'Odds algorithm'-based opportunity-triggered preventive maintenance with production policy

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‘ODDS ALGORITHM’-BASED OPPORTUNITY-TRIGGERED PREVENTIVE MAINTENANCE WITH PRODUCTION POLICY

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Abstract: In the field of manufacturing, the planning of opportunistic preventive maintenance actions adapted to minimise the number of unnecessary interruptions to production remains a major industrial challenge. Indeed, maintenance decisions have to be made in synchronisation with the production demands to eliminate costly unscheduled maintenance shutdowns and to improve productivity as well as quality. To confront this issue, this paper proposes an innovative approach based on the ‘odds algorithm’. The objective is to select, among all the production stoppages already planned, those which will be optimal to develop maintenance actions in time according to the degraded system state. The optimisation phase incorporates criteria such as reliability, maintainability and the duration of production stoppages. The feasibility of the approach is shown in a sample case.

Keywords: maintenance decision making, ‘odds algorithm’ (Bruss algorithm), opportunistic preventive maintenance.

1. INTRODUCTION

With today's growing demand on system productivity, availability and safety, product quality, customer satisfaction and the decrease of profit margins, the importance of maintenance function has increased (Al-Najjar and Alsyouf, 2000). Indeed maintenance process has, at least, a simultaneous impact upon the productivity, the availability of the production tools and the quality of the manufactured products (Waeyenbergh and Pintelon, 2004; Kutucuoglu, *et al.*, 2002). In that way, industrial maintenance process now has to be thought not only from a local point of view (a maintenance action on a component may restore it) but also from a global point of view (the maintenance process is a key tool to increase the residual lifetime and the performances of the system). It implies that maintenance decisions have to be made in synchronisation with the production demands to support global performances.
Nevertheless the global performance is difficult to be controlled since the system environment is changing, its functioning mode is dependent on product flows, and the component ageing modifies continuously system characteristics. Thus most maintenance strategies today are not well adapted to these requirements because purely reactive (fixing or replacing equipment after it fails) or time-scheduled (Wang, 2002). For instance, time-scheduled maintenance is blind because the equipment could be in a perfect operating condition when it is changed. As (Gupta, *et al.*, 2001) notice: ‘Analysis of dynamic preventive maintenance schedules, and their effect on the performance of the system, remains an open problem’. This justifies moving from traditional maintenance towards condition-based or predictive maintenance (Djurdjanovic, *et al.*, 2003) performed only when a certain level of equipment deterioration occurs rather than after a specified period of time. This results in decreasing maintenance expenses while improving global performances (Swanson, 2001).
That means to have at one's disposal not only statistical and historical information about the system operation but also just-in-time information (such as the degradation state of the system) to anticipate failures. It thus will be possible to take into account the dynamics of the production system evolution.
The predictive maintenance relies on processes that perform three essential tasks (Iung, *et al.*, 2005):

- elaborate and evaluate the current state of a component (monitoring process),
- assess the future situation of the system from its present, its past, its degradation laws and the maintenance actions to be investigated (prognosis process, characterised by a degree of uncertainty),
- select, from the results of the different assessed future situations, the most efficient one which guarantees, in the next future, the optimised performances for the system (decision making process).

The implementation of these three processes within a unique system ends in an integrated system of proactive maintenance as proposed by (Léger and Morel, 2001) (Figure 1).

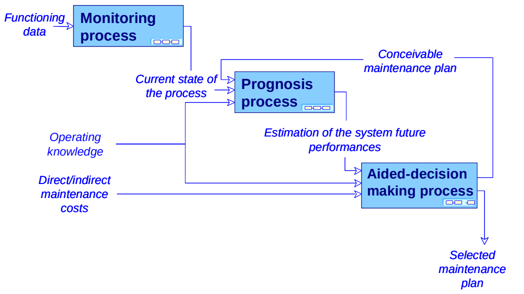

Fig. 1. Integrated system of proactive maintenance.

More precisely, in manufacturing area, the prognosis process aims at foreseeing how a component will evolve, until its failure and then until the system's breakdown (Muller *et al.*, 2004; Wang and Vachtsevanos, 1999). For example the prognosis of the residual lifetime of the components, in the case where no maintenance action is performed, can be obtained, as shown in Fig. 2. A possible warning threshold, an inherent risk, the expected availability of spares and manpower are taken into account to obtain this remaining useful life (*c.f.* ISO 13381). With this information, the main challenge is to plan the most adapted maintenance action which is optimised for the component (and its degradation) and synchronised with production demands.

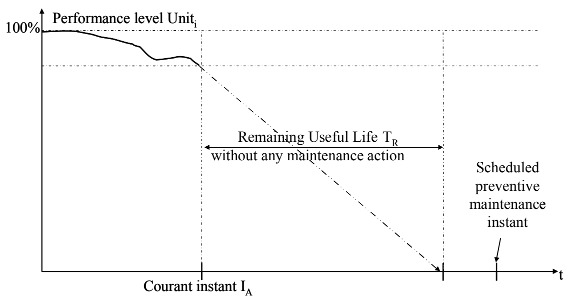

Fig. 2. Remaining useful life of a component, assessed by a prognosis process.

One industrial issue related to the open problem highlighted by (Gupta, *et al.*, 2001) can thus be formulated this way: knowing that a given component has a finite remaining useful life (assessed by the prognosis process and subject to on-line modifications), do the forecast production stoppages enable to perform a (proactive) maintenance action? Some academic work has already been achieved in that direction: (Kianfar, 2005) proposes a simultaneous cost-oriented planning of the production and the maintenance, but does not study the case where production stoppages are imposed. (Rosqvist, 2002) proposes a cost-oriented stopping-time model to maximise the expected lifetime utility of a system based on experts' judgement, but it focuses mainly on cost-oriented issues rather than on maintenance characteristics. (Gharbi and Kenné, 2005) tackles a stochastic optimal control problem related to the production control and preventive maintenance scheduling of manufacturing systems with non identical machines. Their approach focuses on optimal control and on considerations about cost functions rather than on maintenance properties of industrial systems. (Gupta, *et al.*, 2001) proposes a state-dependant preventive maintenance policy that takes into account the realities of production environments: preventive maintenance is triggered when the system reaches a particular state. But polling models are used rather than global planning and maintenance characteristics. Yet they evaluate the impact of the policy on the system performances, measured in terms of tardiness or work-in-process. A thorough study of the impact of a maintenance action upon the failure rate can be found in (Doyen and Gaudoin, 2004) and could be integrated to our study.
In relation to the previous contributions, our approach is more relevant for industrial purpose. Indeed the triggering of maintenance actions depends on the state of the system throughout the prognosis process, and the algorithm used here is easy to use, dynamic and proved to be optimal. Some realistic characteristics such as maintainability and reliability are taken into account. However neither cost-oriented considerations, nor the evaluation of the proposed strategy (over a finite time horizon) on the system global performances, are tackled. Availability of spares, tools and manpower could be taken into account in some future work.
The rest of the paper is organised as follows: section 2 formalises the problem related to production stoppages and then describes the 'odds algorithm' (or Bruss algorithm) able to support selection fulfilment. Section 3 presents the results of the implementation of the algorithm for a numerical example. Then the last section concludes and gives elements for further prospects.

2. PRODUCTION STOPPAGES MODEL

To solve the problem proposed, a mathematical result, issued from F. Thomas Bruss' work and based on the theory of optimal stopping, is used. Explanations about this theory can be found in (Chow, *et al.*, 1991). This method indicates the optimal behaviour in some situations where future is uncertain.

2.1 Formalisation of the production stoppages

The context is the operation phase (particularly the production and the maintenance) of an industrial system. An observation horizon being given, the expert has at his disposal the calendar of every forecast production stoppage (see Fig. 3).

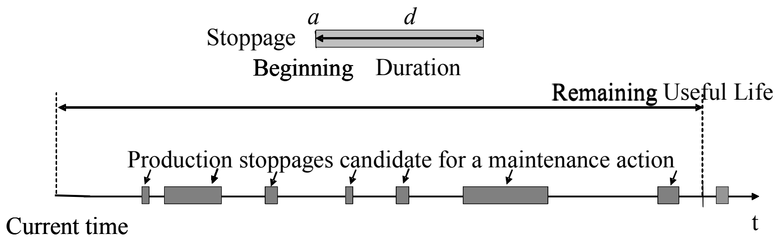

Fig. 3. Forecast production stoppages.

The challenge is the following: the monitoring process provides the state of the system. During the observation horizon, some production stoppages will happen. The prognosis process provides, upon the observation horizon, a set of maintenance actions that can be performed to ensure the well functioning of the system. A maintenance action being given, what is the most appropriate production stoppage (if any) that allows to perform this action?

Let S be a system. The prognosis process provides a remaining lifetime of T time units for this system, which defines a bounded uncertain observation horizon $[0;T]$.

The expert has at his disposal, before time T, the 'production stoppages' defined thanks to 'beginning instants' (a sequence $(a_i)_{1\le i\le n}$ where $n\in\mathbb{N}\setminus\{0\}$ and $\forall i\in[\![1;n]\!],\ a_i\in[0;T[$) and respective 'durations' $(d_i)_{1\le i\le n}$, with $\min_{1\le i\le n} d_i>0$ and $\sum_{i=1}^{n} d_i<T$.

A production stoppage A is thus a couple (a; d), where a is the beginning of the stoppage and d its duration. The end of the stoppage appears therefore at the instant $a+d$. Among these production stoppages, some will be considered by the maintenance experts as acceptable to perform an operation of preventive maintenance, because *(a)* spares and manpower are available, *(b)* the system is reliable at instant a and maintainable during the production stoppage. Those privileged stoppages will be called 'successes' in the following. Hence the issue can be reformulated this way:

Determine the instant a_{k_0}, $1\le k_0\le n$, at which a maintenance action will be performed in order to restore the system or one of its components into a nominal state. Note that this instant is necessarily the beginning of a success.

2.2 Thomas Bruss' results

A way to solve the aforementioned problem is to put forward Thomas Bruss' 'odds-theorem'. This theorem is described here and used thereafter.

Let $(I_i)_{1\le i\le n}$ be $n\in\mathbb{N}\setminus\{0\}$ indicators of random and independent events $(A_i)_{1\le i\le n}$ defined upon the same probability space $(\Omega;G;P)$. We observe $I_1, I_2\ldots$ sequentially and may stop at any of these observations (say I_j, $1\le j\le n$), but may not recall on the previous $(I_k)_{1\le k\le j-1}$. A 'success' is defined as being an observation equal to 1. Let B_k denote the sigma-field generated by $(I_i)_{1\le i\le k}$ and Υ the class of all rules t such that the event $\{t=k\}$ is B_k-measurable, for $k\in[\![1;n]\!]$. The scope is to find an 'optimal rule', *i.e.* a stopping rule $\tau_n\in\Upsilon$ that will maximise $P\left(I_t=1;\ I_j=0, t+1\le j\le n\right)$ over every $t\in\Upsilon$ and its value. This quantity is the probability to stop the observations precisely at the last success.

The following conventions are assumed: an empty sum is equal to zero, an empty product is equal to one, the supremum of the empty set is $-\infty$, and the infimum of the empty set is $+\infty$.

With these, the main result is the 'odds-theorem'. The result, the complete and technical hypotheses as well as the proof can be found in (Bruss, 2000).

Let $(I_i)_{1\le i\le n}$ be a sequence of independent indicator functions. The following quantities will be used: $p_j:=E(I_j)=P(A_j)$, $q_j:=1-p_j$, $r_j:=\dfrac{p_j}{q_j}$, $1\le j\le n$.

The quantities r_j, $1\le j\le n$, are traditionally called the 'odds'.

Then an optimal rule τ_n to find the last success exists and is to stop on the first index (if any) k with $I_k=1$ and $k\ge s$, where

$$s:=\sup\left(1;\sup\left\{k\in[\![1;n]\!]\mid\ \sum_{j=k}^{n} r_j\ge 1\right\}\right).$$

The optimal reward is given by $\left(\prod_{j=s}^{n} q_j\right).\left(\sum_{j=s}^{n} r_j\right)$.

2.3 Application

A key element relevant to the problem stated in this paper is a decision-making tool that is optimal and easy to use. This justifies that the aforementioned theorem is applied to our issue, by considering the universe Ω as the set of all the n planned production stoppages, and G as G_n where G_i is the sigma-field generated by the relative ranking of the i^{th} production stoppage. P is equiprobability upon G. (Bruss, 2000) and (Chow, *et al.*, 1991) provide complete technical details.

In the following the assumption that the production stoppages are independent random variables will hold. In particular, the beginnings of the stoppages and their respective durations are independent. This assumption, which is crucial for the application of the 'odds' method, can be described as follows: the stoppages of the system are subject to many constraints, ranging from production (requirements for the quantity of products to be transformed by the system), to management (requirements for the use of the system) and legal recommendations (to respect safety standards). This multiplicity of the factors, as well as the part of hazards they bear, incite us to believe that this assumption is justified.

The *n* independent random variables A_i, $1 \le i \le n$, are the repartition of the intervals that represent the production stoppages. To apply the theorem it is necessary to assess the probability that these variables are successes. In fact the probability of a success can be assessed by an expert, or by using the reliability and the maintainability of the system, as they are the two major elements to get together to perform a maintenance action. This approach is developed hereafter.

The probability that a production stoppage (a; d) allows to perform a maintenance action is here the product of the probability that the system is reliable at the instant a by the probability that the system is maintainable during the interval [a; $a+d$]. As the stoppages beginnings and their durations are independent events, therefore, so are the reliability of the system at a and the maintainability of the system during d time units. The reliability distribution and the maintainability distribution, which can be known in practice, provide these quantities.

It can be noticed that the expert can modify the evaluation of the quantities by multiplying the probabilities by an *a priori* distribution which would sum up his knowledge, his know-how or his intuition as for the fact that a production stoppage is a potential success.

The next and final step is to compute the index s, thanks to the formula in the theorem, and then to wait for the first success, if any, after A_s. Thus, the scope of the next section is to detail how to calculate this index and to show the feasibility and interest of this mathematical approach with a concrete example for which a reliability distribution and a maintainability distribution are provided.

3. IMPLEMENTATION OF THE ALGORITHM

Bruss algorithm is natural and it is easy to make use of it. First the time from which onwards it is optimal to stop on the first success is calculated. Then the results that can be obtained for a numerical example are presented.

3.1 *How to determine* s *by using the odds algorithm*

It is easy to determine the index s thanks to the formula provided by the theorem. Let us explain a way of doing it. We write successively the p_k, q_k, r_k as follows (we begin from the last one, *i.e.* for $k = n$):

$$p_n,\ p_{n-1},\ p_{n-2} \cdots p_1,$$
$$q_n,\ q_{n-1},\ q_{n-2} \cdots q_1,$$
$$r_n,\ r_{n-1},\ r_{n-2} \cdots r_1.$$

Let us keep in mind that, for each index $k \in [\![1;n]\!]$, we have: $p_k \in [0;1]$, $p_k + q_k = 1$, $r_k = p_k / q_k$.

Now add the numbers r_k from left to right, until the sum is equal to or greater than 1: add, for decreasing values of n, $r_n + r_{n-1} + \ldots + r_s =: R_s$, until the value of R_s exceeds 1. The value of s for which this sum first exceeds 1 is the stopping index we are looking for. But if the value 1 is not reached once all the terms have been added, let s be 1. This is the meaning of the formula provided by the theorem. s is the time from which onwards it is optimal to stop on the first success. The case 's=1' means that the first success that occurs (if any) is utilised to perform a maintenance action.

Now compute the product $Q_s := q_n q_{n-1} \cdots q_s$. The optimal strategy is thus the following: wait for the production stoppage whose index is s, and then perform a maintenance action as soon as a success appears (if any appears). The success probability of this strategy (the win probability) is equal to $Q_s R_s$.

In our application, the probability of a success p_i is defined by the product of the probability that the system is survivor at instant a by the probability that the maintenance action can be performed during d, in concordance with the previous section. Every event A_i of the sequence $(A_i)_{1 \le i \le n}$ is therefore characterised by a couple $(a_i; d_i)$ and:

$p_i = R(a_i).M(d_i)$, $\quad q_i = 1 - R(a_i).M(d_i)$ with $R(a_i)$ the reliability of the component at instant a_i and $M(d_i)$ the maintainability of the component during d_i. The odds can thus be defined by:

$$r_i = R(a_i).M(d_i) / (1 - R(a_i).M(d_i)).$$

Calculate the odds for every single production stoppage and, in the same way as in the general case, sum up the odds from the last one until reaching (or going beyond) the value of 1 (at index s). The first following production stoppage that is a success is the optimal one. The probability v_s that this particular production stoppage is the best one is given by:

$$v_s = \left(\prod_{j=s}^{n} (1 - R(a_i).M(d_i)) \right) \left(\sum_{j=s}^{n} \frac{R(a_i).M(d_i)}{1 - R(a_i).M(d_i)} \right).$$

3.2 *Numerical results for an application case*

Here are reported the results that are obtained by using the odds algorithm for a realistic although academic problem of finding the most suitable moment to perform a maintenance action. The following values for the parameters are considered:

1500h for the observation horizon and 12 production stoppages, planned as indicated in Table 1.

Production stoppage number	Beginning (hours)	Duration (hours)
1	200	3
2	310	2
3	400	4
4	560	2
5	620	1
6	690	4
7	800	4
8	910	2
9	980	3
10	1100	7
11	1250	3
12	1360	4

Table 1. Detail of the twelve production stoppages, in time units (hours)

The reliability function is supposed to be a Weibull distribution, with the following values: (0.8, 1, 1.5, 2, 2.5, 3) for the shape parameter β, (50h, 100h, 200h, 400h, 700h, 1000h, 1500h) for the scale parameter η, and 0 for the location parameter. The maintainability function is supposed to be exponential with parameter μ successively equals to (0.5h^{-1}; 1h^{-1}; 2h^{-1}; 4h^{-1}; 10h^{-1}; 15h^{-1}). From these input data, the sums of odds are computed and given in the next tables (see Tables 2, 3 and 4).

Production stoppage index	Sum of the odds for β equals to 0.8	1	1.5	2	3	5
12	0.19	0.14	0.06	0.02	0.00	0.00
11	0.38	0.29	0.14	0.05	0.00	0.00
10	0.68	0.54	0.29	0.14	0.02	0.00
9	0.79	0.65	0.38	0.20	0.05	0.00
8	**1.02**	0.86	0.54	0.33	0.13	0.02
7	1.42	**1.24**	0.89	0.64	0.37	0.16
6	1.89	1.72	**1.37**	**1.13**	0.86	0.68
5	2.01	1.91	1.57	1.34	**1.11**	0.97
4	2.46	2.31	2.02	1.84	1.72	**1.81**
3	3.30	3.26	3.30	3.50	4.26	6.17
2	3.86	3.94	4.19	4.58	5.64	7.81
1	5.06	5.35	6.19	7.10	8.78	11.26

Table 2. Numerical results: sum of the odds rounded at 0.01, evolution with β.

In Table 2, the fixed parameters have been set to 700h for η and 0.5h^{-1} for μ. As expected, with the increase of β, the optimal instant to perform a maintenance action comes sooner.
As β increases from 0.8 to 5, it can be assessed that the proposed solutions are optimal with respective probabilities 0.41, 0.41, 0.41, 0.42, 0.42 and 0.43. Those rates are in right concordance with Bruss' results and, if they look low, they are not: a typical decision making tool will not give the optimal instant with such a good rate. Here the optimality holds with probability greater than 40% (Bruss, 2003).

Production stoppage index	Sum of the odds for η/100 equals to 1	2	4	7	10	15
12	0.0	0.0	0.0	0.1	0.2	0.6
11	0.0	0.0	0.0	0.1	0.5	**1.1**
10	0.0	0.0	0.0	0.3	0.9	2.2
9	0.0	0.0	0.0	0.4	**1.1**	2.5
8	0.0	0.0	0.0	0.5	1.4	3.2
7	0.0	0.0	0.1	0.9	2.2	4.6
6	0.0	0.0	0.2	**1.4**	3.1	6.3
5	0.0	0.0	0.3	1.6	3.4	6.7
4	0.0	0.0	0.4	2.0	4.1	7.8
3	0.0	0.1	0.9	3.3	6.2	10 .8
2	0.0	0.2	**1.3**	4.2	7.3	12.2
1	0.1	0.6	2.5	6.2	9.8	15.0

Table 3. Numerical results: sum of the odds rounded at 0.1, evolution with η.

In Table 3, the fixed parameters have been set to 1.5 for β and 0.5h^{-1} for μ. The algorithm logically emphasises that with the increase of η, the optimal instant comes later. Here again the results are in right concordance with the theory.

Production stoppage index	Sum of the odds for μ equals to 4	2	1	0.5	0.25	0.125	0.0625
12	0.1	0.1	0.1	0.1	0.0	0.0	0.0
11	0.2	0.2	0.2	0.1	0.1	0.1	0.0
10	0.3	0.3	0.3	0.3	0.2	0.2	0.1
9	0.6	0.5	0.5	0.4	0.3	0.2	0.1
8	0.9	0.8	0.7	0.5	0.4	0.2	0.1
7	**1.3**	**1.2**	**1.1**	0.9	0.6	0.4	0.2
6	1.9	1.8	1.7	**1.4**	0.9	0.5	0.3
5	2.6	2.4	2.1	1.6	**1.0**	0.6	0.3
4	3.6	3.4	2.8	2.0	1.3	0.7	0.4
3	5.4	5.2	4.6	3.3	1.9	**1.0**	0.5
2	8.3	7.9	6.4	4.2	2.4	1.2	0.6
1	14.4	13.9	10.8	6.2	3.2	1.6	0.8

Table 4. Numerical results: sum of the odds rounded at 0.1, evolution with μ.

In Table 4, the fixed parameters have been set to 1.5 for β and 700h for η. As it is well-known, it can be seen that, as μ increases, the optimal instant comes sooner. Here again these rates are in full consistence with the theoretical results.

4. CONCLUSION AND PROSPECTS

Some issues related to the planning of maintenance actions in synchronisation with production have been presented and a dynamic method to solve them has been proposed. This method is proved to be optimal. A numerical example was provided to emphasise the feasibility and the easiness of Bruss algorithm, by using such fundamental maintenance concepts as reliability and maintainability. Yet our main target is to provide maintenance plans to the prognosis process, to assess the future state of the system. This paper brings a relevant although partial answer and should be extended to tackle this goal. A first industrial application concerns the European DYNAMITE project (IP DYNAMITE FP6-IST-NMP-2-017498). Then the following questions are of great interest for scientific purpose: suppose the odds algorithm is applied to every component of the system; how could the expert group the maintenance interventions together in order to minimise the cost of these operations? Can the expert apply the odds algorithm recursively to classify the production stoppages according to their respective relevance? We know that the odds algorithm can be generalised in that sense. How to evaluate then the impact of a maintenance action performed at those production stoppages upon the performances of the whole system? Does the odds algorithm help lower total expected work-in-process and overall tardiness? These questions and others have been partially addressed in (Levrat, *et al.*, 2008), (Iung, *et al.*, 2007) and (Thomas, *et al.*, 2008). Complete answers to these questions would eventually help control the performances of industrial systems.

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