Improving urban public transport performances by tendering lots: a cost function panel data estimation

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Abstract

In order to reduce their public transport costs, some cities want to multiply the number of call for tender they organise, by dividing their network in several lots ("allotment"). In terms of costs-benefit analysis, gains obtained by introducing more competition for the market should be compared with the increasing transaction costs. But cutting a network into several parts have also basic consequences on potential returns to scale. In this paper, we estimate a translog cost function according to a panel of 141 French urban public transport networks. Our main conclusion is that scale economies are exhausted for a production of about one million vehicles-kilometres per year. Therefore, in terms of scale economies, allotment would reduce costs of public transport services for the most of the cities of our sample. The main limit of this result is that we are not strongly consider competitive gain and transaction costs.

Keywords: cost function, return to scale, public transport, allotment, GMM

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1 Introduction

In order to provide public transport services, 90% of French urban transport authorities use a tendering procedure, which is legally required in order to contract with a private operator (Roy & Yvrande-Billon 2007). But while an increasing number of European cities work with more than one operator (London, Stockholm, Helsinki, Copenhagen), none of the French transport authority moved to this form of governance, that is called "allotment"[1]. Typically, public transports in cities like Lyon, Lille, Toulouse or Bordeaux are only operated by one single private operator.

However, during the last few years, the concentration of the industry has reduced the number of potential bidders: the three main companies hold now a 75% market share. And as a consequence or not, urban public transport costs increased dramatically during the same period. One of the possibilities in favour of competitive pressures (including benchmarking) and cost reducing, is to divide networks into small and attractive parts. This is the governance scheme that we want to analyse in this contribution.

Allotment will probably increase transaction costs and reduce market power of local monopolies. But it has also consequences on the cost minimisation process. Unit production costs depends on technological returns to scale. So before studying the other aspects[2], we will consider that an allotment could be desirable if it is not disconnected from the industry natural monopoly frontiers.

Technologically, an industry is said to be a natural monopoly if, over any relevant vector of outputs Y_k, the cost function $C(Y)$ is subadditive:

$$C(\sum_{k=1}^{K} Y_k) < \sum_{k=1}^{K} C(Y_k)$$

So in order to handle the question of urban transport natural monopoly frontiers, the central point deals with the industry cost function structure. The literature contains many single-product[3] estimations for transport industry (Braeutingam 1999, Pels & Rietveld 2000). Some contributions are unavoidable, particularly about rail-

[1] Allotment consists of a fragmentation of the unique call for tender into several ones.

[2] This research has been partly supported by the PREDIT (http://www.predit.prd.fr), and is part of a research program that also discuss the other aspects and consequences of allotment.

[3] A next paper will discuss the more complete but also more complex multiproduct case

ways and airways (Caves, Christensen & Swanson 1980 and 1981, Caves, Christensen & Tretheway 1984), and a large number of econometric estimations have been realised on urban transport samples (Karlaftis 2001, De Borger, Kersterns & Costa 2002).

This paper studies the operating costs of the French urban public transport networks. A translog cost function is estimated with a 1995-2002 panel data. As a result, we measure several significant diseconomies of scale in the production of vehicles-kilometres. So, the classical argument of a local natural monopoly seems to be limited.

We will first (section 2) come back to the essential lessons on returns to scale provided in the literature applied to urban public transport industry. We will then present in section 3 the translog cost function that will be estimated. In section 4, we discuss the econometric methodology chosen, particularly the advantages of panel data. The data are presented in section 5. The results of panel estimations are discussed in section 6.

2 Optimal size: a single-product literature review

Empirical analysis considering an unique output are limited to study the heterogenous production of public urban transport services. However, this kind of econometric studies have some recurrent results that are an important basis in order to understand cost formation and cost functions. Indeed, the literature contains lessons on scale economies in the industry that should not be ignored.

2.1 Definition of optimal size

In the single-product case, returns to scale are generally[4] defined by the ratio between average cost and marginal cost, that is the inverse cost elasticity of production (ϵ_Y):

$$RTS = \frac{C}{Y.\partial C/\partial Y} = \frac{1}{\partial \ln C/\partial \ln Y} = \frac{1}{\epsilon_Y}$$

Consequences are the followings:

- If $RTS < 1$: decreasing returns to scale, diseconomies of scale

[4]Returns to scale are measured in some paper by: $-\frac{\partial \ln(C(Y)/Y)}{\partial \ln Y} = 1 - \epsilon_Y$

- If $RTS = 1$: constants returns to scale
- If $RTS > 1$: increasing returns to scale, economies of scale

In addition, economies of scale in railway industry are classically decomposed into returns to density and returns to size (Keeler 1974, Caves Christensen & Tretheway 1984). Returns to density (RTD) measure the evolution of costs when the level of production change, given a constant network of infrastructures. Returns to size (RTS) measure the evolution of costs when a network is enlarging, given traffic constant per lane. In this objective, returns to density do not include the variation of the length of lines (LL):

$$RTD = \frac{1}{\epsilon_Y}$$

$$RTS = \frac{1}{\epsilon_Y + \epsilon_{LL}}$$

Some urban transport studies include this decomposition (Fazioli, Filippini & Prioni 1993, Levaggi 1994, Matas & Raymond 1998, Gagnepain 1998, Jha & Singh 2001, Karlaftis & McCarthy 2002, Filippini & Prioni 2003), but it is not clearly relevant. Indeed, infrastructure costs are specific only for some modes (subway, tramway, trolleys...), not for buses. And those costs are not supported by the operators (at least in the French urban transport industry). Moreover, the hypothesis of a constant level of infrastructure is only valid in a short term. So we think that this decomposition is not that helpful in our context.

2.2 Returns to scale and output measurement

The literature review presented in the annexed table show substantial differences between the numerous estimations realised. This diversity is in particular due to the variety of methodologies used by authors, and to the different samples considered. Yet, two results seem to be stable.

First, level of returns to scale depends on the output selected. Berechman & Guiliano (1984) observed diseconomies in terms of vehicles-kilometres; while considering receipts per passenger they noticed economies of scale. This result is confirmed in Karlaftis, McCarthy & Sinha (1999a) study, who estimated returns to size and density by passengers and vehicles-miles. So there seems to be higher returns to scale with a demand-oriented outputs (trips, journeys, receipts pere passenger or passenger-kilometres) than with supply-oriented outputs (vehicles-kilmetres or seats-kilometres).

Second, in most of the cases, returns to scale decrease with total production (Viton 1981, Button & O'Donnell 1985, Thiry & Lawarree 1987, Filippini Maggi & Prioni 1992, Fazioli Filippini & Prioni 1993, Matas & Raymond 1998, Karlaftis McCarthy & Sinha 1999a, Jha & Singh 2001), mainly when it deals with supply oriented output. So generally returns to scale are increasing, except for some studies from the 1980's: Williams & Dalal (1981) and Obeng (1984, 1985).

The main characteristics of those empirical studies are given in appendix.

In total, one of the most essential facts in order to discuss returns to scale levels is the choice of output. Some authors argue that demand-related indicators (e.g. passenger-km or number of passengers) are more relevant than pure supply indicators (e.g. vehicle-km or seat km) because they take into account the economic motivation for providing services. Ignoring demand may lead to consider that the most efficient operators are those whose buses are empty.

Vehicles-kilometres operators seem to have smaller optimal sizes than demand-oriented output producers do. So in terms of passengers, networks need to be more integrated, but vehicles-kilometres operation should be done by smaller companies. This may explain why some European cities like Helsinki, London or Stockholm have adopted an institutional scheme with an organising authority controlling all the demand side, while several operators run vehicles on the lots (after a call for tender).

As a conclusion of this literature review, two main points have been identified about returns to scale in the urban transport industry. On the one hand, the optimal size in terms of "demand side output" has a weak probability to be smaller than the whole urban area size. On the other hand, returns to scale in terms of vehicle-kilometres ("supply side output") production has a quite high probability to do not imply a natural monopoly. So we will concentrate our investigations in measuring the optimal size in terms of vehicle-kilometres.

3 The translog total cost function

Standard microeconomics defines the cost function as a minimum cost for each production level, given technology and relative factor prices:

$$C(Y,W) = \min_X W.X \text{ under constraint } Y = f(X)$$

where $C(.)$ represents the cost function, Y is the vector of outputs levels, W is the vector of input prices, X is the vector of input quantities and $f(.)$ is the production function.

This function has the following properties:

- Monotonicity: Cost function is non-decreasing in prices (positive gradient).
- Homogeneity: For a given output, if all the input prices increase in the same proportion, total costs increase by this proportion.
- Concavity: The hessian matrix is negative semidefinite

Cost structure analysis requires a flexible functional form, minimising *a priori* restrictions and in particular unconstrained in terms of returns to scale and elasticity of substitution (Berndt & Khaled 1979). Christensen & Green (1976), in their study applied to electricity production, have for instance shown the translog[5] cost function ability to treat in a relevant way the question of scale economies.

The translog cost function had been used in the large majority of studies (see appendix) needing a flexible cost function.

[5] The *transcendental logarithmic* cost function had been introduced by Berndt, Christensen, Jorgensen & Lau. Guilkey, Lovell & Sickles (1983) demonstrated the reliability of the translog, compared to some other flexible functional forms, by a Monte Carlo simulation. However, the translog is still a second order approximation which could be generalised (Piacenza & Vannoni 2004)

A translog cost function is defined by:

$$\ln C = \alpha_0 + \sum_{k=1}^{K} \beta_k \ln Y_k + \sum_{n=1}^{N} \alpha_n \ln W_n$$
$$+ \frac{1}{2}\sum_{k=1}^{K}\sum_{l=1}^{K} \beta_{kl}(\ln Y_k)(\ln Y_l) + \frac{1}{2}\sum_{n=1}^{N}\sum_{n=1}^{N} \alpha_{nm}(\ln W_n)(\ln W_m)$$
$$+ \sum_{k=1}^{K}\sum_{n=1}^{N} \gamma_{kn}(\ln Y_k)(\ln W_n)$$

where C represents total costs, Y the vector K outputs and W the vector N input prices.

Two more *a priori* conditions are assumed:

- the matrix of coefficients is symmetric (Young theorem): $\beta_{kl} = \beta_{lk}$ and $\alpha_{nm} = \alpha_{mn}$
- homogeneity of degree 1 in price implies the following conditions (Euler theorem): $\sum_{n=1}^{N} \alpha_n = 1$; $\sum_{m=1}^{N} \alpha_{nm} = 0, \forall k$; $\sum_{n=1}^{N} \gamma_{nk} = 0, \forall k$.

Elasticities of scale ϵ_{Y_k} for each output are:

$$\epsilon_{Y_k} = \frac{\partial \ln C}{\partial \ln Y_k} = \beta_k + \sum_{l=1}^{K} \beta_{kl} \ln Y_l + \sum_{n=1}^{N} \gamma_{kn} \ln W_n$$

In addition, from a translog cost function, it is easy to determine the demands of input. The Shephard lemma implies equality between each input cost share (S_n) and the partial logarithmic derivation of cost in its input price:

$$S_n = \frac{\partial \ln C}{\partial \ln W_n} = \alpha_n + \sum_{m=1}^{N} \alpha_{nm} \ln W_m + \sum_{k=1}^{K} \gamma_{kn} \ln Y_k \text{ with } W_n > 0$$

The simultaneous estimation of cost shares and the cost function increases the number of degrees of freedom, and to improve the quality of the estimators. Parameters of the $n-1$ cost share equations are estimated simultaneously (Christensen & Green 1976) by Zellner (1962) method[6] (or SURE: Seemingly Unrelated Regressions).

At last, many studies used variable cost functions which consider a fixed input, typically the fleet size. But despite its apparent relevance, this choice implies serious difficulties. And those difficulties conducted us to reject this approach. Indeed,

[6]Results are asymptotically equivalent to the maximum likelihood estimator.

most of variable cost estimations conduct to a positive effect of fleet variable on cost (Viton 1981, Levaggi 1994, Kumbhakar & Bhattacharyya 1996, Karlaftis & McCarthy 2002, Fraquelli Piacenza & Abrate 2004, Piacenza 2006), which is clearly unrealistic. It means that buying more vehicles conduct to more variable costs! Credible results (a negative effect) are rare (Obeng 1985, Gagnepain 1998), so the literature leads to consider that variable cost function estimations including fleet size have a bias.

The bias seems to come from a misunderstanding about the production capacity choice. Urban transports require different levels of production during the day. And the volume of vehicles is decided in order to have a sufficient capacity during peak hours. As a consequence, "These proxy-variables for the capital stock reflect maximum available production capacity at one particular point in time and, therefore, are generally highly correlated with output increasing" (Filippini 1996). So if a network manager decides an increase in its peak-base ratio (with a constant amount of kilometres produced), both variable costs and fleet will increase. It is probably the effect measured by the fleet coefficient, but this is not what the variable cost function model assumes.

As a conclusion, total cost function seems to be better appropriate to study urban transport industry. The use of a variable cost function with fleet as proxy for capital raises more questions than it solves. Moreover, in the French case, investments (including buses) are directly granted by local governments. Operators produce a service by using public capital. As a consequence, their capital cost share in under 1%[7]. So, it is more adapted to run a total cost function in that case, which include indeed a very small amount of capital.

4 Econometric models and methods

Translog cost functions have been generally used for urban transport estimations (see appendix). Since the very often quoted paper of Viton (1981), econometric methods and models became more refined. On the one hand, specifications estimated

[7]For instance, the firm that operate Lyon's network (buses, subway and tramway) has a positive working capital (inventories - reveivables + payables), no financial debt and about 3 M€of equity. Returns on equity ratio was between 5% and 25% the last few years, which means a capital cost under 0.5% of the sum of labor, energy and maintenance cost (about 260 M€). On the other side, asset amortization is always under 1% of this amount.

are less and less restrictives : linear, Cobb-Douglas then translog. And on the other hand, data are more and more available. It allows to go beyond time series analysis or cross-section studies. As a result, standards are clearly oriented to flexible functions (typically translog) estimations on panel data (Thiry & Lawarree 1987, Kumbhakar & Bhattacharyya 1996, Matas & Raymond 1998, Karlaftis McCarthy & Sinha 1999a, Filippini & Prioni 2003).

Time series models reduce the analysis to a particular network, or imply a macro aggregation. Time series estimations had been historically used during the 1980's, according to macroeconomic data (Berechman 1983, De borger 1984) or local ones (Berechman & Guiliano 1984, Androkopoulos & al 1992). In the urban transport case, time series are standing a relatively small variance and are very sensible to local determinants.

Cross-section estimations give a really more interesting perspective of production structure. The joint study of firms with different sizes is more able than time series analysis to improve our understanding of scale economies. In return, cross-section analysis assume that firms have access to the same technology, produce the same type of services and are facing the same environment, which is sometimes a strong hypothesis and can be ease by using control variables[8] that limit effects of non relevant heterogeneity:

- Average commercial speed : Viton 1992, Levaggi 1994, De Rus & Nombela 1997, Gagnepain 1998, Fraquelli Piacenza & Abrate 2004, Piacenza 2005
- Load factor for a demand-oriented output: Levaggi 1994, Kumbhakar & Battacharyya 1996, Jha & Singht 2001
- Number of stops: Filippini, Maggi & Prioni 1992, Filippini & Prioni 2003
- Urban constraints (density, centrality): Levaggi 1994, Dalen & Gomez-Lobo 2003
- Institutional or organisational design (ownership, contract) : Kumbhakar & Battacharyya 1996, De Rus & Nombela 1997, Gagnepain 1998, Dalen & Gomez-Lobo 2003, Filippini & Prioni 2003, Piacenza 2005

[8]A screaming of the data sample (Williams & Dalal 1981), or a cluster design (Karlaftis & McCarthy 2002) can also be used

- Peak-base ratio: Button & O'Donnell 1985, Viton 1992, Karlaftis McCarthy & Sinha 1999a.

Estimations on panel data tend to stand out, thanks to their double individual and time dimensions. Panel models gather individual effects (for each network) in a specific term. So estimated coefficients are free of individual effects. And moreover, panel data allow us to take into account the influence of unobservable information. The additional hypothesis we need to assume is that unobservable characteristics of networks are constant during the time period considered. In our sample considering urban area, it is quite realistic to assume that some local components (geography, housing, urban structure) are constants in a short run. So panel data estimations have attractive properties and are quite adapted to the urban problematic.

5 Data

We used an unbalanced panel data of 959 observations on 141 French urban transport networks[9], for the years 1995 to 2002. This data set was gathered thanks to the annual surveys conducted by the *Centre d'Etude et de Recherche du Transport Urbain* (CERTU), a ministerial agency.

For a purpose of homogeneity, we have excluded non-urban traffics, and the small networks (under 30,000 inhabitants) that are assumed to have a different production function. In addition, several observations (network-year) were discarded because some data were missing or were suspected to be wrong after a careful scanning of the data. This database is the biggest and the most updated on the French urban public transport system. Its physical and institutional variables were used in a recent paper (Roy & Yvrande-Billon 2007). Since, we did collect and treat accounting data.

5.1 Prices and costs

In this data base, private firms itemised their costs, at least between labour and puchases. It is not individual input factor costs as we would have like to treat: labour, fuel, material, maintenance and administrative costs. But this is nevertheless better than lump-sum costs that are reported for instance the National Transit Database in the US.

[9]Data on Paris region are not available and included

Labour is the main input factor of the operators. Its price W_L is obtained by dividing annual labour expenses by the number of equivalent full time employees[10]. So labour price W_L is actually an average cost of labour for each network, ignoring the differences of wage structure.

The others expenses considered are dealing with energy and maintenance. Price of energy and maintenance W_A is the ratio between the total amount of purchases (including taxes) and the total number of kilometres covered by the vehicles of the network during the year.

We are not considering any price of capital, and cost of capital (typically amortisation and depreciation expenses) is not added to the cost variable C. Mainly, this kind of data is not properly filled out. And moreover, buses are generally owned by the organising authority. At last, we assume that capital cost is the same everywhere in France, and by the way it will not explain differences in costs.

In total, costs considered are only operating costs (€2002) in their main dimensions: labour, energy and maintenance. Table 1 shows the 959 observations panel we use. Around 50% of firms operating costs are between 1.5 millions€and 10 millions €. The last quartile is very large, as it includes networks with more than 100 millions €of expenses: Lille, Marseille (> 130 millions €) and Lyon (> 220 millions €). The mean price of labour is 33,400 €and the mean purchases price is 0.69€.

Table 1: Descriptive statistics

Variables	Min.	1st Quartile	Median	Mean	3rd Quartile	Max.
C: operating costs (€2002)	404,000	1461,000	3,480,000	12,430,000	10,810,000	226,100,000
W_L: labour price (€2002)	19,900	30,400	33,600	33,400	36,700	48,700
W_A: purchases price (€2002)	0.274	0.549	0.66	0.692	0.774	2.08
Km	206,000	619,600	1,320,000	3,434,000	4,005,000	45,390,000
- KmL: Light Rail Transit systems	0	0	0	219,500	0	10,950,000
- $KmBA$: articulated buses	0	0	0	487,800	393,500	6,745,000
- KmP: microbus and short buses	0	0	42,750	134,400	162,200	1,612,000

5.2 Output: vehicles-kilometres

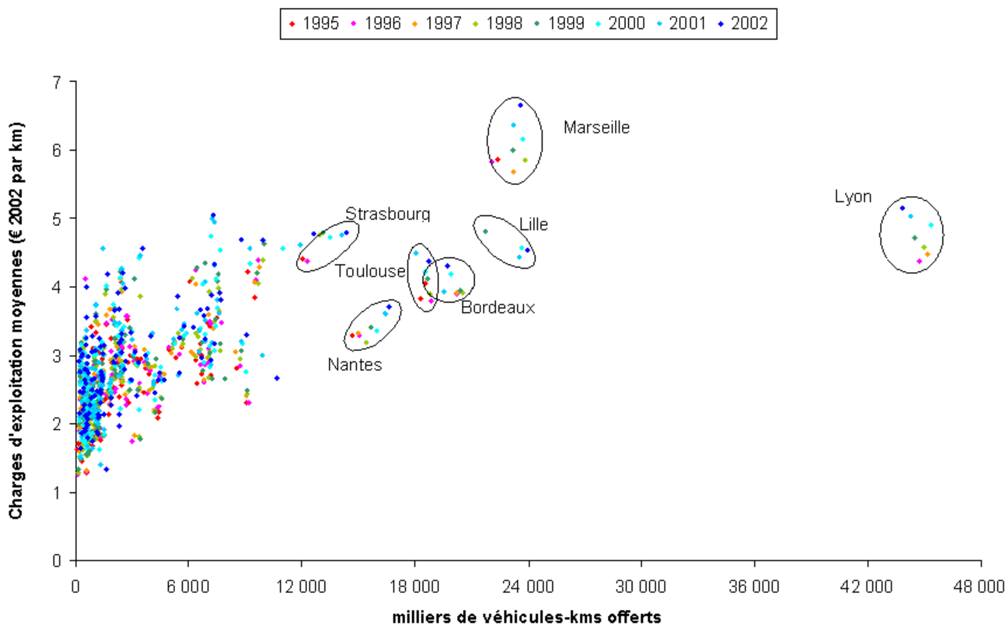
The definition of outputs had been largely debated in transportation literature as explained earlier. The output we retain in this paper is supply-orientated; it is the num-

[10] Including temporary work, excluding subcontracting personnel, and with no distinction between driving labour and non-driving labour.

ber of vehicle-kilometres. The main argument explaining our choice is that demand-related output will certainly conduct to a larger natural monopoly as we explained it in section 2.2.

The variable Km includes different kind of vehicles-kilometres. Some of them, KmL, are realised by different light rail transit (LRT) systems, mainly subway and tramway. We also distinguish in Table 1 kilometres produced with short buses or microbuses (KmP), and articulated buses ($KmBA$). Table 1 shows that only a few number of networks use the whole diversity of vehicles. Close to 50% of networks declare to do not operate articulated buses or small buses. Figure 1 shows the "gross" cost function extracted from those data.

Figure 1: Structure of the panel



Data used are not perfect, but the sample is large and quite complete. They will allow us to estimate returns to scale in the French urban transport industry, according to the advanced standards of a translog function on panel data.

6 Estimation results

We use a one way error component specification :

$$y_{it} = \beta' x_{it} + \epsilon_{it} \text{ with } \epsilon_{it} = \mu_i + \eta_{it}$$

- μ_i is a time invariant individual effect : regulation contracts, industrial group the network belongs to, geographic and climatic conditions, costs not directly related to output ("administrative costs", advertising, tickets selling, ...)
- η_{it} is the idiosyncratic error term : it may include for exemple the impact of technical problems or strikes on costs.

To estimate consistently the cost function, one has to investigate whether one or both of the effects are correlated or not with the explanatory variables.

- if "administrative costs" grow faster than production, the individual effect is positively correlated with the production : $\frac{\partial \mathrm{E}(\mu|x)}{\partial x} > 0$
- technical problems will result in an increase in costs but also maybe in a decrease of production. Therefore, the idiosyncratic term may be negatively correlated with the production $\frac{\partial \mathrm{E}(\eta|x)}{\partial x} < 0$

Supposing for the moment that $\mathrm{E}(\eta \mid x) = 0$, two models are available :

- the fixed effects models where the individual effect μ_i are estimated (or eliminated via a suitable transformation),
- the random effect model where the variance of the individual effects are estimated.

This two models deal with two different regression functions : $E(y \mid x, \mu)$ for the first one and $E(y \mid x)$ for the second one.

So the first model is consistent whether μ is correlated with x or not, but the second is consistent only if $E(\mu \mid x) = 0$. In the latter case, the random effects model is more efficient than the fixed effects model. Results are given in table 2. The following cost function is estimated at the mean values:

$$\begin{aligned}\ln c_{it} =& \beta_k \ln Km_{it} + \frac{1}{2}\beta_{kk}(\ln Km_{it})^2 + \alpha_p \ln P_{it} \\ &+ \frac{1}{2}\alpha_{pp}(\ln P_{it})^2 + \gamma_{kp}(\ln Km_{it})(\ln P_{it}) \\ &+ d_t + \epsilon_{it}\end{aligned}$$

where

- d_t: time effects
- $P_{it} = \frac{W_{L,it}}{W_{A,it}}$ and $c_{it} = \frac{C_{it}}{W_{A,it}}$ (price homogeneity condition)

Table 2: Estimation of the fixed and random effects model

	Random Effects		Fixed Effects	
	Estimate	t-value	Estimate	t-value
β_k	1.06	95.47	0.74	19.28
β_{kk}	0.07	5.45	0.01	0.36
α_p	0.55	45.02	0.58	48.86

The first order production coefficient (β_k) corresponds to cost elasticity of production (ϵ_Y) at the mean level of output. And returns to scale are defined as follow[11] for this translog specification:

$$RTS = \frac{1}{\epsilon_Y} = \frac{1}{\beta_k + \beta_{kk}\ln(Km_{it}) + \gamma_{kp}\ln(P_{it})}$$

The two models give very different results. In this case, it is generally suspected that the random effect model is inefficient because of the correlation of the individual effects with some of the explanatory variables. A Hausman test can be computed. The value of the statistic is 135.9 (10 degrees of freedom), so the hypothesis of consistency of the random effect model is clearly rejected.

There seems to be here a positive correlation, which induces that the production coefficient is much smaller in the within model, that “administrative” costs increase faster than production.

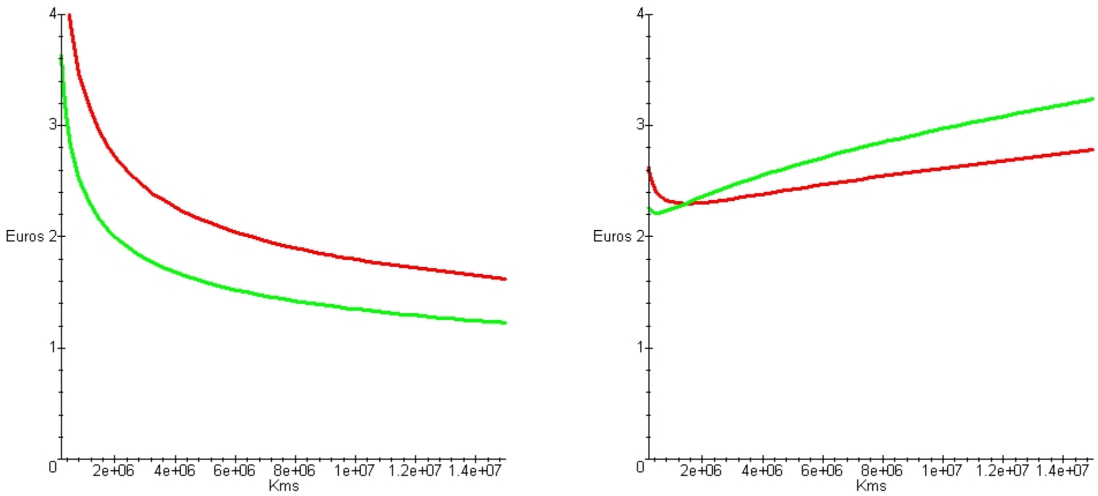
[11] It is in fact more simple for our estimations, as γ_{kp} is not significantly different from 0

But, from an economic point of view, the results of the within model are very questionable. The scale elasticity has a very high value (1/0.74 = 1.35) and is constant. The results of the random effects model seems to be much more relevant with a scale elasticity, which is decreasing and has a unit value at the sample mean.

The explanation of these unusual results may be that both models are inconsistent because they do not take into account the fact that the idiosyncratic term may be correlated with the production.

Figure 2: Average (red) and marginal (green) cost functions at the input price mean point

a) Within effects b) Random effects



If there is a negative correlation between the idiosyncratic term and the production and a positive correlation between the individual effects and the production:

- The production coefficient is clearly underestimate in the within model because only the second source of inconsistency is taken into account,
- For the random effect model, none of both sources of inconsistency are controlled, which means that the sign of the bias is not obvious.

To estimate consistently this model, one has to control both sources of inconsistency. We are here in a situation very similar to dynamic panel models, where one of the explanatory variable (the lag dependent variable) is correlated with the error term. These models are estimated by general method of moments estimator. First, the equation is transformed to get rid of the individual effect:

$$y_{it} = \beta' x_{it} + \mu_i + \epsilon_{it}$$

The within transformation is not suitable here because the error term in the transformed equation would contain the error of every periods:

$$y_{it} - \bar{y}_{i.} = \beta'(x_{it} - \bar{x}_{i.}) + (\epsilon_{it} - \bar{\epsilon}_{i.})$$

One uses instead the first difference transformation :

$$y_{it} - y_{i(t-1)} = \beta'(x_{it} - x_{i(t-1)}) + (\epsilon_{it} - \epsilon_{i(t-1)})$$

If no external instruments are available, one can use lag values of the production. All lags from $(t-2)$ are uncorrelated with the error term of the transformed equation and therefore can be used as instruments.

To this equation in differences with instruments in level, one can had an equation estimated in levels with instruments in differences to improve the efficiency of the estimator. Lags up to $(t-4)$ are used as instruments. The results of the two step estimator, using robust standard errors are are given in table 3.

Table 3: Blundel and Bound's GMM estimator

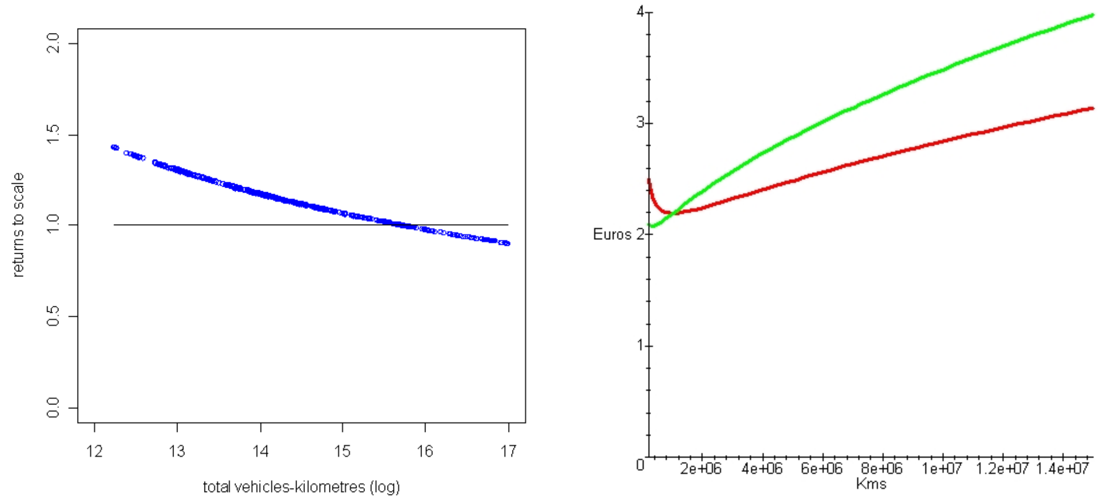
	Estimate	z-value
β_k	1.12	45.02
β_{kk}	0.10	2.17
α_p	0.58	12.27

Sargan Test: $\chi^2(43)$ = 57.87227 (p.value=0.06436798)
Autocorrelation test (1): normal = -4.798029 (p.value=8.011742e-07)
Autocorrelation test (2): normal = -1.268115 (p.value=0.1023784)
Wald test for coefficients: $\chi^2(3)$ = 11072.87 (p.value=0)
Wald test for time dummies: $\chi^2(6)$ = 59.32911 (p.value=6.159606e-11)

At the 5% level, the hypothesis of no overidentification and of no autocorrelation of order 2 are not rejected.

The results indicate a scale elasticity of about 0.9 at the sample mean. Moreover the second order coefficient for the production is significantly positive, which implies that the average cost curve is U–shaped and that there is an optimal size for the urban transportation firm. This optimal size is of 1,079,207 kilometers, which is close to the median production (1,320,000).

Figure 3: Returns to scale and cost curves, GMM estimator results,
a) Returns to scale b) AC (red) and MC (green) functions



7 Conclusion

In order to study the opportunity of urban public transport allotment, the aims of this paper was to evaluate the natural monopoly frontiers in this industry in France. According to a review of literature that mainly shows that a supply-oriented output was more appropriate to fragment networks, we estimated a translog cost function.

Within and random model were estimated on a large panel of French data but we showed that both models are inconsistent because of the correlation of the error terms with the production.

We therefore use a GMM estimator which induce a U shape for the average cost and an optimal size for urban transit firms of about 1 million vehicules-kilometers.

However, this study is restricted to the production side, and other determinants like transaction, and information costs and benefits are not taken into account. Further researches into those directions should give us a clearer idea of what is desirable. We are also investigate the field of multiproduct cost functions in order to find more detailed results.

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Appendix

Table 4: A cost function (single output) estimation survey

Authors	Models estimated	Data	Outputs (average) [range]	Main results[1] (average) [range]
Viton (1981) *Journal of Industrial Economics*	Translog Variable cost SURE Cross-section	54 operators 1975 USA Urban + periphery	Vehicles-miles (11.73 millions) [0.168 to 88.5] + fleet	RTD^{CT}= (1.78) [1.67 to 1.93] RTD^{LT}= [1.16 (small) to 0.87 (big)] ϑ_{LF} = [0.22 to 0.56] η_{Lw} = [-0.03 to -0.19] η_{Fe} = [-0.19 to -0.57]
Williams & Dalal (1981) *Journal of Regional Science*	Translog Total cost SURE Cross-section	20 operators publics 1976 Illinois USA Bus	Vehicles-miles Small and medium networks: < 4 millions	RTS [0.60 (small) to 2 (medium)] ϑ_{LF}= [ns to 0.060] ϑ_{LM}= [-2.02 to -2.07] ϑ_{KM}= [2.03 to 2.26] $\vartheta_{LK} = \vartheta_{MF} = \vartheta_{FK}$ = ns
Berechman (1983) *Journal of Transport Economics and Policy*	Translog Total cost SURE Time series	Quarterly national data 1972-1979 Israel Urban and interurban	Gross receipt (millions of shekels 1969) [69.7 to 103]	RTS^{LT}= (1.85) ϑ_{LK}=[-0.024 to -0.214] η_{Lw}=[-0.007 to -0.046] η_{Kr}=[-0.432 to -0.451] η_{Lr}=[-0.015 to -0.157] η_{Kw}=[-0.008 to -0.056]
De Borger (1984) *Journal of Industrial Economics*	Translog Variable cost SURE Time series	Annual data 1951-1979 Belgium SNCV: regional buses	Seats-kilometres	RTD^{CT} = [0.34 to 5.29] ϑ_{LF} = [0.316 to 0.703] η_{Lw} = [-0.135 to -0.023] η_{Fe} = [-0.568 to -0.293]
Berechman & Giuliano (1984) *Transportation Research Part B*	Translog Total cost SURE Time series	Quarterly data 1972-1979 San Francisco USA	Vehicles-miles then receipt per passenger 800 bus	RTS^{V-M}= (0.696) $RTS^{R/P}$= (1.22) ϑ_{LF} = [-0.03 to 0.11] η_{Lw} = [-0.002 to -0.04] η_{Fe} = [-0.05 to -0.12]
Button & O'Donnell (1985) *Scottish Journal of Political Economy*	Translog Total cost SURE Cross-section	44 networks 1979-1980 United-Kingdom 44 districts	receipt per passenger + peak/base ratio and density	RTS=[0.9 (big) to 1.4 (small)] ϑ_{LK}=(0.305) ϑ_{LM}=(0.657) ϑ_{MK}=(-0.339) Weak price elasticities
Obeng[2] (1985) *Journal of Transport Economics and Policy*	Translog Variable cost Cross-section	62 operators 1982 USA Urban + periphery	Passengers-miles Firms from 25 to 600 vehicles	RTS^{CT}=[0.75 (small) to 4.17 big)] RTS^{LT}=[0.55 to 0.72] ϑ_{LK}= [0.497 to 0.708] η_{Lw}= [-0.164 to -0.218] η_{Fe}= [-0.441 to -0.474] η_{Le}= [-0.087 to -0.328] η_{Fw}= [-0.379 to -0.46]

Quantities: L = labour, F = fuel, M = maintenance and K = capital; Prices: w = travail, e = fuel and r = capital; ϑ Allen's elasticity of substitution; M Morishima's elasticity of substitution; η price-elasticity of input demand; SFA: Stochastic Frontier Analysis; TFP: Total Factor Productivity Analysis; $SURE$: Seemingly Unrelated Regressions; ML: Maximum Likelihood

Previous table continuation

Authors	**Models estimated**	**Data**	**Outputs (average) [range]**	**Main results**[1] **(average) [range]**
Berechman (1987) *Regional Science and Urban Economics*	Translog Total cost ML Time series	Quarterly national data 1972-1981 Israel Urban, suburban and inter-urban buses	Vehicles-kilometres (93.9 millions in 1972) Journeys (7.93 millions in 1972)	RTS^{VK}= [1.7 to 2] ϑ_{LF} = [-1.6 to ns]; ϑ_{MK} = [-0.80 to ns] RTS^{Jou}= [1.2 to 2.86] ϑ_{LK} = [ns to 0.32]; ϑ_{MF} = [0.39 to 0.76]; ϑ_{ML} = [-0.25 to 0.25] ϑ_{FK} = [0.26 to 0.91]
Thiry & Lawarree (1987) *Annales de l'économie publique, sociale et coopérative*	Translog Variable cost SURE Panel	5 operators 1962-1986 Belgium Bus, tramway, subway and trolley	Seats-kilometres (5 034 to 0.310 millions in 1986)	RTS^{LT} = RTS^{CT}=[0.89 to 4] ϑ_{LF} = [0.57to 0.67] η_{Lw} = [-0.03 to -0.06] η_{Fe} = [-0.50 to -0.61]
Andrikopoulos, Loizidis & Prodromidis (1992) *International Journal of Transport Economics*	Translog Total cost SURE Time series	Annual data 1960-1986 Athens Subway, bus and rail separately	Passengers From 95 (1960) to 104 (1986) millions	RTS^{Sub} = RTS^{rail} = (0.41) RTS^{bus}= (0.68) ϑ_{EK} = [0.99 to 1.5] η_{Lw} = [0 to -0.21] η_{Fe} = [-0.26 to -0.46] η_{Kr} = [0 to -1.15]
Delausse, Perelman & Thiry (1992) *Economie et Prévision*	Cobb-Douglas Variable cost SURE Panel	13 operators (all) 1978-1987 Belgium Urban, SNCV and regions	Seats-kilometres and Passengers	RTS^{P} = 0.685 Weak complementary between labour and fuel
Filippini, Maggi & Prioni (1992) *Annals of Public and Cooperative Economics*	Translog Total cost SURE Cross-section + trend	62 operators 1986-1989 Switzerland bus	Seats-kilometres (7.3 millions) + number of stops Passengers-km (2.1 millions) + number of stops	RTS^{SKO}= (1.16) [1.50 (small) to 1.00 (big)] RTD^{SKO}= (1.45) [1.78 (small) to 1.28 (big)] RTS^{PK}= (1.24) . RTD^{PK}= (2.19)
Fazioli, Filippini & Prioni (1993) *International Journal of Transport Economics*	Translog Total cost SURE Cross-section + trend	40 operators 1986-1990 Emilia Romagna (Italy) Bus	Seats-kilometres(18.4 millions) + length of lines (34 kilometres)	RTD^{LT}=[2.47 (big) to 2.64 (small)] RTS^{LT}=[1.68 (big) to 2.11 (small)]
Levaggi (1994) *Studi Economici*	Translog Variable cost SURE Cross-section	55 operators 1989 Italy Bus	Passengers-kilometres + length of network, density, average speed and load factor	RTS^{CT} = (0.92); RTD^{CT} = (0.89) RTS^{LT} = (1.43); RTD^{LT} = (1.38) ϑ_{LF} = (−0.30) Cost-elasticity to speed: -0.017

Quantities: L = labour, F = fuel, M = maintenance and K = capital; Prices: w = travail, e = fuel and r = capital; ϑ Allen's elasticity of substitution; M Morishima's elasticity of substitution; η price-elasticity of input demand; SFA: Stochastic Frontier Analysis; TFP: Total Factor Productivity Analysis; $SURE$: Seemingly Unrelated Regressions; ML: Maximum Likelihood

Previous table continuation

Authors	**Models estimated**	**Data**	**Outputs (average) [range]**	**Main results**[1] **(average) [range]**
Kumbhakar & Bhattacharyya (1996) *Empirical Economics*	Translog Variable cost TFP SURE Panel (random)	31 operators publics 1983-1987 India Bus	Passengers- kilometres + fleet use, load factor and ownership	RTD = (2.38)
De Rus & Nombela (1997) *Journal of Transport Economics and Policy*	Translog Total cost ML Cross-section	35 operators 1992 Spain Bus	Vehicles-kilometres (3 304 thousand) + average speed (12.5 km/h) et ownership (12 publicly owned)	$RTS = 1$ Weak ϑ's $\eta_{Lw} = (-0.235)$ $\eta_{Fe} = (0.091)$
Matas & Raymond (1998) Transportation	Translog Total cost OLS Panel (random)	9 networks 1983-1995 Spain Mains cities	vehicles-kilometres (22.723 millions) + length of network (377 kms)	$RTD = 2$ RTS^{CT}= [0.91(big) to 2.25 (small)] RTS^{LT}= [0.70(big) to 1.29(small)]
Gagnepain (1998) *Economie et Prévision*	Translog Variable cost ML Cross-section + trend	60 operators 1985-1993 France Urban and periphery (without Lyon, Paris and Marseille, > 100 000 inhabitants)	vehicles-kilometres (5.4 millions) + average commercial speed (16.7 km/h), length of network and type of contract	$RTD^{CT} = 2.60; RTD^{LT} = 0.87$ $RTS^{CT} = 2.42; RTS^{LT} = 0.80$ $\eta_{Lw} = (-0.015)$ $\eta_{Fe} = (-0.134)$ $\eta_{Le} = (0.149)$ $\eta_{Fw} = (0.149)$ Cost-elasticity to speed:(-0.13)
Karlaftis, McCarthy & Sinha (1999a) *Journal of Transportation Engineering*	Translog Variable cost SURE Cross-section + trend	18 networks 1983-1994 Indiana (USA) Fixed-route systems	Vehicles-miles (0.73 millions) [2.9 to 0.155] +age fleet, ratio peak/base et Saturday . Passengers + age fleet, ratio peak/base & Saturday	RTS^{V-M}=[> 1 (small) to < 1 (big)] RTD^{V-M}=[> 1 (small) to < 1 (big)] ϑ_{LF} = [0.197 to 0.222] $\eta_{Lw} = (-0.08)$ η_{Fe}=[-0.447 to -0.418] . $RTD^{pass} > 1$
Karlaftis, McCarthy & Sinha (1999b) *Journal of Transportation and Statistics*	Translog Variable cost ML Monthly series	60 observations 1991-1995 Indianapolis (USA)	Vehicles-miles	$RTS = (1.05)$ $RTD = (1.75)$
Jha & Singh (2001) *International Journal of Transport Economics*	Translog Total cost SFA ML Cross-section + trend	9 operators publics 1983-1997 India Bus	Passagers-kms, + length of lines, load factor and rate of bus use	10 billions passengers-kms $RTS^{small} = (1.036)$ 27 billions de passengers-kms $RTS^{medium} = (0.898)$ 50 billions de passengers-kms $RTS^{big} = (0.799)$

Quantities: L = labour, F = fuel, M = maintenance and K = capital; Prices: w = travail, e = fuel and r = capital; ϑ Allen's elasticity of substitution; M Morishima's elasticity of substitution; η price-elasticity of input demand; SFA: Stochastic Frontier Analysis; TFP: Total Factor Productivity Analysis; $SURE$: Seemingly Unrelated Regressions; ML: Maximum Likelihood

Previous table continuation

Authors	Models estimated	Data	Outputs (average) [range]	Main results[1] (average) [range]
Karlaftis & McCarthy (2002) *Transportation Research Part E*	Translog Variable cost Cluster SURE Cross-section + trend	256 networks 1986-1994 USA Urban + periphery	Vehicles-miles (5.1 millions) [67.8 to 0.408] + length of network	RTS^{CT} = (1.28) [0.99 to 11] RTD^{CT} = (1.33) [0.99 to 20] $\vartheta_{LF} = (-0.55)$ [0.63 to -0.53] $\eta_{Lw} = (-0.17)$ [-0.16 to -0.24] $\eta_{Fe} = (-0.45)$ [-0.45 to -0.17]
Filippini & Prioni (2003) *Applied Economics*	Translog Total cost SURE et ML Panel + trend	34 operators 1991-1995 Switzerland Regional buses	Bus-kms (421,000) + length of lines Seats-kilometres (29 millions) + number of stops and ownership	RTS^{BKO} = 1.04 - RTS^{PKO} = 1.17 RTD^{BKO} = 1.37 - RTD^{PKO} = 1.97 $\vartheta_{LF} = (0.007)$ $\vartheta_{LK} = (2.52 - 2.65)$
Dalen & Gomez- Lobo (2003) *Transportation*	Cobb-Douglas Total cost SFA ML Panel + trend	142 operators 1987-1997 Norway Bus	Urban vehicles-kms and inter-urban vehicles-kms + density, centrality et industry index	$RTD_k^{CT} = 1.038$ RTD higher with inter-urban traffic. Cost-complementarity: 0.013
Fraquelli, Piacenza & Abrate (2004) *Annals of Public and Cooperative Economics*	Translog Variable cost SURE Cross-section	45 operators 1996-1998 Italy Urban (without Rome, Milan and Naples), inter-urban and regional railways	Seats *vehicles-kms (437,709 millions) [36 to 8,156,709] + commercial average speed (23.12 km/h) [13km/h to 45km/h] and type of service	$RTD^{CT} = 2.09$ $RTD^{LT} = 1.85$ M_{LF} = [0.30 to 0.35] $\eta_{Lw} = (-0.11)$ $\eta_{Fe} = (-0.32)$ Cost-elasticity to speed:(-0.22)
Piacenza (2006) *Journal of Productivity Analysis*	Translog Variable cost SFA ML Cross-section + trend	45 operators 1993-1999 Italy Urban, inter-urban and regional railways	Seats *vehicles-kms (542,216 millions) + average commercial speed (23.3 km/h) [13 km/h to 45.5 km/h], type of service and type of contract	$RTD^{CT} = 1.89$ $RTD^{LT} = 1.83$ Some restrictions accepted Cost-elasticity to speed: (-0.18)

Quantities: L = labour, F = fuel, M = maintenance and K = capital; Prices: w = travail, e = fuel and r = capital; ϑ Allen's elasticity of substitution; M Morishima's elasticity of substitution; η price-elasticity of input demand; SFA: Stochastic Frontier Analysis; TFP: Total Factor Productivity Analysis; $SURE$: Seemingly Unrelated Regressions; ML: Maximum Likelihood

[1]Returns to scale are recalculated when they are defined by $1 - e_Y$, instead of $1/e_Y$.
[2]Results close to Obeng (1984) ones.