INTERNAL WAVE GENERATION THEORY: FROM LIGHTHILL ONWARDS

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A major landmark in internal wave theory has been the publication in 1978 of the book *Waves in Fluids* by Sir James Lighthill. With this book, for the first time a general theory of internal wave generation was available, considering the three basic types of forcing (transient, oscillatory and travelling) and introducing a single tool for their study (the wavenumber surface, a geometrical representation of the dispersion relation). Before that, only *ad hoc* approaches had been used, specific to particular problems. Lighthill's theory has since proved invaluable for the analysis of internal wave fields. However, it has met mitigated success for their calculation, owing to analytical complexity and to restriction to the above three types of forcing.

Starting from the early 1980s in Russia, an alternative theory has been developed based on the Green's function method, allowing easier implementation and consideration of a greater variety of forcing mechanisms. Suffice it to mention the derivation of the Green's function by Sekerzh-Zen'kovich (1979) and Teodorovich & Gorodtsov (1980), its application to travelling sources by Gorodtsov & Teodorovich (1980, 1981) for uniform translation and Sturova (1980) and Gorodtsov & Teodorovich (1983) for accelerated motion, and to initial disturbances by Sekerzh-Zen'kovich (1983). The approach has been extended and systematised later by Voisin (1991*a*, *b*, 1994) and is still used and developed to date (Scase & Dalziel 2004, 2006).

One type of forcing, though, has resisted investigation: oscillatory forcing, obtained in the laboratory with oscillating bodies (Gostiaux *et al.* 2007) and observed in the ocean through ebb and flow of the surface tide over the continental slope, generating the so-called internal tide (Gostiaux & Dauxois 2007). Difficulties arise from the peculiar nature of monochromatic internal waves: the waves propagate in beams, on a St Andrew's cross in two dimensions and on a double cone in three dimensions; the frequency of oscillation determines the inclination of the beams, but not the structure of the waves inside them; this structure is determined by the size and shape of the forcing, the viscosity of the fluid, and the interference with transients generated by the start-up (Voisin 2003).

Again, Russia has been at the forefront of research on this topic: while the influences of the size and shape of the forcing on one hand, and of viscosity on the other hand, had been known for some time (Lighthill 1978; Gorodtsov & Teodorovich 1986), it was Makhortykh & Rybak (1990) who first pointed out the influence of transients, and Ivanov (1989) and Makarov *et al.* (1990) who first pointed out the separation of the wave field into zones, where each influence dominates in turn. With reference to the transverse amplitude profiles through the beams, the terms *bimodal* were introduced for the zone governed by the size and shape of the forcing, and *unimodal* for the other zones. Further experiments have confirmed both the influence of transients (Ermanyuk & Gavrilov 2005) and the existence of bimodal and unimodal zones (Il'inykh *et al.* 1999*a*, *b*; Sutherland *et al.* 1999, 2000, 2003; Sutherland & Linden 2002; Flynn *et al.* 2003), consistent with numerical simulation (Javam *et al.* 2000).

The present communication is organised in two parts: first a general perspective on internal wave generation theory is offered, then more attention is paid to the generation of monochromatic waves by oscillating bodies. Two parameters are introduced which quantify the relative influence of the three phenomena affecting the modality of the waves (i.e. the size of the body, the viscosity of the fluid, and the time elapsed since the start-up), and a parameter space diagram is presented (figure 1). Two novel aspects are exhibited: for the waves, the importance of near-field effects; for the body, the importance of added-mass effects.

Near-field effects induce an asymmetry of the transverse amplitude profiles through the wave beams, a result validated by comparison with experiment and also visible in the calculations of Chashechkin *et al.* (2004). Added-mass effects induce a reduction of the wave amplitude and the occurrence of maximum power output at a frequency effectively independent from the direction of oscillation (figure 2). These results are consistent with the direct measurements of added mass by Ermanyuk (2000, 2002) and Ermanyuk & Gavrilov (2002*a*, *b*, 2003), based on a preliminary version of the analysis (Voisin 1999).

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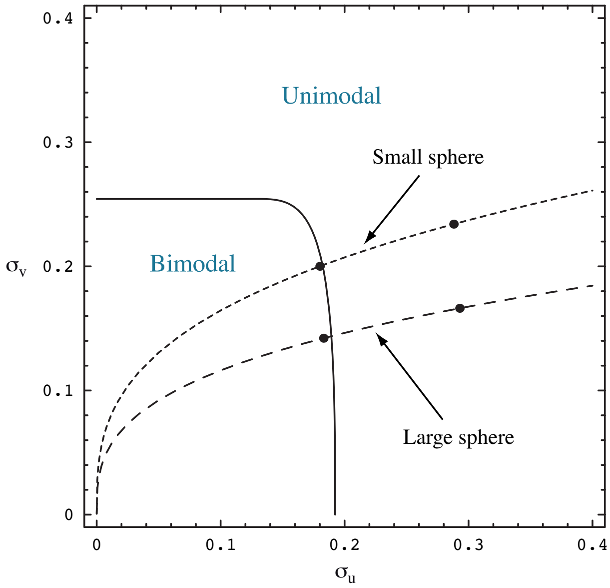

Figure 1: Regimes of internal wave propagation for the vertically oscillating spheres in the experiments of Flynn *et al.* (2003). The parameters σ_u and σ_v are defined as $\sigma_u = r/(\omega t\ a \tan\theta)$ and $\sigma_v = [r/(Re\ a \tan\theta)]^{1/3}$, with a the radius of the sphere, ω its frequency of oscillation, N the buoyancy frequency, $\theta = \arccos(\omega/N)$ the inclination of the wave beams to the vertical, r the distance from the centre of the sphere along the beam axes, t the time elapsed since the start-up, ν the kinematic viscosity and $Re = 2\omega a^2/\nu$ the oscillatory Reynolds number (sometimes called a Stokes number). The dashed lines represent the trajectory of the experiments in parameter space as the distance from the sphere varies.

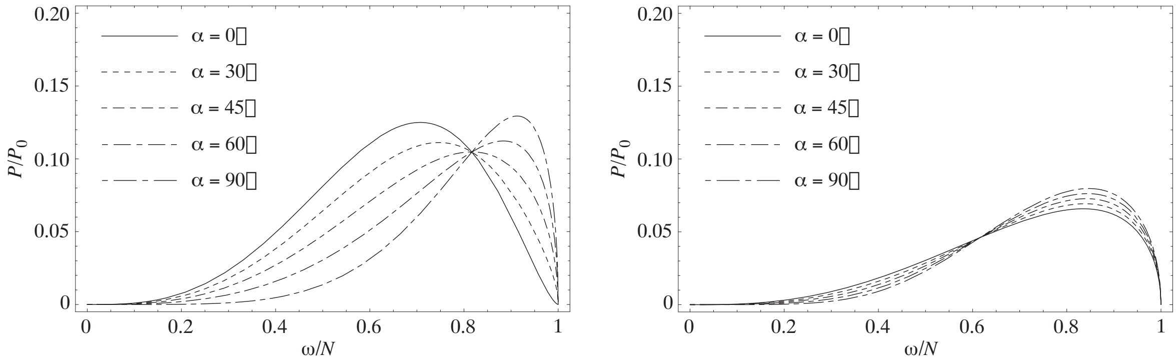

Figure 2: Power output of a sphere oscillating at the angle α to the vertical versus frequency. On the left, the variations of the added mass of the sphere with frequency are ignored, and the sphere is assumed to have instead the same coefficient of added mass ½ as in a homogeneous fluid (Gorodtsov & Teodorovich 1986). On the right, the variations of added mass with frequency are taken into account; as α varies, the maximum output is seen to occur at a practically constant fraction, varying between 0.83 and 0.85 only, of the buoyancy frequency.