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# Plant Control using Digital Sensors that introduce a Delayed and Sampled Output

Afzal Chamroo, Christian Vasseur and Haoping Wang

**Abstract:** *This article deals with the control of linear plants that deliver delayed and sampled output. We propose a control theory which makes use of a class of piecewise continuous systems and that enables sampled tracking. A mathematical approach of the dynamics of our control command is given. This approach uses only the delayed and sampled output vector as feedback. Moreover, a computer simulation example is presented so as to enhance the theoretical aspect. The method shows reliability and robustness against slight time-variations of the plant's parameters.*

**Keywords:** *tracking, sampled tracking, sampled feedback, delayed output, piecewise continuous systems*

## Introduction

The goal of the present article is to develop a control strategy that enables tracking on linear plants in cases where the only available feedback is the plant's delayed and sampled output vector. This is often the case when we deal with control architectures that make use of digital calculators and digital sensors that are time consuming in what concerns step calculations.

The particular type of output that is studied here brought us to consider a sampled type control technique. In that point of view, Kabamba [3] was the first to propose the sampled control commands for linear plants using a generalised sampled-data hold function. Urikura and Nagata [11] developed an optimised control method for sampled-data systems, by reducing ripples between sampling instants. Otherwise, Yamamoto [13] proposes an optimisation using a hybrid control system, based on the notion of functions defined piecewisely.

Considering the hybrid notion introduced by Decarlo [2], Koncar and Vasseur [4-6] have established, more recently, a class of control systems whose evolution is described by switchings. The chronology of these switchings is defined by a set of sampling instants  $S = \{t_k, k=0,1,2,\dots\}$  called "switching instants". These controllers are referred to as piecewise continuous systems (PCS). In this approach, the control input of the plant is defined from two input spaces: the first space  $U^r$  allows control between switching instants, while the second input space  $V^s$  enables control at the switching instants. Referring to the classification of Tittus and Egardt [10], this class of control systems has hybrid properties and extends the concept of compound control realised by Laurent [7] and Vasseur [12]. According to Branicky's taxonomy of hybrid systems [1], these control

units are characterised by autonomous switchings and controlled impulses.

It is well established in [6] that the use of PCS controllers enables sampled tracking on linear plants by undertaking a *state* feedback. However, in our case, we must adapt appropriately the method in [6] so as to define a controller no longer based on state feedback, but on an *output* feedback, which is, moreover, delayed and sampled.

In this paper, we start by defining the particular nature of the output signal considered for feedback. An overview of the construction and behaviour of a PCS is given in the second section, as well as its use in sampled tracking. Then we show how the use of a PCS controller can be adapted in our case. Moreover, the equations of the whole closed loop structure are established in view of taking into account the delayed nature of the output. The reader can find at the end of the paper a typical example raising delayed and sampled output while controlling a mobile cart by camera.

## The Particular Sensor Output

The plant which we consider in our study is a usual general linear  $n$ th order system that we represent by its state and output equations as follows:

$$(1a) \quad x'(t) = Ax(t) + Bu(t),$$

$$(1b) \quad y(t) = Cx(t),$$

with  $A \in \mathfrak{R}^{n \times n}$ ,  $B \in \mathfrak{R}^{n \times r}$  and  $C \in \mathfrak{R}^{m \times n}$  being the real, characteristic matrix of the system, and  $x(t) \in \Sigma^n$ ,  $u(t) \in U^r$  and  $y(t) \in Y^m$  representing respectively the state, the input and the output of the plant.

However, in our case, we consider that neither the state  $x(t)$ , nor the output  $y(t)$  of the plant is available. The only data to which we can access comes from a digital sensor that "observes" the output  $y(t)$  and delivers it in a sampled and delayed format. The sampling period of the sensor being  $t_e$  and the delay associated being  $N.t_e$ , we define the sensor data as such:

$$(1c) \quad z(t) = y^*(t - N.t_e),$$

with  $N$  being a positive integer and (\*) representing sampling with a period of  $t_e$ .

In our study, we assume that  $N$  and  $t_e$  are known constants.

An illustrating example can be the case where processed data accessed from a digital camera constitute

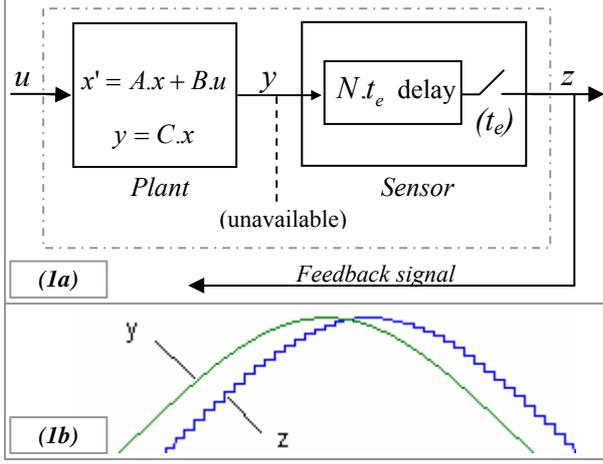


Fig. 1a. The plant-sensor entity,

Fig. 1b. Output signals for  $N=4$ .

the output  $z(t)$  of a “visual” sensor. In that case, the  $t_e$  sampling period corresponds to the delivery of image information where  $t_e$  represents the time for an image shooting. For what concerns the delay, it represents the time necessary for image processing.

In this study, we assume that this delay is a multiple of the sampling period. This means actually that  $N$  snapshots are necessary to obtain the required data.

The whole statement of this section is summarised in Fig.1 above, where  $N = 4$ .

### Piecewise Continuous Systems as Controllers

The principle of PCS control is to build an associated PCS system whose output constitutes the input of the plant. According to [6], we make use of a PCS system to define a particular PCS controller whose behaviour can be summarised as follows:

- (i) The state of the PCS controller is switched to forced values at regular intervals of period  $t_e$ , which is the sample rate of the sensor. The corresponding switching set is represented by  $S = \{k.t_e, k = 0, 1, 2, \dots\}$ .
- (ii) The equations describing the behaviour of the controller are:

$$(2a) \quad \lambda'(t) = \alpha \lambda(t), \quad \forall t \in ]k.t_e, (k+1)t_e],$$

$$(2b) \quad \lambda(k.t_e^+) = \delta \psi(k.t_e), \quad \forall k = 0, 1, 2, \dots,$$

$$(2c) \quad w(t) = \gamma \lambda(t), \quad \forall t.$$

Equation (2a) describes the continuous evolution of the controller's state  $\lambda(t) \in \Sigma^{\hat{n}}$  upon  $]k.t_e, (k+1)t_e]$ ,  $\alpha \in \mathcal{R}^{\hat{n} \times \hat{n}}$  being the state matrix of the controller. The only parameter that defines the behaviour of the controller's state in this interval of time is  $\alpha$  which can take an arbitrary value. Usually, it is fixed such that the PCS is stable between switching instants.

Equation (2b) defines the controller's state at switching instants, by means of a bounded discrete input

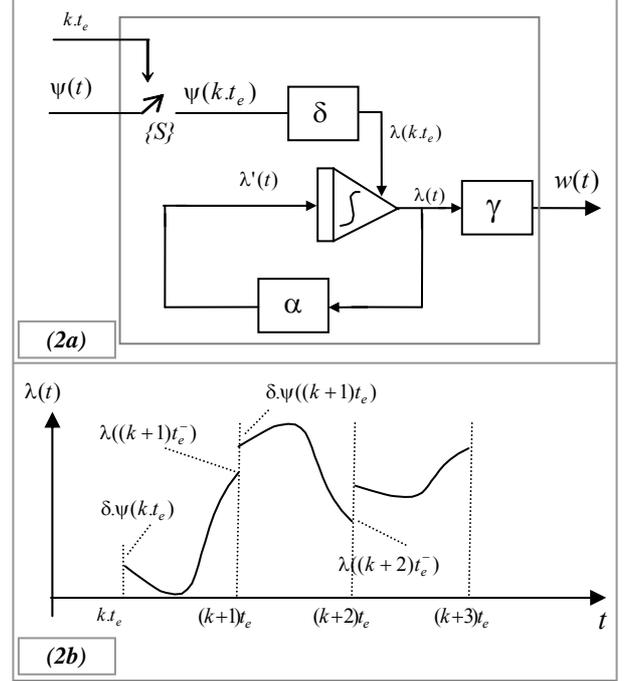


Fig. 2a. Realisation diagram,

Fig. 2b. State evolution of a PCS controller

$\psi(k.T_e) \in V^s$ , and according to the linear relationship characterised by the matrix  $\delta \in \mathcal{R}^{\hat{n} \times s}$ .

Equation (2c) is the output equation of the controller, characterised by the full rank matrix  $\gamma \in \mathcal{R}^{\hat{m} \times \hat{n}}$ . The output  $w(t) \in Y^{\hat{m}}$  constitutes the input command to be fed to the plant.

Fig.2a gives the realisation diagram of a PCS controller and Fig.2b shows its state's evolution.

It is shown in [6] that if the state of the plant is available, it is possible to define  $\psi(t)$  and  $\delta$  so as to achieve discrete tracking of a  $c(t)$  state trajectory by the plant's state  $x(t)$  at each switching instant and with one sampling period delay:  $x((k+1)t_e) = c(k.t_e)$ ,  $\forall k = 0, 1, 2, \dots$ . The parameters  $\alpha$  and  $\gamma$  are chosen arbitrarily.

Note that from now on, the discrete values of every function will be considered as being sampled at  $t_e$  period and to simplify the notations, any time function  $f(t)$  at a given  $k.t_e$  instant will be written as  $f(k.t_e) = f_k$   $\forall k = 0, 1, 2, \dots$ . Moreover, dealing with PCS gives rise to discontinuous signals. Thus, if any signal  $f(t)$  is discontinuous, we shall consider the right value at the discontinuity since the switchings at each  $k.t_e$  imply consequences occurring at every  $k.t_e^+$ . However, for simplification sake, the notation  $f_k$  will be used, instead of the strict one:  $f_k^+ = f(k.t_e^+)$ .

### Control Strategy

The idea is to make use of the PCS controller defined above in order to achieve sampled tracking of a state trajectory  $c(t)$  by the plant's inaccessible state  $x(t)$  with a delay equal to a period of the sensor's sample time. Hence, we will define a control strategy to ensure  $x_{k+1} = c_k \forall k = 0, 1, 2, \dots$  by using only  $z(t)$ , defined in (1c), as feedback signal.

Thus, the method we'll propose should allow us to observe:  $z_{k+N+1} = C.c_k \forall k = 0, 1, 2, \dots$ , with  $C \in \mathbb{R}^{m \times n}$  being the output matrix of the plant as described formerly.

For better comprehension, we will proceed by step:

- (i) The PCS controller playing the main role of our method, we will first show how to define its matrix  $\psi(t)$  and input  $\delta$  to achieve  $x_{k+1} = c_k$  in cases where *state* feedback is possible.
- (ii) Then, we'll describe how the same goal can be achieved with a *delayed state* feedback.
- (iii) Thus we only need to reconstitute the delayed state from the *delayed output* and use the approach of step (ii).

In every case, the controller's output is linked to the plant's input, thus  $u(t) = w(t)$ .

### State Feedback

In this case, the behaviour of the closed loop system can be given by the following equation set:

$$(3a) \quad x'(t) = A.x(t) + B.u(t), \quad \forall t,$$

$$(3b) \quad \lambda'(t) = \alpha.\lambda(t), \quad \forall t \in ]k.t_e, (k+1)t_e],$$

$$(3c) \quad u(t) = \gamma.\lambda(t), \quad \forall t,$$

$$(3d) \quad \lambda_k = \delta.\psi_k, \quad \forall k = 0, 1, 2, \dots$$

By integration, the first three equations allow us to write in a sampled format, the next step value  $x_{k+1}$  of the state as a function of its previous one  $x_k$ :

$$(4) \quad x_{k+1} = f.x_k + M.\lambda_k, \text{ with}$$

$$f = e^{A.t_e} \text{ and } M = f.\int_0^{t_e} e^{-A\tau} B.\gamma.e^{\alpha\tau} d\tau.$$

In order to realise the discrete tracking which is defined above, we only have to fix down the tracking condition which is  $x_{k+1} = c_k$ , where  $c(t)$  is the desired state trajectory. Thus, from (4) we have:

$$(5) \quad \lambda_k = M^{-1}\{c_k - f.x_k\}$$

Equation (5) gives the switching value of the controller's state, under the condition that  $M^{-1}$  exists [6]. Hence, in this case, we are able to define the PCS controller with:

$$\delta = M^{-1} \text{ and } \psi(t) = c(t) - f.x(t),$$

$\alpha$  and  $\gamma$  chosen arbitrarily.

### Delayed State Feedback

Here, we assume that the most recent information available on the state at current time is  $x(t - N.t_e) = x_{k-N}$ , due to an imaginary sensor that delivers the state with a delay of  $N.t_e$ . We can

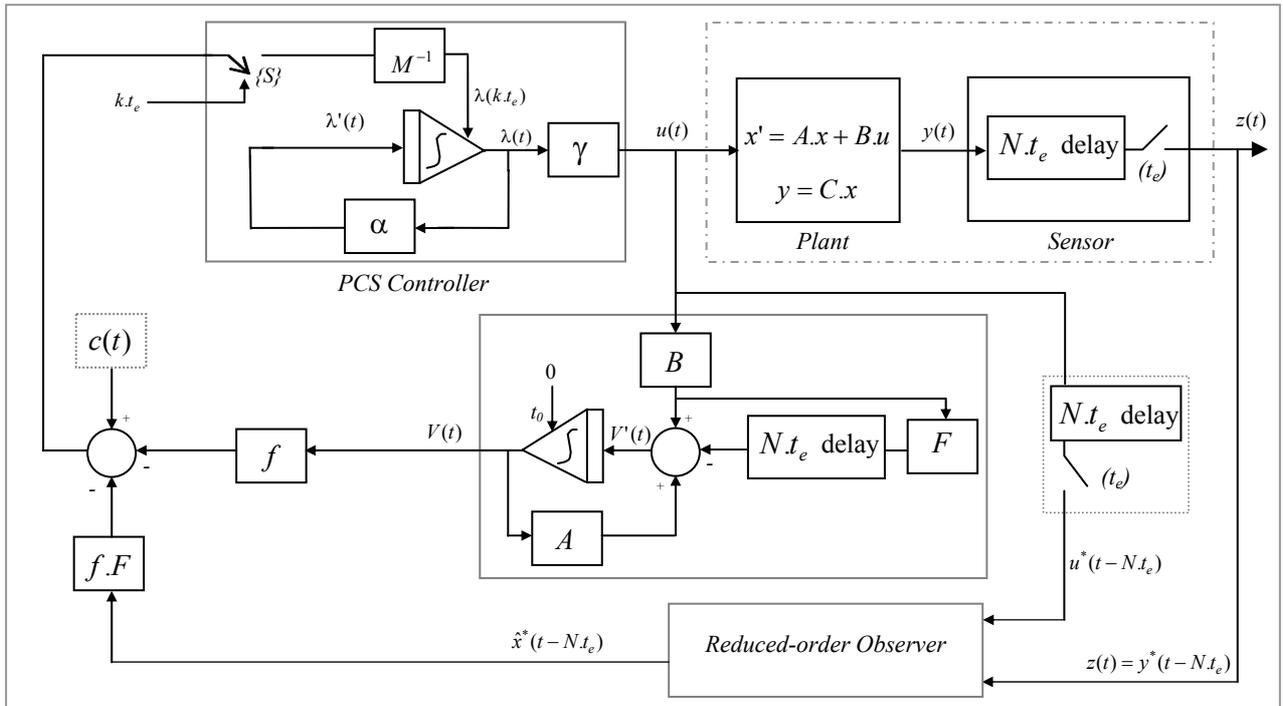


Fig.3. Block diagram of the whole closed loop structure using delayed output feedback

nevertheless write down mathematically equation (4) as before. However,  $x_k$  being unavailable, it must be calculated from  $x_{k-N}$  as follows:

$$(6) \quad x_k = F.x_{k-N} + V_k, \text{ with}$$

$$F = e^{A.N.t_e} \text{ and } V_k = F. \int_{(k-N)t_e}^{kt_e} e^{-A\tau} B.u(\tau).d\tau.$$

Note that in practice,  $V_k$  can be calculated as being the solution to the following differential equation:

$$(7) \quad V'(t) = A.V(t) + B.u(t) - F.B.u(t - N.t_e), \text{ with } V(0) = 0.$$

Thus, by replacing the calculated value of  $x_k$  (6) in equation (4), we obtain:

$$(8) \quad x_{k+1} = f.(F.x_{k-N} + V_k) + M.\lambda_k$$

Now, with the same tracking condition as in the former case, and assuming that  $M$  is invertible, we define the PCS controller with:

$$\delta = M^{-1}$$

$$\text{and } \psi(t) = c(t) - f.(F.x(t - N.t_e) + V(t))$$

### Delayed Output Feedback

At this stage, we only have to reconstitute the delayed plant's state  $x_{k-N}$  using the available sensor data  $z(t)$  so as to bring us back to the previous case.

This is easily realised using a discrete reduced-order observer [8] [9], that takes as input signals  $z(t) = y^*(t - N.t_e)$  and  $u^*(t - N.t_e)$ . The observer thus outputs an estimate of the delayed plant's state,  $\hat{x}^*(t - N.t_e)$ .

Using the same tracking condition  $x_{k+1} = c_k$ , we can therefore ensure  $z_{k+N+1} = C.c_k \forall k = 0,1,2,\dots$

Fig.3 represents the detailed architecture of the closed loop structure in the case of sampled and delayed output feedback. Note that the sampled nature of the delayed output is supported by the PCS controller which switches at the sample rate of the digital sensor.

### Computer Simulation Example

In view of validating our method we have simulated, by means of Matlab®/Simulink®, the behaviour of the whole closed loop structure shown in Fig.3. This computer simulation reflects the control of a real system which is described below. As shown in Fig.4, this system consists of the visual position control of a moving cart.

### The Plant

The plant which is considered here is a cart that moves along a horizontal and straight line segment. The cart is powered by an electric motor by means of a notched belt. The plant's state is composed of the real

position and speed of the cart, while its output is given by the real position only:

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \text{real position} \\ \text{real speed} \end{bmatrix}, y = x_1,$$

The motor is of a brushless type. It is driven in +/-10V by a dSpace® computer input/output card via a power amplifier. Supplied with 240V (mono), it can offer a nominal couple of 3.0Nm with a power of 200W. Identification with a second order approximation of the amplifier-motor-cart set has shown a time constant of 8.3ms and an overall gain of 2.9m/S/V.

Hence, we assume that the plant can be defined by the following state representation:

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -120 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 350 \end{bmatrix} \text{ and } C = [1 \quad 0]$$

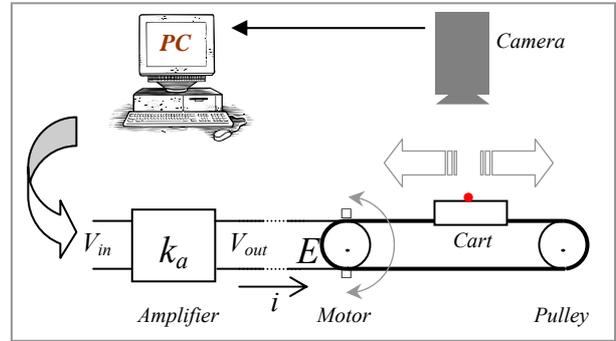


Fig.4. Visual control

### The Sensor

The aim of the experiment is to realise a visual position control of the cart. Thus, the sensor is a "artificial vision" system that observes an infrared LED fixed on the cart, as shown in Fig.4. This vision system is constituted of a motionless digital infrared CCD camera connected to a computer allowing image processing. The camera is positioned above the cart and observes its motion. Thus, after a location operation, the artificial vision system outputs the position of the cart in a  $t_e$ -sampled format, with a delay equal to  $N.t_e$ :

$$z(t) = x_1^*(t - N.t_e), \text{ with } (*): \text{ sampling at } t_e.$$

### The Associated PCS Controller

In this simulation, we make use of the strategy for delayed output feedback as explained formerly. The delayed state is reconstituted by using the output  $z(t)$  of the artificial vision system. The appropriate inputs (as defined in the "strategy" section) are fed to the PCS controller, whose internal parameters are chosen as such:

$$\alpha = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.2 \end{bmatrix}, \gamma = [1 \quad 1]$$

### The Aim of the Experiment

In the present example, the goal is to be able to realise sampled position tracking of a desired trajectory by the cart. According to our method's requirement, we have to define a state trajectory, which is here chosen to be:

$$c(t) = \begin{bmatrix} c_1(t) \\ c_2(t) \end{bmatrix} = \begin{bmatrix} \sin(\omega.t) \\ \omega.\cos(\omega.t) \end{bmatrix} \text{ with } \omega = 2 \text{ rad/s}$$

In this example,  $c_2(t)$  is bound to be the derivative of  $c_1(t)$ , since they represent the desired speed and position trajectories.

As specified previously, our method should ensure  $x_k = c_{k-1}$  and thus  $z_k = C.c_{k-N-1} \forall k = 0,1,2,\dots$ . The results obtained are shown in Fig.5 and Fig.6.

Note that in order to better illustrate the results, the values of  $t_e$  have been chosen to be relatively big as compared to real cases. Moreover, to show that the tracking occurs at each switching instant, the desired trajectory has been delayed appropriately, for comparison sake.

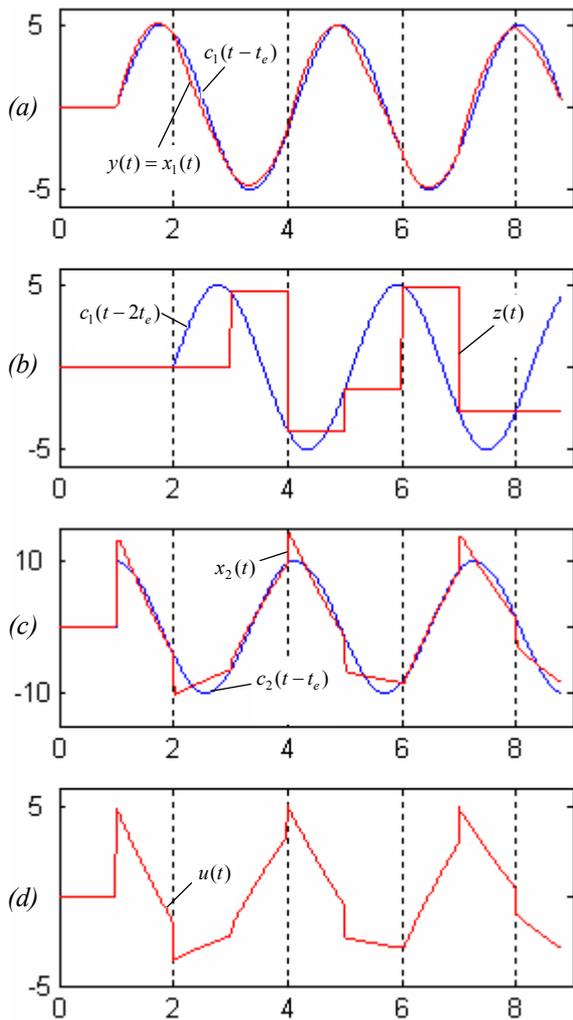


Fig.5. Tracking,  $t_e = 1s$  and  $N = 1$

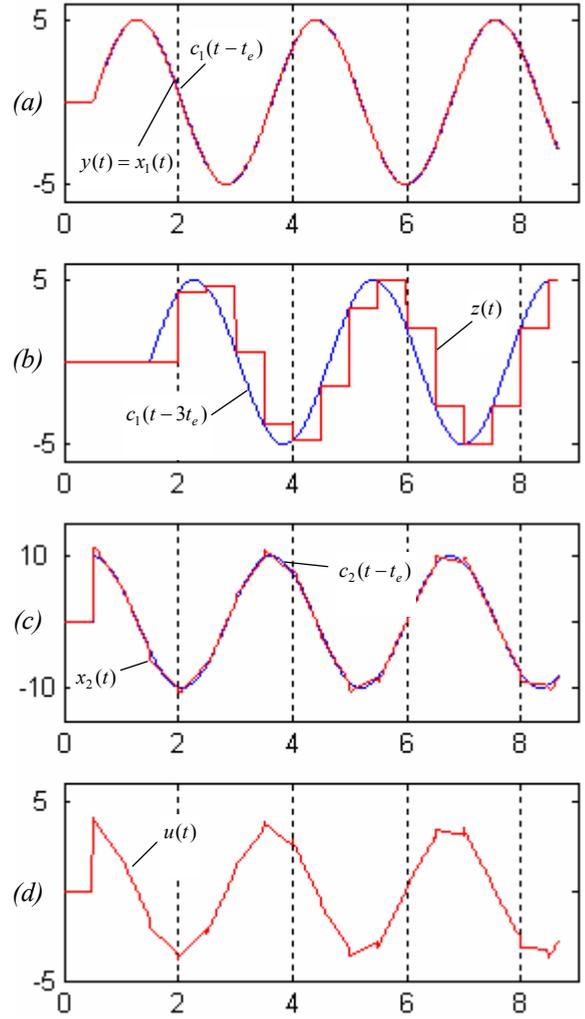


Fig.6. Tracking,  $t_e = 0.5s$  and  $N = 2$

### Results Comment

Fig.5 and Fig.6 illustrate the result of sampled tracking using two different type of sensors. In both cases, the sensor gives rise to a delay of 1s, but the values of  $t_e$  and  $N$  differ.

Fig.5a and Fig 6a show sampled tracking of  $c_1(t)$  by the plant's output  $y(t)$  which is here equal to the real cart position  $x_1(t)$ . Note that the  $y(t)$  curve intersects that of  $c_1(t - t_e)$  at every  $k.t_e$ , thus showing  $t_e$ -sampled tracking with a delay of  $t_e$ . Similarly, Fig.5c and Fig 6c show how the second state variable (speed) reaches its desired trajectory at switching instants.

In the same way, Fig.5b and Fig 6b, represent sampled tracking of  $c_1(t)$  by  $z(t)$  with a delay of  $(N + 1).t_e$ .

Fig.5d and Fig 6d illustrate the control command fed to the plant. Coming out of a PCS controller, we can notice its piecewise continuous nature.

Note that though we have shown the state's evolution for demonstration sake, we do not use it for feedback, since we assume it to be unavailable.

## Conclusion

The method that we present in this paper is appropriate for control of linear plants in cases where the only available feedback comes from a sensor delivering the plant's output vector in a delayed (of  $N.t_e$ ) and sampled (at  $t_e$ ) format. The control unit is based on a PCS controller whose switching matches the sample rate  $t_e$  of the sensor, thus designed for sampled feedback. Moreover, the use of PCS control has been adapted in cases this sampled feedback is delayed. The method, thus ensures sampled tracking at every  $k.t_e \forall k = 0,1,2,\dots$ , with a delay of  $t_e$  concerning the state and one of  $(N+1).t_e$  concerning the sensor's output such that  $x_k = c_{k-1}$  and  $z_k = C.c_{k-N-1} \forall k = 0,1,2,\dots$ .

Computer simulations showed that the method is reliable and moreover robust against time-variations of the plant's parameters ( $A$  and  $B$ ).

Although the main goal of the paper is to deal with a delayed and sampled *output* vector, the reader can also find techniques in cases of *state* and delayed *state* feedback. Those techniques are easily implemented, and also proved to be robust, even in the presence of noise in the state signal.

Note that in every case, the PCS controller show better efficiency for small values of  $t_e$ , which is the period at which the PCS controller's state switches. However, in cases of sampled feedback, the best that can be done to optimise the controller is to switch at a period equal to the sampling period of the feedback data.

As a perspective of our study, works are presently being carried out to optimise the PCS controller to ameliorate its behaviour between switching instants so as to enhance the tracking in this interval, especially in cases where  $t_e$  is relatively high. This optimisation is based on that given in [6].

Furthermore, we are undertaking real time experiments to test the present method on the real system of Fig.4.

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