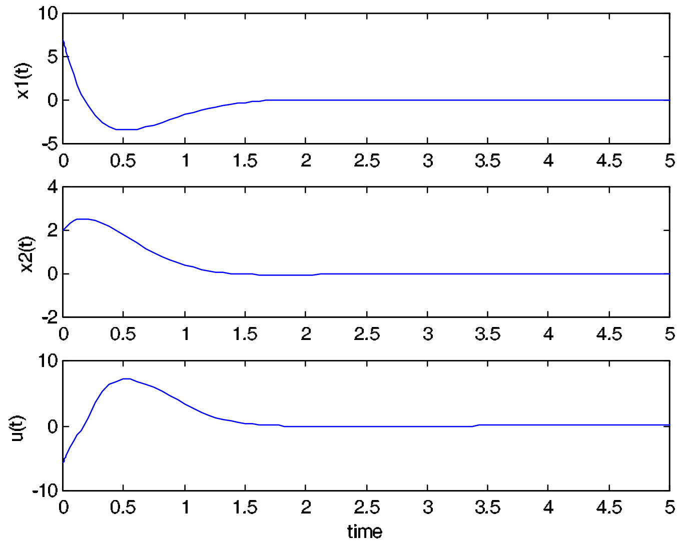

Figure 3. Closed loop model (35) with initial value $x(0) = (7, \; 2)$

6 Conclusion

This paper presents static output feedback controller for nonlinear system described by T-S models. We have shown that the OPDC controller with poles placement can be formulated as the solution of LMIs set. Another way to deal with the static output feedback control for T-S models is to transform the synthesis of the output feedback into a cone complementarity problem, which constitutes our future researches.

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