Performance Analysis of OFDM-CDMA Systems with Doppler Spread

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***Abstract*—Multi carrier modulation are very sensitive to rapid time-varying multi-path channel characterized by Doppler spread. Although progress has been made in the description of the time variation, there is still considerable gaps in its effect especially on diversity gain acquired by time selectivity. This paper models a general case of time-varying channel effect on the OFDM-CDMA performance. This performance is measured through the Signal to Interference and Noise Ratio SINR at the output of the detector and the Bit Error Rate BER at the output of the channel decoder. The originality of the paper is twofold. First, we propose a simple tool to evaluate an analytical expression of the SINR independently on the spreading codes while taking into account their orthogonality. Second, we adapt a new technique to predict the BER at the output of the channel decoder from the link level simulation expressed in terms of the SINRs. We show by simulation the validity of our analytical models. We show also that the time variation of the channel would be favourable for system performance in MC-DS-CDMA system and QPSK constellation however it is destructive with other simulation assumptions.**

Keywords: *Multi carrier spreading systems, Large system analysis, SINR, Doppler spread, EESM technique.*

1. INTRODUCTION

Recently, Orthogonal Frequency and Code Division Multiplexing (OFCDM) access technology has been investigated for the next generation of mobile communication systems [1][2]. It is a combination of Orthogonal Frequency Division Multiplexing (OFDM) and Code Division Multiple Access (CDMA). To achieve high spectrum efficiency, these systems will implement a large number of sub-carriers. By using cyclic prefix CP, the sub-carriers in an OFDM system are orthogonal in a time-invariant multipath channel. However, the orthogonality is destroyed when the channel is time variant throughout the duration of an OFDM symbol. The variances of the channel time variations can be modelled by the Doppler spread. The Doppler spread is the difference in Doppler frequencies between different channel paths. It reduces the useful energy in each subcarrier and introduces Inter Carrier Interference ICI [3]. Nevertheless, the time variation of the channel introduces time selectivity during transmission. It offers higher degree of diversity which can be exploited by the channel decoder to improve system performance. In the literature, the Doppler effect was introduced in several papers for OFDM [3][4] and MC-CDMA systems [5]. However, [5] assumes an infinite number of sub-carriers to derive an analytical expression of the local-mean Bit Error Rate BER deduced from a local-mean Signal to Interference and Noise Ratio SINR expression. The instantaneous SINR is not accounted for.

This article presents the effect of Doppler spread in an OFDM-CDMA system [1][2] while spreading is done in both time and/or frequency domains. It extends to a 2 Dimensional OFDM-CDMA context the work of [5] which was done for a MC-CDMA system. It is also based on our previous works done for different synchronization errors in OFDM-CDMA systems [6][7][8]. The interest of 2D spreading system is to combine in a flexible way the benefits of time and/or frequency spreading in order to optimize the trade-off between diversity gain and interference loss due to channel selectivity. The merits of this paper consist into the combination of the asymptotic instantaneous SINR computation of an OFDM-CDMA systems with Doppler spread and the Effective Exponential SINR Mapping EESM technique. The asymptotic SINR is computed at the output of the receiver detector by using some properties of random matrix and free probability theories [9][10][11]. The SINR formula is independent of the actual values of spreading codes while taking into account their orthogonality. The asymptotic regime is attained when both the number of users and the spreading factor become large while their ratio remains constant. It has been validated thanks to the link level simulations. However, it is desirable to evaluate the system level performance after channel decoder in terms of the Block/Bit Error Rate BER by using an analytical expression. Thus, we propose to adapt an Exponential Effective SINR Mapping EESM method initially validated within 3GPP for the OFDM study item to an OFDM-CDMA context. The EESM method is currently used to provide a reliable interface between the link level and the system level simulations.

This article is organized as follows. Section 2 describes the system model of the OFDM-CDMA system including transmitter and receiver. Section 3 gives an asymptotical expression of the estimated SINR. Section 4 describes the system performance without channel coding. In section 5, we derive a mapping technique used for the evaluation of the system level simulations while section 6 presents system performance with channel coding. Eventually, conclusions are drawn in section 7.

2. SYSTEM MODEL

In this section, the generalized framework describing an OFDM-CDMA system is presented. Figure 1 shows a 2 dimensional (2D) spreading OFDM-CDMA transmitter/receiver chain for a downlink communication with N_u users [1]. The bits $b_m[s]$ of each user m are sent to a channel coding module which performs Forward Error Correction FEC encoding. Then the coded bits are modulated into QAM complex symbols. For each user, each symbol is first spread by a Walsh-Hadamard WH sequence of N_c chips, and scrambled by a portion of N_c chips of the cell specific long pseudo random sequence. This scrambling code is used to minimize the multi-cell interference. The chips are then allocated on the time/frequency grid as shown on Figure 2. Assuming an IFFT of N sub carriers, each user transmits S=N/N_F data symbols $a_m[s]$ of the m^{th} user on the j^{th} OFDM-CDMA block composed of N_T OFDM symbols. N_F and N_T are respectively the frequency and time domain spreading factors. The system is thus a general case of the MC-CDMA [5] and MC-DS-CDMA systems [12]. The scheme is identical for other users using one specific WH sequence by user, with $N_u \leq N_c$.

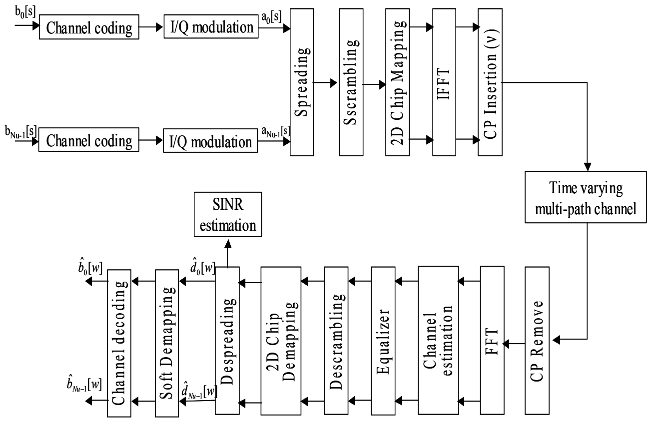

Figure 1: OFDM-CDMA Transceiver.

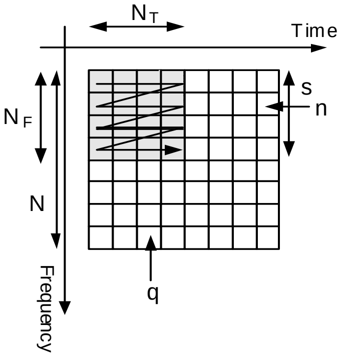

Figure 2: Time Frequency grid

Eventually, the signal at the IFFT input is:

$$d_q[sN_F+n]=\sum_{m=0}^{N_u-1}\sqrt{P_m}\,a_m[s]C_{m,s}[nN_T+q] \qquad (1)$$
$$q=0,\dots,N_T-1;\ s=0,\dots,S-1,\ n=0,\dots,N_F-1$$

s is the index referring to the sub band used for the transmission of the symbol $a_m[s]$ of the m^{th} user. P_m is its transmit power which is identical in all sub-bands, $C_{m,s}$ represents its spreading sequence (chip by chip multiplication of the user assigned WH sequence and the cell specific scrambling code).

At the output of the IFFT, a cyclic prefix CP of ν samples is inserted to discard the Inter Symbol Interference ISI. The signal is then convoluted by the time-varying multipath channel which is modelled by its Channel Impulse Response CIR $g_q(\tau,t)$ where t is the time and τ is the lag. Hence, $g_q(.,t)$ is the CIR as seen by the receiver at time t. In our simulation results, we will assume that each macro path is formed by I_l micro paths each one characterized by a maximum Doppler frequency $F_d=V_M/C$ where V_M is the Mobile Velocity and C is light speed. It is given by

$$g_q(\tau;t)=\sum_{l=0}^{L-1}\sum_{i=0}^{I_l-1}\alpha_{li}\exp\left[j\left(2\pi f_{dli}t+\varphi_{li}\right)\right]\delta(\tau-\tau_{li}) \qquad (2)$$

Where f_{dli}, φ_{dli}, τ_{dli}, α_{dli} are respectively the Doppler frequency, the arrival phase, the delay, and the module of the micro path i of the macro path l. L is the number of macro paths which corresponds to W samples with a sampling frequency F_s. Then, W represents the delay spread assumed less than to the cyclic prefix (W≤ν). For very small Doppler frequencies (versus the sampling frequency), the CIR becomes independent on the time variations. In our model description, we will use a general discrete representation of the CIR given by $g_q[k;u]$ where k represents the delay and u the time.

The signal at the channel output is then corrupted by an AWGN with variance σ^2. At the receiving side, the received samples after the CP remove can be written as

$$r_q[u]=\sum_{k=0}^{W-1}g_q[k;u]x_q[u-k]+n_q[u]=\sum_{k=-\nu}^{N-1}g_q[u-k;u]x_q[k]+n_q[u] \qquad (3)$$
$$\mathit{with}\quad u=0,\dots,N-1$$

After the FFT operation, the value of $(wN_F+p)^{th}$ sub-carrier of the q^{th} OFDM symbol of an OFDM-CDMA block is:

$$R_q\left[wN_F+p\right]=\sum_{s=0}^{S-1}\sum_{n=0}^{N_F-1}b_q\left[sN_F+n\right]\phi(w,s,p,n,q)+N_q\left[wN_F+p\right] \qquad (4)$$

w is the desired sub-band index (w = 0,…, S-1) and p is the index of a sub-carrier in the w^{th} sub-band (p = 0,…, N_F-1). ϕ(w,s,p,n,q) is the "frequency side-to-side" equivalent channel transfer function (IFFT-channel-FFT). It is given by

$$\phi(w,s,p,n,q)=\frac{1}{N}\sum_{u=0}^{N-1}h_q[sN_F+n;u]\exp\left(-j2\pi\frac{(w-s)N_F+p-n}{N}u\right) \qquad (5)$$

Where $h_q[sN_F+n;u]$ is the FFT over the delay k of the CIR $g_q[k;u]$. Equation (5) shows that for w=s and p=n (interesting $(wN_F+p)^{th}$ sub-carrier), ϕ(w,w,p,p,q) characterizes the mean of the FFT of the CIR over one OFDM symbol duration.

Without loss of generality, we assume that one is interested by the symbols of user 0. In order to write the received signal with matrix-vector notation, the following matrices are defined: P = diag(P_0,…, P_{Nu-1}) is the N_uxN_u diagonal matrix which entries are the power allocated to each user, Q = diag(P_1,…,P_{Nu-1}) is the (N_u-1)x(N_u-1) diagonal matrix containing the powers of the interfering users, C[s] = (C_0[s], C_1[s],…, C_{Nu-1}[s]) is the N_cxN_u matrix containing all the spreading codes used in the s^{th} sub-band and U[s] = (C_1[s],…, C_{Nu-1}[s]) is the N_c x (N_u-1) matrix containing the codes of the interfering users in the s^{th} sub-band. Both matrices C[s] and U[s] depend on sub-band index s because of the long scrambling code. We also define the vectors $a[s]=\left(a_0[s],\dots,a_{N_u-1}[s]\right)^T$ and $\tilde{a}[s]=\left(a_1[s],\dots,a_{N_u-1}[s]\right)^T$ corresponding to the symbols of all and interfering users respectively transmitted in the s^{th} sub-band. The estimated symbol of the reference user on the sub-band w after equalization and despreading is then:

$$\begin{aligned}
\hat{a}_{j,0}[w] &= I_0+I_1+I_2+I_3+I_4\\
I_0 &= \sqrt{P_0}\,C_0^H[w]Z[w]H[w,w]C_0[w]a_0[w]\\
I_1 &= C_0^H[w]Z[w]H[w,w]U[w]Q[s]\tilde{a}[w]\\
I_2 &= \sum_{\substack{s=0\\ s\neq w}}^{S-1}C_0^H[w]Z[w]H[w,s]C[s]P[s]a[s]\\
I_3 &= C_0^H[w]Z[w]N[w]
\end{aligned} \qquad (6)$$

I_0 represents the useful signal, I_1 the Multiple Access Interference (MAI) of the same sub band w, I_2 the interference generated by all users from other sub bands (Inter Band Interference IBI) and I_3 the noise. H[w,s] is a $N_c \times N_c$ matrix which components represent the frequency channel coefficients depending on time-varying CIR coefficients.

$$H_{p,n}[w,s] = \begin{pmatrix} A[0,0] & \cdots & \cdots & A[0,N_F-1] \\ A[1,0] & \ddots & & A[1,N_F-1] \\ \vdots & \vdots & A[p,n] & \vdots \\ A[N_F-1,0] & \cdots & \cdots & A[N_F-1,N_F-1] \end{pmatrix}$$

$$A[p,n] = \begin{pmatrix} \phi(w,s,p,n,0) & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \phi(w,s,p,n,N_T-1) \end{pmatrix} \tag{7}$$

Z [w] is a $N_c \times N_c$ diagonal matrix which equalizes the diagonal elements of H[w,s].

3. SINR EVALUATION

The data symbols are assumed i.i.d. having zero mean and unit variance. The SINR for every sub band is deduced from (6) by calculating the expectation values of I_0, I_1, I_2 and I_3. It is given by

$$SINR = \frac{E|I_0|^2}{E|I_1|^2 + E|I_2|^2 + E|I_3|^2} \tag{8}$$

The expectations values in (8) over the random data symbols are given by

$$\begin{aligned} E|I_0|^2 &= P_0 \left| C_0^H[w]Z[w]H[w,w]C_0[w] \right|^2 \\ E|I_1|^2 &= C_0^H[w]Z[w]H[w,w]U[w]QU^H[w]H[w,w]^H Z[w]^H C_0[w] \\ E|I_2|^2 &= \sum_{\substack{s=0 \\ s\neq w}}^{S-1} C_0^H[w]Z[w]H[w,s]C[s]PC^H[s]H[w,s]^H Z[w]^H C_0[w] \\ E|I_3|^2 &= \frac{\sigma^2}{N_c} tr\left(Z[w]Z[w]^H\right) \end{aligned} \tag{9}$$

These expressions show that the SINR depends on a complex way of the actual spreading codes. (9) can not be used practically due to its complexity. Thus, using some properties of random matrix and free probability theories [11][10][9][14], we can show as in our previous works [6][7][8] that the dependence of the SINR on the spreading codes was vanishing in the asymptotic regime (N_c and $N_u \rightarrow \infty$ while the ratio $\alpha = N_u/N_c$ is kept constant). The performances only depend on the system load α, noise variance and the power distribution. With these assumptions, (9) yields for given CIR time variations (see Appendix):

$$\begin{aligned} E|I_0|^2 &= P_0 \left| \frac{1}{N_c} tr\left(Z[w]H[w,w]\right) \right|^2 \\ E|I_1|^2 &= \alpha\overline{P} \left(\frac{1}{N_c} tr\left(Z[w]H[w,w]H[w,w]^H Z[w]^H\right) - \left| \frac{1}{N_c} tr\left(Z[w]H[w,w]\right) \right|^2 \right) \\ E|I_2|^2 &= \frac{\alpha\overline{P}}{N_c} \sum_{\substack{s=0 \\ s\neq w}}^{S-1} tr\left(Z[w]H[w,s]H[w,s]^H Z[w]^H\right) \\ E|I_3|^2 &= \frac{\sigma^2}{N_c} tr\left(Z[w]Z[w]^H\right) \end{aligned} \tag{10}$$

$\alpha = N_u/N_c$ is the system load and $\overline{P} = \frac{1}{N_u - 1} \sum_{m=1}^{N_u - 1} P_m$ is the average power of the interfering users.

We will now exploit (10) to expand an analytical expression of the asymptotical SINR. First, we assume that the receiver uses a MMSE equalizer which coefficients are given by:

$$z_q[wN_F + p] = \frac{\phi^*(w,w,p,p,q)}{\left|\phi(w,w,p,p,q)\right|^2 + \gamma}$$

γ is the inverse of the SNR per sub-carrier : $\gamma = \frac{\alpha\overline{P}}{\sigma^2}$.

Expanding the expressions of (10), one obtains:

$$\begin{aligned} E|I_0|^2 &= P_0 \left| \frac{1}{N_c} \sum_{p=0}^{N_F-1} \sum_{q=0}^{N_T-1} \frac{|\phi(w,w,p,p,q)|^2}{|\phi(w,w,p,p,q)|^2 + \gamma} \right|^2 \\ E|I_1|^2 &= \alpha\overline{P} \left(\frac{1}{N_c} \sum_{p=0}^{N_F-1} \sum_{n=0}^{N_F-1} \sum_{q=0}^{N_T-1} \left| \frac{\phi(w,w,p,p,q)}{|\phi(w,w,p,p,q)|^2 + \gamma} \right|^2 |\phi(w,w,p,n,q)|^2 - \left| \frac{1}{N_c} \sum_{p=0}^{N_F-1} \sum_{q=0}^{N_T-1} \frac{|\phi(w,w,p,p,q)|^2}{|\phi(w,w,p,p,q)|^2 + \gamma} \right|^2 \right) \\ E|I_2|^2 &= \frac{\alpha\overline{P}}{N_c} \sum_{\substack{s=0 \\ s\neq w}}^{S-1} \sum_{p=0}^{N_F-1} \sum_{n=0}^{N_F-1} \sum_{q=0}^{N_T-1} \left| \frac{\phi(w,w,p,p,q)}{|\phi(w,w,p,p,q)|^2 + \gamma} \right|^2 |\phi(w,s,p,n,q)|^2 \\ E|I_3|^2 &= \frac{\sigma^2}{N_c} \sum_{p=0}^{N_F-1} \sum_{q=0}^{N_T-1} \frac{|\phi(w,w,p,p,q)|^2}{\left| |\phi(w,w,p,p,q)|^2 + \gamma \right|^2} \end{aligned} \tag{11}$$

4. SYSTEM PERFORMANCE WITHOUT CHANNEL CODING

In this section, we will validate our theoretical model given previously by means of simulations. We first compare the SINR computed with (8) and (11) with the instantaneous SINR measured via Monte Carlo simulations. However, for a selective fading physical channel and due to the variations of the channel caused by the mobile speed, we give the comparison between the average SINRs. The simulations assumptions in the sequel are the followings:

- FFT size: N=64, Spreading factor: N_c= 32 chips, QPSK modulation, sub-carrier spacing=312.5KHz.
- Scrambling code: concatenation of 19 Gold codes of 128 chips each.
- Spreading schemes: MC-CDMA: (N_F=32, N_T=1), OFDM-CDMA: (N_F=8, N_T=4), MC-DS-CDMA: (N_F=1, N_T=32).

Figure 3 illustrates the comparison between theoretical and simulated average SINRs for a BRAN A channel [15], a mean ratio E_b/N_0=20dB and a full load system. The SINRs have been measured in different sub-bands. Figure 3 shows that our theoretical model matches perfectly with simulations, even for a relatively small spreading factor (N_c=32). Moreover, it shows that the SINR sensitivity to Doppler spread becomes noticeable for a relative (to sub-carrier spacing) Doppler spread NF_dT_s=0.03.

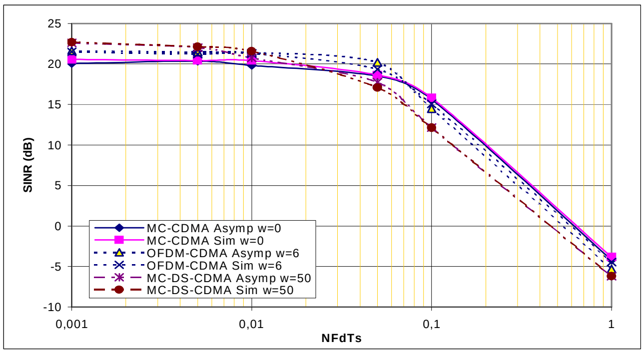

Figure 3: validation of theoretical model (BRAN A).

Since the theoretical model has been validated, we will exploit equation (11) to give more insights of the Doppler spread. Thus, we define the instantaneous degradation by $Deg_{(dB)} = 10 * \log_{10}\left(\frac{SINR_{\max}}{SINR}\right)$ where $SINR_{\max}$ is the SINR obtained for a slowly time-varying channel. Figure 4 gives the comparison between degradations of different spreading schemes to Doppler spread for a BranA channel model, a mean ratio E_b/N_0=20dB, and a full load. It is easy to conclude that the sensitivity of the 3 spreading schemes to Doppler spread is comparable. For a fixed Doppler spread, this sensitivity increases with the number of subcarriers N and a loss of 1dB appears for NF_dT_s=0.03.

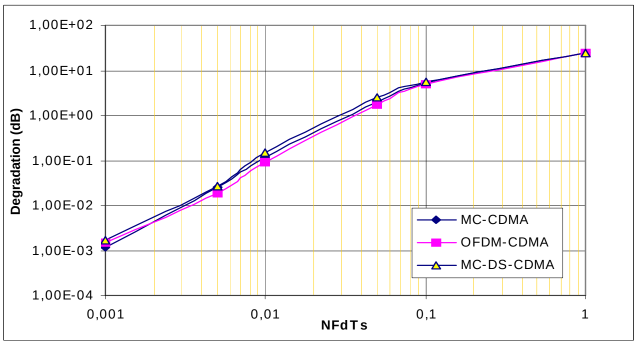

Figure 4: comparison between degradations of different spreading schemes.

5. THE EESM TECHNIQUE

In previous section, we derived a simplified formula for the estimation of the SINR at the detector output. However, it is desirable to evaluate the system level performance after channel decoding in terms of the Block/Bit Error Rate BER. Surely, explicit Bit or Block level simulation of each mobile in every cell of the network would be forbidding time consuming. Thus, it is very important to accurately abstract link level performance into system level studies. Usually, the performance of radio links has been evaluated in terms of BER as a function of the SINR averaged over the all channel realizations for a given channel model. This relationship has widely be used as the interface between the link and the system level simulations. Nevertheless, this relationship is not always true, the specific channel realization may result in a performance which is different from that predicted from the average SINR. Therefore, an accurate relationship between the SINRs obtained at the output of the detector and the BER performance at the output of the channel decoder must be identified.

In an OFDM system, it was concluded that the key issue to accurately estimate the system level performance is to be able to establish a relationship between the instantaneous SINR to a corresponding block error probability [16]. Instead of directly finding the block or bit error probability, it was proposed to use an effective SINR mapping which would be able to map a set of instantaneous SINRs (such as SINRs on different subcarriers) to an instantaneous effective SINR. The effective SINR will then be used in combination with AWGN curves to determine the appropriate Block/Bit Error Rate BER after channel decoding. [17] proposes an Exponential Effective SINR Mapping EESM technique which is based on the Chernoff Union bound [18] to find the effective SINR. The key EESM technique expression relevant to an OFDM system is given by

$$SINR_{eff} = -\lambda \ln\left(\frac{1}{N}\sum_{n=0}^{N-1}\exp\left(-\frac{SINR[n]}{\lambda}\right)\right) \qquad (12)$$

Where N is the number of sub-carriers, SINR[n] is the SINR obtained over the n^{th} sub-carrier and λ is a unique parameter which must be estimated from the system level simulations for every Modulation and Coding Scheme MCS. The uniqueness of λ for every MCS is derived from the fact that the effective SINR must fulfill the approximate relation

$$BER\left(\{SINR[n]\}\right) = BER_{AWGN}\left(SINR_{eff}\right) \qquad (13)$$

Where BER is the actual Block/Bit error probability for the instantaneous SINR set {SINR(n)} and BER_{AWGN} is the Block/Bit error probability for the AWGN channel which depends only on the MCS.

In our study, the EESM technique must be adapted to an OFDM-CDMA context. First, a unique effective SINR (called $SINR_{eff}$) is computed for each coded set of J "2D-symbols", each composed of S QAM symbols (**Figure 5**), accordingly to

$$SINR_{eff} = -\lambda \ln\left(\frac{1}{JS}\sum_{j=0}^{J-1}\sum_{w=0}^{S-1}\exp\left(-\frac{SINR_j[w]}{\lambda}\right)\right) \qquad (14)$$

Where $SINR_j[w]$ is the SINR of the w^{th} QAM symbol transmitted on the w^{th} sub-band of the j^{th} 2D symbols. J is the number of 2D symbols used once for decoding. The $SINR_{eff}$ is then used to estimate the BER at the output of the channel decoder by using a simple Look-Up Table LUT. This LUT is obtained analytically or by simulations for a AWGN channel. The parameter λ can be obtained by simulations for every MCS. Due to the relation (13) which uses the AWGN BER curve as a reference curve, λ is independent on the channel type. Moreover, it is independent of the Doppler spread or any synchronization error type. The choice of λ can be seen as a verification of Minimum Mean Square Error criterion according to the relation:

$$\lambda_{opt} = Arg \min\left\{\overline{BER_{sim}} - \overline{BER}(\lambda)\right\} \qquad (15)$$

Where K is the number of useful simulated blocks of J "2D-symbols", $\overline{BER}_{sim}$ and $\overline{BER}(\lambda)$ are respectively the obtained by simulation over the channel realizations and the averaged BER for a given λ. Using the asymptotic SINR formula given in (11) in combination with the EESM technique (14), we are now able to predict the BER at the output of the channel decoder without achieving system level simulations.

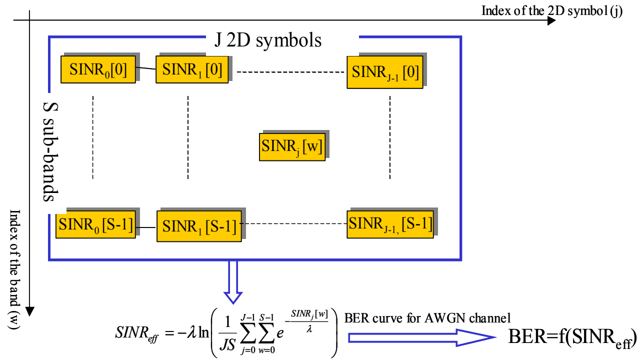

Figure 5: Effective SINR computation scheme.

6. System Performance with Channel Coding

In this section, we validate our modified EESM technique by system simulations. The simulation assumptions are the same of the previous ones to which we add the followings:

- QPSK and 16 QAM modulation.
- Convolutional Codes with coding rate R=1/2.
- Channel coding Memory equal to 6.

Figure 6 and Figure 7 give the comparison between results with channel coding obtained with EESM technique using the asymptotic SINRs and Monte Carlo simulations for a BranA channel, and different values of relative Doppler spread. It shows that the results obtained with EESM technique perform very well with Monte Carlo simulations. Moreover, the parameter λ is independent of the spreading system and the Doppler spread value. One can easily remark that the performance of MC-CDMA becomes worsen for higher Doppler spread. Surprisingly, the performance of MC-DS-CDMA is improved for higher Doppler spread. This can be explained as follows. In a frequency selective fading channel, frequency domain spreading (like MC-CDMA) leads to MAI that degrades performance while code orthogonality is maintained with time domain spreading (like MC-DS-CDMA) for slowly fading channel (small Doppler value). For fast time-varying channel, the MC-CDMA suffers more from the MAI while for the MC-DS-CDMA, the time diversity outperforms MAI and IBI. This diversity is exploited by the channel decoder to improve system performance. The OFDM-CDMA system presents an intermediate position i.e. the system performance is conserved until a certain value of Doppler spread and thus its sensitivity increases with higher Doppler spread as depicted with Figure 8.

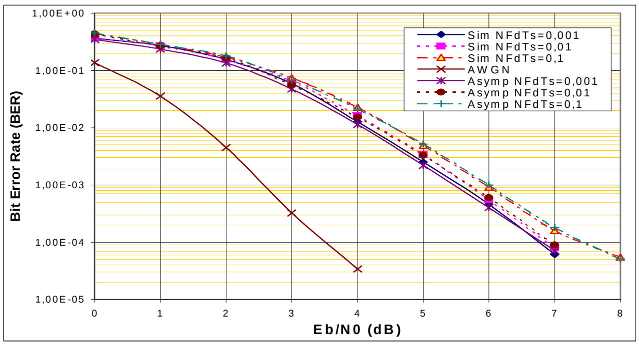

Figure 6: Validation of the EESM technique (MC-CDMA, QPSK).

For higher constellations, it is known that spreading systems are very sensitive to MAI. That is, increasing Doppler spread means an increasing of the MAI value. Figure 9 gives the sensitivity of MC-CDMA system to Doppler spread with the EESM technique and Monte Carlo Simulation. A loss of 0.4dB appears when the Doppler spread passes from NF_dT_s=0.001 to NF_dT_s=0.01at a BER=10^{-4}. This loss increases enormously for higher Doppler spread. For a MC-DS-CDMA system, a similar result to the MC-CDMA system appears since the MAI is the dominating term. Thus, the sensitivity increases with Doppler spread as depicted in Figure 10. A loss of 1dB appears when the Doppler spread passes from NF_dT_s=0.001 to NF_dT_s=0.01at a BER=10^{-4}.

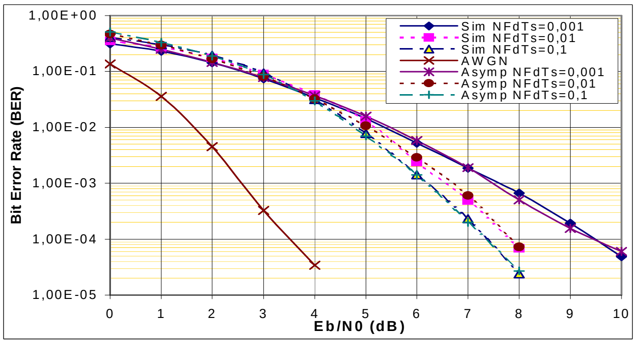

Figure 7: Validation of the EESM technique (MC-DS-CDMA, QPSK).

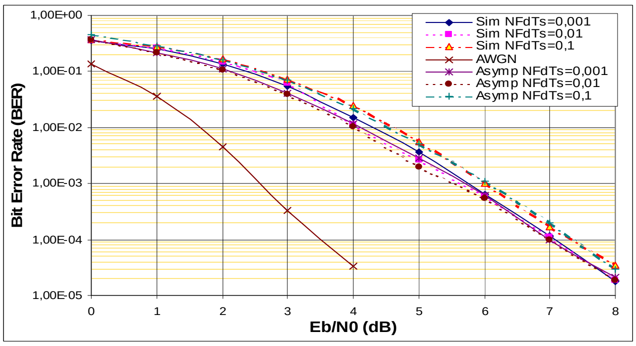

Figure 8: Validation of the EESM technique (OFDM-CDMA, QPSK)

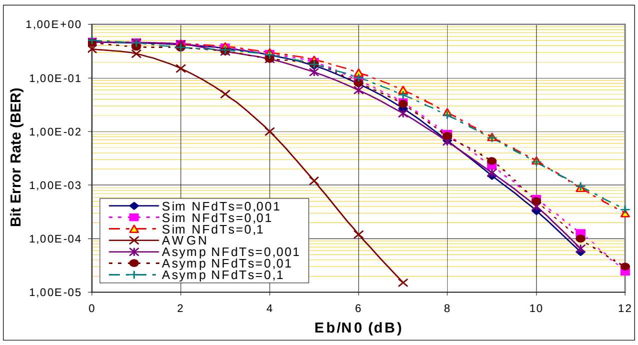

Figure 9: Validation of the EESM technique (MC-CDMA, 16-QAM).

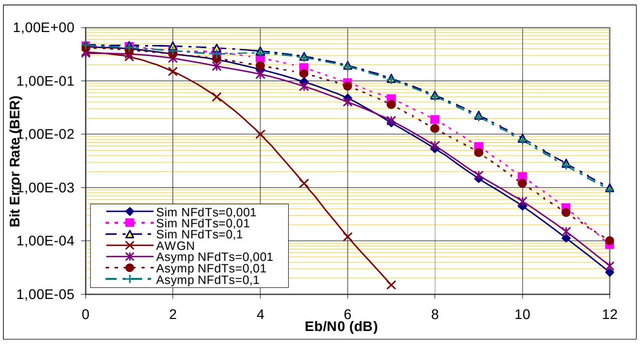

Figure 10: Validation of the EESM technique (MC-DS-CDMA, 16-QAM).

7. CONCLUSION

In this article, we have investigated the effect of time-varying multipath channel on the performance of 2D OFDM-CDMA spreading schemes. We have determined an analytical expression of the SINR which is independent of the spreading codes and showed that its results are conformed to Monte Carlo simulations. Also, we presented helpfully technique to establish the relation between the link level and the system level simulations. Eventually, we showed that different spreading schemes are sensitive to channel time variation. This time variation would be beneficial in MC-DS-CDMA with QPSK modulation however it introduces a performance loss in other simulation assumptions.

8. APPENDIX

In this section, the 3 properties from the random matrix and free probability theories are first defined. Then their application for the computation of (11) is detailed.

Property 1: If A is a $N_c \times N_c$ uniformly bounded deterministic matrix and $C_m = \frac{1}{\sqrt{N_c}}(c_m(0),\ldots, c_m(N_c-1))$ where $c_m(l)$'s are iid complex random variables with zero mean, unit variance and finite eighth order moment, then [9]:

$$C_m^H A C_m \xrightarrow[N_c \to \infty]{} \frac{1}{N_c} tr(A) \quad (16)$$

C_0 is obtained by the multiplication of a Walsh-Hadamard sequence with a long scrambling code. Hence, the assumptions needed for (16) are easily satisfied. This property is used to evaluate $E|I_0|^2$, $E|I_1|^2$, $E|I_2|^2$ and $E|I_3|^2$.

Property 2: Let C be a Haar distributed unitary matrix of size $N_c \times N_u$ [10]. $C=(C_0,U)$ can be decomposed into a vector C_0 of size N_c and a matrix U of size $N_c \times (N_u-1)$. Given these assumptions, it is proven in [11] that:

$$UQU^H \xrightarrow[N_c \to \infty]{} \alpha \overline{P}\left(I - C_0 C_0^H\right) \quad (17)$$

$\alpha = N_u/N_c$ is the system load and $\overline{P} = \frac{1}{N_u - 1}\sum_{m=1}^{N_u-1} P_m$ is the average power of the interfering users. This property is used to evaluate $E|I_1|^2$.

Property 3: If C is generated from a $N_c \times N_c$ Haar unitary random matrix then matrices $Z[w]H[w,s]H^H[w]Z^H[w]$ and $C[s]QC^H[s]$ are asymptotically free almost everywhere [13]. In other words, verifying the above conditions, one concludes:

$$\begin{aligned}&\frac{1}{N_c} tr\left(Z[w]H[w,s]C[s]PC^H[s]H[w,s]^H Z[w]^H\right) \xrightarrow[N_c \to \infty]{} \\ &\frac{1}{N_c} tr\left(Z[w]H[w,s]H[w,s]^H Z[w]^H\right) \times \frac{1}{N_c} tr\left(C[s]PC^H[s]\right)\end{aligned} \quad (18)$$

For definition of freeness, the reader may refer to [10] for more details.

Assuming that $C_0[w]$ is random (16) is used to evaluate $E|I_0|^2$. Since we use a long scrambling code, $C_0[w]$ and $C[s]$ are independent for $w \neq s$. Using (16), $E|I_2|^2$ becomes :

$$E|I_2|^2 = \sum_{\substack{s=0 \\ s \neq w}}^{S-1} tr\left(Z[w]H[w,s]C[s]PC^H[s]H[w,s]^H Z[w]^H\right) \quad (19)$$

Applying (17) for the computations of $E|I_1|^2$ and (18) for the computations of $E|I_2|^2$ in (19), (11) is obtained.

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