Comparison of structures for TD-CDMA multi-user detector on the mobile

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Abstract

In downlink TD-CDMA multiple access scheme, the use of multi-user and multi-sensor detection gives great improvement compared to the Rake receiver. Nevertheless, the algorithmic ressources are limited on the mobile and that is why we study and compare some strategies for the practical implementation of the well known linear detector. We consider two linear discrete-time structures with finite number of taps, called Tc-structure and Ts-structure. The first is a wide-band and free structure which performs one single fractional filtering per sensor. The second is an imposed structure consisting of a bank of Matched Filters followed by a bank of discrete equalizers working at symbol time. The determination of the coefficients is based on a temporal optimization. By means of simulations, these two practical detectors are studied for a small number of coefficients, which show very different behavior. Using a complexity analysis, the Ts-structure is shown to be more advantageous globally.

1 Introduction

TD/CDMA system, as TDD-UMTS [9] manages the transmission, during each TDMA slot, of K simultaneous bursts of QPSK symbols, for one or several users. Unfortunately, the multi-path propagation breaks the orthogonality of the K waveforms and results in Multiple Access Interference (MAI) and Inter-Symbol Interference (ISI) when a conventional matched filter detector as Rake receiver is used. In a specific downlink situation, several authors ([3] for instance) propose firstly a channel equalization to restore the orthogonality of the actives codes, followed by correlation with the desired user's code. The concept of this sub-optimum linear detector seems very simple. However, for inverting the wide-band selective channel, the wide-band discrete filter needs a large number of taps, and leads to an high noise amplification compared to optimum theoretical linear detector.
That is why, in downlink high bit rate situations, the use of multi-user detection algorithms and multi-sensor antenna on the receiving mobile is recommended, since it provides a significant capacity improvement [2, 6]. A synthesis of joint detection has been realized in [2] with high complexity temporal block methods. Approximate solutions are proposed in [4]. Detection structures using only linear filters are generally the less complex. Their theoretical Signal to Interference and Noise Ratio (SINR) performance, considering infinite length, are revisited in [6]. However, in a practical design with finite number of taps, the resulted performance and complexity are very dependent on the choice of the structure and of the method of computing the coefficients.
In this paper, we analyse two practical structures derived from the well known theoretical linear scheme: the former, called Tc-structure is a wide-band and free structure; whereas the latter, called Ts-structure, combines wide-band Matched Filters (MF) and narrow-band filters at symbol time. We focus on the exact computation (*i.e.* without approximative methods) of the coefficients, called "temporal optimization" method, relative to the Zero-Forcing (ZF) and Minimum Mean Squared Error (MMSE) criteria. We give the complexity (only the number of multiplications per second) of the two structures for the detection of symbols and for the coefficients computation. The complexity orders are illustrated with the TDD-UMTS parameters. The evaluation of complexity is described with more details in [7]. The behaviour of the two detectors, in term of interference cancellation as well as noise amplification, is studied by means of simulations. Section 2 describes the multi-user transmission system, section 3 and 4 introduces the two practical detectors and section 5 discusses the performance.

2 K-user transmission system

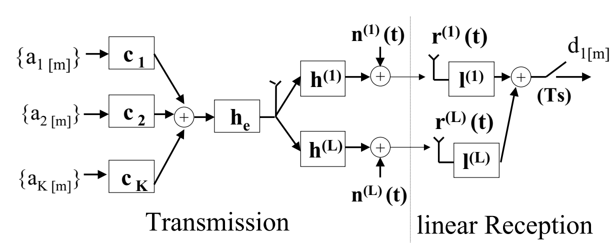

Figure 1: Multi-user transmission and linear reception

During one time slot in downlink, the baseband received signal at the l^{th} sensor is modeled as:

$$r^{(l)}(t) = T_s \sum_{k=1}^{K} \sum_{m} a_{k[m]} g_k^{(l)}(t - mT_s) + n^{(l)}(t) \quad (1)$$

where,
-$Ts = QTc$ is the symbol time, with Tc the chip time and Q the spreading factor,

-$a_{k[m]}$ are iid QPSK symbols of the k-th code, with zero-mean and power A^2. The desired code is code number $k_u = 1$. However in the complexity evaluation, we will consider the detection of $K_u \leq K$ desired codes for the mobile, supposing these codes have the numbers $k_u = 1 ... K_u$
-$g_k^{(l)} = c_k * h_e * h^{(l)}$ represents the convolution between the code $c_k(\tau) = \sum_{q=0}^{Q-1} c_{k[q]}\ \delta(\tau - qTc)$ of user k, the 1/2 Nyquist filter (Root Raised Cosine filter "RRC" with roll off 0,22) and the l^{th} sensor's deterministic channel with L_t paths $h^{(l)}(\tau) = \sum_{i=1}^{L_t} \alpha_i^{(l)}\ \delta(\tau - \tau_i)$, where τ_i are the delays of the paths, and $\alpha_i^{(l)}$ their complex attenuations, different from one sensor to another ($l = 1...L$),
-W_s will be the entire number of symbols covering the temporal spread of the channel ($\tau_{L_t} \leq W_s Ts$). Note that $g_k^{(l)}(\tau)$ is causal and covered by $W_s + 1$ symbols,
-$n^{(l)}(t)$ is a complex AWG Noise with two sided psd $2N_0$, spatially white and uncorrelated with the $\{a_k\}$,
-$\gamma_{ik[n]} = Ts. \sum_{l=1}^{L} g_i^{(l)} * g_k^{(l)H}(\tau)_{|\tau = nTs}$ will be [1] the Ts sampling of the wide-band cross-correlation function between users "i" and "k" waveforms (with the hermitian symmetry $\gamma_{ik}^H = \gamma_{ki}$). The need and the difficulty of the equalization is conditioned by the values $\gamma_{ik[n]}$, ideally equal to $\delta_{ik[n]}$,
-$E_b = \frac{1}{4} A^2 Ts\ \gamma_{11[0]}$ will be the average input bit energy (over L sensors) for user number one.
The channel is supposed time invariant during the slot. Perfect channel estimation, timing synchronization and knowledge of the active codes are assumed.
A symbol by symbol detector forms a decision variable $d_{1[m]}$ with an output Mean Squared Error defined by $(MSE) \triangleq E\{|\ d_{1[m]} - a_{1[m]}|^2\}$.
As usual, MSE is minimized without any constraint on MMSE detector whereas the ZF detector minimizes MSE under constraint of cancelling the ISI and MAI. The expressions and SINR performances for the theoretical ($i.e.$ infinite length) ZF and MMSE linear detectors (right part of figure 1) are described in [6].

3 Tc-Structure

3.1 Detection with Tc-structure

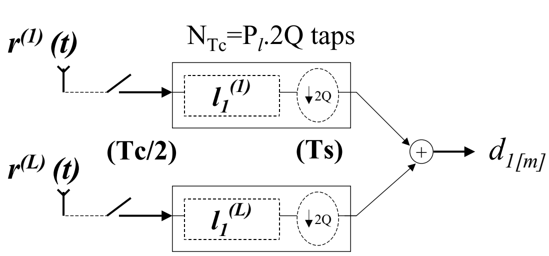

Figure 2: Tc-Structure

This structure, called Decorrelating Receiver in [1] and Row Equalizer in [4], makes directly the linear combination of the received signal samples using discrete multi-rate filters, with the input sampled at Tc/S and the output at Ts. We assume an ideal anti-aliasing analog filter. The oversampling $S = 2$ assures a correct sampling. In order to perform comparison with the Ts-structure, the length of the detector is chosen a multiple of Ts, $i.e.$ $P_l Ts$ where P_l is a positive integer. Moreover, we are particularly interested in a length more than or equal to that of the MF, $i.e.$ $(W_s + 1)$Ts. So, we define $P_l = W_s + P$ with $P \geq 1$ an integer so that $(P-1)Ts$ is a surplus compared to the temporal spread waveform (or MF). $N_{Tc} = SQP_l$ is the number of coefficients per sensor. The decision variable for code "1" is obtained by an inner product ($LSQ.P_l$ multiplications):

$$d_{1[m]} = \underline{l_1}^T \underline{r}_{[m]}$$

where,

$$\underline{l_1} = [l_1^{(1)}{}_{[-SQP_{l_1}]}, .., l_1^{(L)}{}_{[-SQP_{l_1}]}, ..., l_1^{(1)}{}_{[+SQ(P_{l_2}+1)-1]}, ..., l_1^{(L)}{}_{[+SQ(P_{l_2}+1)-1]}]^T$$

$$\underline{r}_{[m]} = [r^{(1)}_{((m+P_{l_1})Ts)}, ..., r^{(L)}_{((m+P_{l_1})Ts)}, ..., r^{(1)}_{((m-P_{l_2}-1)Ts+\frac{Tc}{S})}, ..., r^{(L)}_{((m-P_{l_2}-1)Ts+\frac{Tc}{S})}]^T$$

$P_l = P_{l_1} + 1 + P_{l_2}$, where $P_{l_1} \geq 0$ is the anti-causal depth (delay of $P_{l_1} Ts$ in practice) of the detector.
Note that the detection of symbols $a_{k_u[m]}$ of another code "$k_u \neq 1$" requires to duplicate this structure.

Detection complexity: for each complexity evaluation, we take the TDD-UMTS parameters as: 1 slot by frame for the desired user (thus, 100 slots per second), M = 138 symbols per slot, K=12 codes, Q=16, L_t=6 paths. We evaluate just the number of complex multiplications (denoted MAC).
The complexity of the symbol detection alone does not depend on K, but is linear with the total number of coefficients $LSQ.P_l$, and the number of codes to detect K_u. It is expressed as: $M.K_u.LSQ(P+W_s)$ MAC/frame. To give the size order, we define situations "short" where ($W_s = 1$) and "long" where ($W_s = 5$). The number in tables will always be given in units of millions per second (MMAC/s):

	$K_u = 1$			$K_u = 12$		
$short_{,P=1}$	0.9	1.8	2.7	11	21	32
$long_{,\ P=2}$	3.1	6.2	9.3	37	74	111
$long_{,\ P=8}$	6	11	17	69	138	207
	L=1	L=2	L=3	L=1	L=2	L=3

3.2 Computation of the detector coefficients

The received signal can be expressed in function of symbols $\underline{a}_{[m,P_{l_1}]}$ grouped in a $K(P_l + W_s)$ vector (beginning with $a_{1[m+P_{l_1}]}$) from a $LSQ.P_l \times K(P_l + W_s)$ filtering (Sylvester generalised) matrix $\underline{\underline{\tau_{(g)}}}$:

$$\underline{r}_{[m]} = SQ.\underline{\underline{\tau_{(g)}}}\ \underline{a}_{[m,P_{l_1}]} + \underline{n}_{[m]} \qquad (2)$$

[1] $(.)^H$ denotes the hermitian transformation for a function h: $h^H(\tau) = h^*(-\tau)$, and the hermitian transposition for vectors.

By interlacing the symbols of the different codes in $\underline{a}_{[m,P_{l_1}]}$, $\underline{\underline{\mathcal{T}_{(g)}}}$ makes use of a block $[LSQ \times K]$ Toeplitz structure:

$$\underline{\underline{\mathcal{T}_{(g)}}} \triangleq \begin{bmatrix} \underline{\underline{g}}_{[0]} & \underline{\underline{g}}_{[1]} & \cdots & \underline{\underline{g}}_{[W_s]} & 0 & \cdots & 0 \\ 0 & \underline{\underline{g}}_{[0]} & \underline{\underline{g}}_{[1]} & \cdots & \underline{\underline{g}}_{[W_s]} & 0 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & \cdots & 0 & \underline{\underline{g}}_{[0]} & \underline{\underline{g}}_{[1]} & \cdots & \underline{\underline{g}}_{[W_s]} \end{bmatrix}$$

$\underline{\underline{g}}_{[n]} = [\underline{g}_{1[n]}, \underline{g}_{2[n]}, ..., \underline{g}_{K[n]}]$ is the nTs delay block,

$\underline{g}_{k[n]} = (\frac{Tc}{S}).[g_{k}{}^{(1)}_{((n+1)Ts-\frac{Tc}{S})}, ..., g_{k}{}^{(L)}_{((n+1)Ts-\frac{Tc}{S})}, ..., g_{k}{}^{(1)}_{(nTs)}, ..., g_{k}{}^{(L)}_{(nTs)}]^T$ contains samples taken with $\frac{Tc}{S}$ intervals (in anti chronological order) of the k-th global waveform for the delay between $(n+1)Ts$ and nTs, with an interleaving for the different sensors.

$\underline{a}_{[m,P_{l_1}]} = [a_{1[m+P_{l_1}]}, .., a_{K[m+P_{l_1}]}, ..., a_{1[m-(P_l+W_s-P_{l_1}-1)]}, .., a_{K[m-(P_l+W_s-P_{l_1}-1)]}]^T$,

$\underline{n}_{[m]}$ contains the samples of the noise (with the same arrangement as $\underline{r}_{[m]}$).

This formulation is in fact similar to those of multichannel systems. Note that CDMA is a fractional system since "one symbol is transmitted through Q chips", reinforced by oversampling and the use of multiple sensors with an overall degrees of freedom of LSQ to compensate channel spread, W_s. The only modification made here is to take into account the supplementary dimension K of the system, considering $(K-1)$ interfering codes.

The MMSE vector detector is given from the Wiener solution and can be written as:

$$\underline{l_1}^T = \frac{1}{SQ}.\mathbf{1}^T_{\Delta_l}[\underline{\underline{\mathcal{T}_{(g)}}}^H \underline{\underline{\mathcal{T}_{(g)}}} + \sigma_0^2 \underline{\underline{I}}_{K(P_l+W_s)}]^{-1} \underline{\underline{\mathcal{T}_{(g)}}}^H \quad (3)$$

where $\sigma_0^2 = (\frac{N_0}{2E_b})$, $\underline{\underline{I}}_N$ is the $N \times N$ identity matrix, $\mathbf{1}^T_{\Delta_l} \triangleq [\underbrace{\cdots, 0, 1}_{\Delta_l}, 0, \cdots]$, $\Delta_l = K.P_{l_1} + k_u$.

Under current CDMA situation, the number of degrees of freedom is such that the matrix $\underline{\underline{\mathcal{T}_{(g)}}}$ is more high than large:

$$LSQ.P_l \geq K(P_l + W_s) \quad (4)$$

The squared hermitian matrix $\underline{\underline{\psi_{(g)}}} = (\underline{\underline{\mathcal{T}_{(g)}}}^H \underline{\underline{\mathcal{T}_{(g)}}})$ is nonsingular if $\underline{\underline{\mathcal{T}_{(g)}}}$ is of full rank $K(P_l + W_s)$, which is generally true. **In noise-free situations**, MSE can be completely cancelled but the Wiener solution is not unique (the correlation matrix $E\{\underline{r}_{[m]}.\underline{r}^H_{[m]}\}$ is singular): equation (3) gives then the particular solution with the minimal norm.

The ZF solution is precisely given by (3) by replacing σ_0^2 by zero. It consists simply of the pseudo-inverse of the matrix $\underline{\underline{\mathcal{T}_{(g)}}}$. It can be easily verified that $\underline{l_1}^T.\underline{\underline{\mathcal{T}_{(g)}}}.SQ = \mathbf{1}^T_{\Delta_l}$, corresponding to a unitary global gain for the desired code with interference (ISI and MAI) forced to zero. This particular interference canceller is the solution which minimizes the MSE reduced to the output power of the noise (the pseudo-inverse warrants the minimum norm solution for the vector $\underline{l_1}$).

Note 1: $\underline{\underline{\psi_{(g)}}}$ has a block structure [K×K] non-Toeplitz. Indeed, the blocks on the center rows are formed by Ts cross-correlation $\gamma_{ik[n]}$, $for\ i,k = 1...K$ but on the bottom and the top of the matrix, the W_s rows of blocks use partial correlation, which breaks the Toeplitz structure.

Note 2: the exact cancellation of interference could be achieved even in a situation with no upsampling ($S = 1$) and with a single sensor ($L = 1$). In this case, the detector temporal depth P_l can only be equal to the channel spread W_s, as long as $K \leq \frac{1}{2}Q$ (equation (4) is satisfied). In other words, $K \leq 8$ should be taken with the TDD-UMTS parameters.

Coefficient computation complexity [7]:

The coefficient computation is performed under the assumption of perfect knowledge of the communication channel, *i.e.* coefficients $(\tau_i, \alpha_i^{(l)})$ and ratio $\frac{E_b}{N_0}$ (needed for an exact MMSE solution). Profiting from the discrete-time nature of the channel model, we compute (3) for ZF or MMSE solutions using successive computation steps, as follows:

-(Tc1): formation of $\underline{\underline{\mathcal{T}_{(g)}}}^H$ from $\alpha_i^{(l)}, \tau_i, (h_e * c_k)$:
$L(SQ + 7S).L_t.K$ (MAC/frame),
-(Tc2): computation of γ_{ik} to form $\underline{\underline{\psi_{(g)}}}$:
$L.(\frac{L_t(L_t-1)}{2} + 1) + \frac{K}{2}(K+1).(2W_s + 1)(\frac{L_t(L_t-1)}{2} + 1)$,
-(Tc3): computation of K_u rows (one row per desired code) of the inverse of the non-negative definite matrix $\underline{\underline{\psi_{(g)}}}$ (with addition of the term σ_0^2 for the diagonal terms in the case of MMSE). An efficient method to do this task is the Cholesky decomposition, followed by K_u solving of equations with unit column vectors [8], which needs:
$\frac{1}{6}[(P + 2W_s)K]^3 + K_u.\frac{2}{3}[(P + 2W_s)K]^2$,
-(Tc4): computation of the LN_{Tc} coefficients for K_u codes. From the elements computed in (Tc1) and (Tc3), we can obtain K_u vectors $\underline{l_1}^T$ in applying (3):
$K_u.LSQ.(P + W_s) \times K(P + 2W_s)$.

The total complexity for coefficients computation from (3) is given (MMAC/s) in the table below:

	$K_u = 1$			$K_u = 12$		
$short_{,P=1}$	1.5	2.0	2.6	4.9	8.0	11.1
$long,\ _{P=2}$	55	58	62	105	144	183
$long,\ _{P=8}$	181	190	199	313	422	530
	L=1	L=2	L=3	L=1	L=2	L=3

In appropriate situations with this structure ($K_u = 1$) and for channels with high dispersion, the majority of the calculation (about 90 %) is due to Cholesky decomposition of the matrix $\underline{\underline{\psi_{(g)}}}$, which is not Block Toeplitz.

4 Ts-Structure

4.1 Detection with Ts-structure

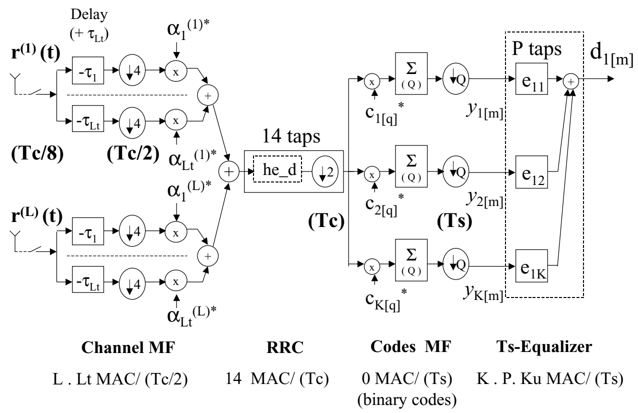

Figure 3: Ts-Structure (with $\gamma_{11[0]} = 1$)

This structure is a truncated version of the infinite length theoretical linear detector: it performs a front-end bank of MF followed by a bank of synchronous discrete-time transverse filters (1 sample per symbol). We make use of the commutativity of the convolution to form the bank of MF as seen in figure 3. Like for the Rake receiver, this structure takes advantage of the discrete paths nature of the channel: the computation of the channel MF needs only one multiplication per sensor and per path at rate $Tc/2$ **after quasi continuous delay compensation** (upsampling of 8 on the figure). The RRC discrete filter h_{e}_d works with input at every $Tc/2$ and output at every Tc, and with a usual number of coefficients of 14. As the complex codes are binary, the codes correlations are simply performed by the additions and subtractions of Q samples during one Ts.
The "Ts-equalizer" linearly combines the outputs $\{y_{k[m]}\}$ of the K branches after passing through a "discrete filters" bank $\{e_{1k[n]}\}_{,k=1...K}$ to obtain the decision variable by:

$$d_{1[m]} = \frac{1}{\gamma_{11[0]}} \, \underline{e_1}^T . \underline{y_{[m]}}$$

with:
$\underline{e_1} = [e_{11[-P_1]}, .., e_{1K[-P_1]}, ..., e_{11[+P_2]}, .., e_{1K[+P_2]}]^T$,
$\underline{y}_{[m]} = [y_{1[m+P_1]}, .., y_{K[m+P_1]}, ..., y_{1[m-P_2]}, .., y_{K[m-P_2]}]^T$,
The number of coefficients on each branch of the equalizer is $P = P_1 + 1 + P_2$, *assuring, for a same P, a same duration $P_l Ts$ for the global detector impulse response as in the Tc-structure.* P_1 is the number of anticausal coefficients on each branch logically chosen around the integer part of $\frac{P}{2}$ because of an hermitian symmetry of the coefficients: $e^*_{ik[-n]} = e_{ki[n]}, \quad \forall i, k = 1...K, \quad \forall n$.

Detection complexity: the complexity of the global MF does not depend on the channel spread, W_s, but only on the number of paths. Only the Ts-equalizer, which necessitates a slower processing, depends on W_s (via P), K and K_u. The total detection complexity is: $M\{L2Q.L_t + 14Q + KPK_u\}$ MAC/frame,
and is given in the following table (MMAC/s):

	$K_u = 1$			$K_u = 12$		
short$_{,P=1}$	05.9	08.5	11.2	07.7	10.4	13.0
long$_{,\ P=8}$	07.1	09.7	12.4	21.6	24.3	26.9
	L=1	L=2	L=3	L=1	L=2	L=3

4.2 Computation of the equalizer coefficients

The samples of $\underline{y}_{[m]}$ can be expressed directly as a function of the transmitted symbols from a $KP \times K(P + 2W_s)$ filtering matrix (Sylvester generalized) $\underline{\underline{\tau_{(\gamma)}}}$ with a block $[K \times K]$ Toeplitz structure, and an additive noise:

$$\underline{y}_{[m]} = \underline{\underline{\tau_{(\gamma)}}} \; \underline{a}_{[m,P_1+W_s]} + \; \underline{\eta}_{[m]}$$

$$\underline{\underline{\tau_{(\gamma)}}} \triangleq \begin{bmatrix} \underline{\underline{\gamma}}_{[-W_s]} & \cdots & \underline{\underline{\gamma}}_{[W_s]} & 0 & \cdots & 0 \\ 0 & \underline{\underline{\gamma}}_{[-W_s]} & \cdots & \underline{\underline{\gamma}}_{[W_s]} & 0 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & \cdots & 0 & \underline{\underline{\gamma}}_{[-W_s]} & \cdots & \underline{\underline{\gamma}}_{[W_s]} \end{bmatrix}$$

where $\underline{\underline{\gamma}}_{[n]}$ is the block corresponding to a delay of nTs:

$$\underline{\underline{\gamma}}_{[n]} \triangleq \begin{bmatrix} \gamma_{11[n]} & \cdots & \gamma_{K1[n]} \\ \cdots & \cdots & \cdots \\ \gamma_{1K[n]} & \cdots & \gamma_{KK[n]} \end{bmatrix} = (\gamma^H_{ik[n]}) \quad {}_{i,k=1..K}$$

The MMSE vector equalizer for the Ts-structure is given from the Wiener solution by :

$$\frac{1}{\gamma_{11[0]}} \; \underline{e_1}^T = \mathbf{1}^T_\Delta \underline{\underline{\tau_{(\gamma)}}}^H [\underline{\underline{\tau_{(\gamma)}}} \; \underline{\underline{\tau_{(\gamma)}}}^H + \sigma_0^2 \underline{\underline{\tau_{tn(\gamma)}}}]^{-1} \qquad (5)$$

where $\Delta = K(P_1 + W_s) + k_u$, and $\underline{\underline{\tau_{tn(\gamma)}}}$ is a square $KP \times KP$ matrix obtained by truncating $\underline{\underline{\tau_{(\gamma)}}}$, with the first block $\underline{\underline{\gamma}}_{[0]}$ (on the left top of the matrix). This unique solution is always well conditioned, even if σ_0^2 is zero, as long as $\underline{\underline{\tau_{(\gamma)}}}$ has a full rank KP, which is generally true (the correlation matrix $E\{\underline{y}_{[m]} . \underline{y}^H_{[m]}\}$ is nonsingular). Notice the hermitian symmetry of the equalizer taps, which may be exploited when the mobile decodes several (K_u) desired codes.

The ZF approximate solution corresponds precisely to putting $\sigma_0^2 = 0$ in the equation (5). In general situations where channel is not a one path channel ($W_s \neq 0$), the matrix $\underline{\underline{\tau_{(\gamma)}}}$ is more large than high ($KP < K(P + 2W_s)$), and the ZF approximation with this structure minimizes the interference but does not force it to zero. It warrants also, with this specific imposed structure, the minimal amplification of noise constrained to the minimal power of interference. This result can be interpreted as follows: the MF and Ts-sampler bank has compressed the information (in a sufficient statistic to detect the symbols...) without conserving any degree of freedom. Then with a simple linear equalizer (which is not the optimal digital processing...) working at Ts, we must theoretically have an infinite impulse response to inverse the system, *i.e.* to cancel the interference. Nevertheless, in many applications, the exact interference cancelling is not required and this ZF approximation may be satisfactory.

Finally, the coefficients computation needs to inverse the block [K×K] Toeplitz matrix of dimension $KP \times KP$ (or after adding $\sigma_0^2 \underline{\underline{\tau_{tn(\gamma)}}}$ for the MMSE solution):

$$\underline{\underline{\phi_{(\gamma)}}} \triangleq \underline{\underline{\tau_{(\gamma)}}}\,\underline{\underline{\tau_{(\gamma)}}}^H = \begin{bmatrix} \underline{\underline{\phi}}_{[0]} & \underline{\underline{\phi}}_{[1]} & \cdots & \underline{\underline{\phi}}_{[P-1]} \\ \underline{\underline{\phi}}_{[-1]} & \underline{\underline{\phi}}_{[0]} & \underline{\underline{\phi}}_{[1]} & \cdots \\ \cdots & \cdots & \cdots & \underline{\underline{\phi}}_{[1]} \\ \underline{\underline{\phi}}_{[-P+1]} & \cdots & \underline{\underline{\phi}}_{[-1]} & \underline{\underline{\phi}}_{[0]} \end{bmatrix}$$

where $\underline{\underline{\phi}}_{[n]} \triangleq (\phi_{ik[n]})$ is the block for the nTs delay, with the symmetry $\phi_{ik[n]} = \phi^H_{ki[n]}$, and obtained from the discrete convolution:

$$\phi_{ik[n]} = \sum_{j=1}^{K} \sum_{u} \gamma_{ji[u]} . \gamma^H_{jk[n-u]} \qquad (6)$$

Coefficients computation complexity [7]:
From (5) for ZF (and similarly MMSE):
-(Ts1): computation of $\gamma_{ik[p]}$: idem (Tc2).
-(Ts2): computation of $\phi_{ik[p]}$; the property of symmetry permits us to compute it only for $_{i=1..K,k=i..K,n=0..P-1}$ with a summation for "u" in (6) on (W_s+1-n) values. The total complexity depends on if $P \geq W_s$ ($\delta_{(P\geq W_s)} = 1$) or not ($\delta_{(P\geq W_s)} = 0$) and is expressed in MAC/frame by: $\frac{K(K+1)}{2}.P.K.\{\frac{(2W_s+1).(2W_s+2)}{2} - \frac{P.(P+1)}{2}.\delta_{(P\geq W_s)}\}$
-(Ts3): Cholesky Decomposition of $\underline{\underline{\phi_{(\gamma)}}} = \underline{\underline{T}}\,\underline{\underline{T}}^H$ where $\underline{\underline{T}}$ is lower triangular with positive elements on the diagonal. The P rows of the blocks of $\underline{\underline{T}}$ can be computed using the algorithm proposed in [5], taking into account the block $[K \times K]$ Toeplitz structure of $\underline{\underline{\phi_{(\gamma)}}}$: $P.K(P)^3$,
Notice that approximate solutions could also be derived since the blocks rows of the Cholesky decomposition $\underline{\underline{T}}$ converge [5, 4], as a consequence of the block Toeplitz structure of $\underline{\underline{\phi_{(\gamma)}}}$.
-(Ts4): computation of the KP coefficients for K_u codes by solving (in 2 steps) the equations $\frac{1}{\gamma_{11[0]}} \underline{\underline{T}}^* \underline{\underline{T}}^T . \underline{e_1} = (\underline{\underline{\tau_{(\gamma)}}}^* . \underline{\mathbf{1}_\Delta})$, which requires: $K_u.(KP)^2$.

The total complexity for the coefficients computation from (5) has a little dependence on the number of sensors L, and is given (in MMAC/s) by:

	$K_u = 1$	$K_u = 12$
$short_{,P=1}$	0.5	0.7
$long_{,\ P=8}$	23.4	33.6
	L=1 to 3	L=1 to 3

4.3 Complexity comparison of Ts- and Tc-Detector

For the same global detector impulse response length, the complexity is compared for the two following tasks:
- **detection:** for a great number K_u of desired codes to detect, the Tc-structure is of course less adequate than the Ts-structure which keeps the same receiver front with any desired codes and with any number of active codes (same channel in downlink). Moreover, even if $K_u = 1$, the Ts-structure takes the advantage when the channel is long, thanks to the "Rake" structure of the front of the receiver and to the symbol time cadence of the equalizer bank: the detection complexity grows very weakly with P in this structure.
- **coefficients computation:** first, we notice that for TDD-UMTS parameters, the coefficients computation task (with "temporal optimization") has a complexity not negligible, but about or greater than the symbols detection task. The coefficients computation is still much less complex with the Ts-structure thanks to a smaller dimension of the system to be inverted (degrees of freedom compacted in the symbol time description), reinforced by a block Toeplitz formulation.

So, for the TDD-UMTS case, for a long temporal spread of the channel, and for a given global detector impulse response length, we can conclude that the total complexity of detection and coefficients computation favours the Ts-structure. Nevetheless, we can not conclude on the better performance / complexity compromise without checking the behaviour of the detectors. Indeed, the performance of the Tc-structure is theoretically better since the linear optimization with finite length is performed without imposing the structure.

5 Simulations

On figure 4, we plot the inverse of the output SINR, noted INSR, reached by the Tc-structure and the Ts-structure, versus P, the length parameter of the detector. Recall that $(P-1)Ts$ is the surplus relative to the MF, of the detector impulse response duration, equal to $P_l Ts$ where $P_l = P + W_s$. The results are given for ZF and MMSE criteria, and for two $\frac{E_b}{N_0}$ ratios, respectively 30 dB and 10 dB. The channel model is one particular realization of a vehicular B model [10], with long temporal spreading ($W_s = 5$). TDD-UMTS parameters [9] are used with 12 active codes, only one desired code and with a single reception sensor to underscore the need for highest detectors' length. The SINR is chosen to measure the detectors' quality since it can be directly computed from matrix formulation (3, 5) and is directly linked to the Bit Error Rate when the interference is totally cancelled or when the residual interference is approximately gaussian.

For the Zero-Forcing criterion, the **Tc-structure** with short P_l can perform exact inversion but the noise amplification is very high, for instance 26 dB for $P = 0$, *i.e.* $P_l = W_s$. For comparison, the noise amplification with theoretical, *i.e.* infinite length, linear ZF receiver is around 2.5 dB, leading to an INSR of around -30.5 dB when $\frac{E_b}{N_0} = 30dB$. The theoretical performance is approached for P above $2W_s$. Moreover, the ZF Tc-detector is very sensitive to the choice of the delay of the detector, P_{l_1}, and we plot only the best results. The **Ts-structure** avoids this large noise amplification but does not completely cancel the interference, as we can see from the high $\frac{E_b}{N_0}$. The decrease of residual interference to a negligible value, for instance 20 dB less than the useful power, needs an equalizer depth of $P > 2W_s$. **For the MMSE** criterion, the **Ts-detector** behaves similarly as with the ZF criterion. On the contrary, the **Tc-detector** is able to reach theoretical values of INSR with very short depth, for instance $P = 2$. For equivalent results with one sensor and under MMSE criterion, the detection complexity in the Tc-detector is less than in the Ts structure but globally, with the coefficient computation, the Ts-structure remains more attractive. Bear in mind that the transverse filter bank requires only $P.K$ coefficients in the Ts-structure whereas the Tc-structure requires $(P + W_s).2Q$ coefficients.

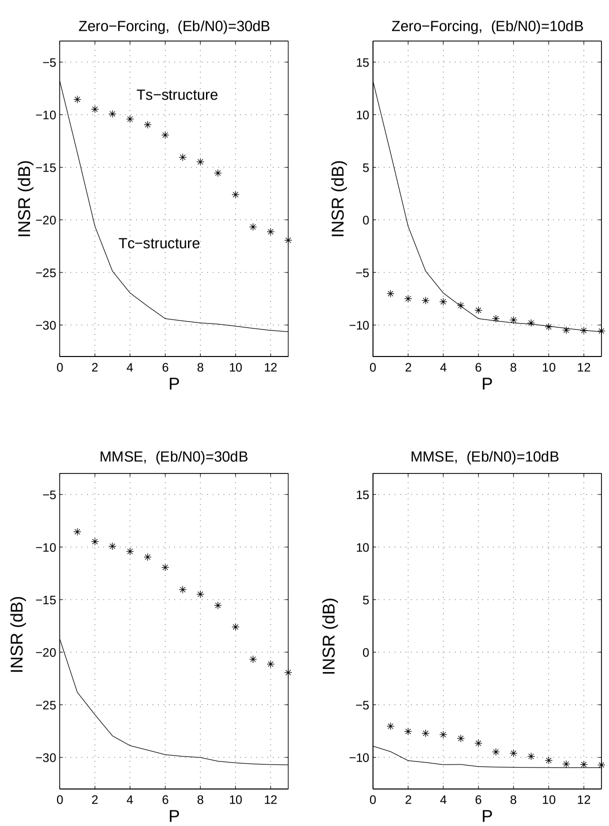

Figure 4: Performances of the Tc- and Ts-structures with a long channel ($W_s = 5$), versus P, for ZF and MMSE criteria and for high and weak $\frac{E_b}{N_0}$.

6 Conclusion

This paper has compared the complexity and behaviour of two practical multi-user linear detectors, through ZF and MMSE criteria with temporal optimization. When the temporal depth of the detectors is very large, the two structures tend to the same performance. For a given short finite length, the Tc-structure is a free linear structure which achieves better results. Under ZF criterion, it performs an exact interference cancellation, but with a large noise amplification if the detector length is very short. Under MMSE criterion, the Tc-detector is able, with just a short length (a little bit above the channel length), to obtain a performance comparable to those of the theoretical, *i.e.* with infinite length, MMSE detector. Nevertheless, the global complexity of this detector is very high, principally because of the "temporal optimization" of the coefficients and it would be interesting to study approximate solutions for the coefficients computation. On the contrary, the Ts-structure is less complex and presents a different behaviour: it needs more temporal depth to decrease the residual interference, but always controls the amplification of the noise. In regard to both performance and complexity, the Ts-detector remains globally more advantageous (especially with a long spread channel and multi-sensor) and adequate for multi-code reception. Nevetheless, as this structure doesn't have the finite length complete interference cancelling property, it will be interesting to investigate new medium structures which keep the advantages of the Tc- and the Ts-structure.

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