Dynamic Axisymmetric and Non-Axisymmetric Buckling of Finite Cylindrical Shells in Propagating and Reflecting of Axial Stress Waves

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Abstract. In this paper, the axisymmetric and non-axisymmetric buckling of finite elastic cylindrical shell, which is impacted on the end by axial step loads, is discussed with the aid of the stress wave propagating and reflecting. By solving the disturbed equations, the bifurcation condition of the dynamic buckling, critical buckling load and buckling mode are obtained. The results show that when the thickness is not very small, non-axisymmetric critical buckling load is higher than axisymmetric one; that when the thickness is very small, non-axisymmetric buckling can occur first and that since the wave is reflected on the other end of the shell, the critical buckling load decreases further. The results are in agreement with the physical phenomenon in experiments.

Résumé: Dans cet article on discute, à l'aide de la propagation et de la réflexion des ondes, le flambage symétrique axial et non-symétrique d'une coque cylindrique élastique et de dimensions finies, impactée à une extrémité par un échelon de force axiale. En résolvant les équations de perturbations, on obtient la condition de bifurcation du flambage dynamique, la charge critique de flambage et le mode de flambage. Les résultats montrent que dans le cas où l'épaisseur de la coque n'est pas très mince, un flambage non symétrique peut d'abord apparaître et lorsque l'onde est réfléchie à l'autre extrémité de la coque, la charge critique de flambage continue de décroître. Les résultats théoriques sont en bon accord avec le phénomène physique dans les expériences.

1. INTRODUCTION

Since the 60s, the elastic dynamic buckling of shells has been paid more attentions than state problem. The axial impact problem has been studied concentrically in the dynamic buckling, and many properties have been found. Coppa [1] discovered in experiment that the mode of dynamic buckling is similar to that of static buckling. Almroth [2] and Lindberg [3] found that the buckling corrugations of shell take their forms in the initial stage. Their results implicate that no unloading exists when the buckling begins in originally plastic state. Zimcik [4], Tamura [5], Fisher [6] and Simitses [7] investigated the axial critical impact load under the condition of different initial imperfections. Xu [8] put a method to determine critical buckling loads of elastic-plastic shells with the aid of waves. Gordienko [9] took notice of non-axisymmetric buckling of shells subjected to impact axial load in the experiment. Since the ratio of the wall thickness to the radius of the shells he studied is only $R/h > 50$, it is the non-axisymmetric mode that buckling takes place with. Wang [10] conducted systematic experiments to investigate the dynamic plastic buckling of cylindrical shells under axial impact. They obtained some important and useful results, e.g. the existence of the first and the second critical velocities and the conclusion that, if the impact velocity is lower than the first critical velocity, the shell was uniformly compressed only, if it is lower than the second one and reaches the first one, it corresponds to a uniformly axisymmetric buckling mode, and if it is greater than or equal to the second critical velocity, non-axisymmetric and non-uniform buckling mode will appear. Han [11] pointed that with the increasing of ratio the critical buckling velocities decrease and approach to elastic one and that when the ratio reaches some value, non-axisymmetric buckling modes appears only. Chen [12] considered the effect of the stress wave in the experiment and their result showed that: the axisymmetric buckling modes occurs first, then two blade and three blade non-axisymmetric modes do and so on.

In this paper, we consider that the assumption of the initial imperfection is not indispensable. The bifurcation phenomenon and the effect of the propagation and the reflection of waves are important factors on the dynamic buckling of perfect cylindrical shells impacted by an axial load. On building the disturbance equations in propagating and reflecting of longitudinal waves, we can solve and obtain general solutions of the equation. Considered boundary conditions and the consistence conditions at wave fronts, the bifurcation condition of the dynamic buckling is obtained. Thus the condition can be used for critical buckling loads and the solution for buckling modes corresponding to a particular time. The results are in agreement with the physical phenomenon above.

2. DISTURBANCE EQUATIONS AND SOLUTIONS

Consider a the finite elastic cylindrical shell which is impacted by the axial step load on the one end ($x = 0$). The thickness is h, the medium surface radius R, the density ρ, the elastic modulus E and Poison ratio υ. The axial step load $N_x(0,t) = -N_0 H(t)$ where N_0 is a constant, t denotes time and $H(t)$ is step function: $H(t) = 1$ if $t \geq 0$; $H(t) = 0$ if $t < 0$. The axial wave induced by the impact load is longitudinal wave Let $x_e = c_e t$ be elastic wave fronts, where c_e is wave speed, and it can be written as [9] $c_e = [E/\rho(1-\upsilon^2)]^{1/2}$. Let us consider first reflecting only. For the ideal perfect shell the axial internal force, in the process of the propagation and reflection of wave, can be expressed as [9]: $N_x = N \quad 0 \leq x \leq x_e, t \leq l/c_e$; $N_x = 0 \quad x_e < x \leq l$, $t \leq l/c_e$; $N_x = N \quad 0 \leq x < x_r$, $t > l/c_e$; $N_x = 2N \quad x_r \leq x \leq l$, $t > l/c_e$. The governing equation expressed by the radial displacements $w(x,\theta,t)$ is

$$D\frac{\partial^4 w}{\partial x^4} + N_x\frac{\partial^2 w}{\partial x^2} + \frac{Eh}{R^2}w - \frac{\upsilon}{R}N_x + \frac{D}{R^4}\frac{\partial^4 w}{\partial\theta^4} + \frac{2D}{R^2}\frac{\partial^4 w}{\partial x^2\partial\theta^2} = 0, \tag{1}$$

where $D = Eh^3/12(1-\upsilon^2)$. Considering that in the impact period before the buckling occurs, the shell is a initial equilibrium and well-distributed stage, $w_0(x,\theta,t) = \upsilon RN_x/Eh$. The radial displacement can be expressed as $\hat{w}(x,\theta,t) = w(x,\theta,t) - w_0(x,\theta,t)$, where $w(x,\theta,t)$ is disturbed displacement. Let $W = \hat{w}/l$, $X = x/l$, $T = c_e t/l$, $\lambda = \beta\sigma_x/Eh$, $\mu = \beta^2/12(1-\upsilon^2)$, $\sqrt{\alpha\beta} = h/l$ and $\sqrt{\beta/\alpha} = R/l$. Eq.(1) can be rewritten as

$$\mu\frac{\partial^4 W}{\partial X^4} + \frac{\lambda}{\alpha}\frac{\partial^2 W}{\partial X^2} + W + \mu\left(\frac{\alpha}{\beta}\right)^2\frac{\partial^4 W}{\partial\theta^4} + 2\mu\frac{\alpha}{\beta}\frac{\partial^4 W}{\partial X^2\partial\theta^2} = 0. \tag{2}$$

In disturbed initial stage, $W(X,\theta,T) = \partial W(X,\theta,T)/\partial T = 0$. Let $W(X,\theta,T) = F(X,T)\cdot g(\theta,T)$. Considering the fact that $\theta = 0$ and $\theta = 2\pi$ are one position, the continuous conditions about function $g(\theta,T)$ are

$$g(0,T)=g(2\pi,T);\ \frac{\partial}{\partial\theta}g(0,T)=\frac{\partial}{\partial\theta}g(2\pi,T);\ \frac{\partial^2}{\partial\theta^2}g(0,T)=\frac{\partial^2}{\partial\theta^2}g(2\pi,T);\ \frac{\partial^3}{\partial\theta^3}g(0,T)=\frac{\partial^3}{\partial\theta^3}g(2\pi,T). \quad (3)$$

$g(\theta,T)$ can be expressed as

$$g(\theta,T)=C_1\cos(\eta\theta)+C_2\sin(\eta\theta) \quad (4)$$

where $\eta_n=n\ (n=0,1,2,\cdots)$, $C_k=C_k(T)\ (k=1,2)$. Equation (2) in terms of $W(X,\theta,T)$ is transformed into an equation in terms of $F(X,T)$:

$$\mu\frac{\partial^4 F}{\partial X^4}+(\frac{\lambda}{\alpha}-2\frac{\alpha}{\beta}\mu n^2)\frac{\partial^2 F}{\partial X^2}+[1+(\frac{\alpha}{\beta})^2\mu n^4]F=0 \quad (5)$$

When $n=0$, g is a constant and Eq.(4) degenerates into the axisymmetric problem [9]. The solution of Eq.(5) can be given by

$$F=\begin{cases}a_1\cos(\alpha_1 X)+a_2\sin(\alpha_1 X)+a_3\cos(\beta_1 X)+a_4\sin(\beta_1 X) & (\lambda\ge\lambda_q)\\ a_1 e^{\alpha_2 X}\cos(\beta_2 X)+a_2 e^{\alpha_2 X}\sin(\beta_2 X)+a_3 e^{-\alpha_2 X}\cos(\beta_2 X)+a_4 e^{-\alpha_2 X}\sin(\beta_2 X) & (\lambda<\lambda_q)\end{cases} \quad (6)$$

where $\beta_1,\alpha_1=\{(\lambda/\alpha-2\alpha\mu n^2/\beta)\pm[(\lambda/\alpha-2\alpha\mu n^2/\beta)^2-4\mu(1+\alpha^2\mu n^4/\beta^2]^{1/2}\}^{1/2}/(2\mu)^{1/2}$, $\beta_2,\alpha_2=[(1+\alpha^2\mu n^4/\beta^2)^{1/2}\pm(\lambda/\alpha-2\alpha\mu n^2/\beta)]^{1/2}/(2\mu)^{1/2}$, $\lambda_q=2\alpha\mu^{1/2}[1+(\alpha/\beta)^2\mu n^4]^{1/2}+2\mu n^2\alpha^2/\beta$, a_k $(k=1,2,3,4)$ is a function of time T.

3. BOUNDARY, CONSISTENCE AND BIFURCATION CONDITIONS

When the step load impacts the end of the shell, the shell is partitioned into several regions for propagating and reflecting of the wave. Before the reflection, the solution is equal to zero in the non-disturbed region of the wave since wave does not reach, i.e. $W_0(X,\theta,T)=0$, or $F_0(X,T)=0$. In the region of impacted end, the solution (6) should satisfy the boundary conditions. Considered simple support in this paper, the boundary condition can be expressed as $W(0,\theta,T)=0$, $\partial^2 W(0,\theta,T)/\partial X^2=0$, or

$$F(0,T)=0\ ,\quad \frac{\partial^2}{\partial X^2}F(0,T)=0\ . \quad (7)$$

Owing to Eq.(6), (7) can be combined into

$$\mathbf{B}_1\,\mathbf{a}_1=\mathbf{0} \quad (8)$$

where $\mathbf{a}_1=(a_1,a_2,a_3,a_4)^{\mathrm{T}}$, $\mathbf{B}_1$ is the matrix of coefficients determined by Eqs. (6) and (7). At the wave front X_e , The consistence conditions about F and F_0 can be obtained

$$F(X_e,T)=0\ ,\quad \frac{\partial}{\partial X}F(X_e,T)=0\ , \quad (9)$$

and written in a simple form as

$$\mathbf{B}_2\,\mathbf{a}_1=\mathbf{0}\ , \quad (10)$$

$\mathbf{B}_2=\mathbf{B}_2(\lambda,X_e)$. After reflection of wave, there is no harm in taking the end conditions of the reflection as:

$$\hat{F}(1,T)=0\ ,\quad \frac{\partial}{\partial X}\hat{F}(1,T)=0\ , \quad (11)$$

$\mathbf{B}_3 = \mathbf{B}_2(\hat{\lambda}, 1)$, $\hat{\lambda} = 2\lambda$, $\mathbf{a}_2 = (a_5, a_6, a_7, a_8)^{\mathrm{T}}$, and written as

$$\mathbf{B}_3\,\mathbf{a}_2 = \mathbf{0} \quad . \tag{12}$$

On the wave front X_r of the reflection, the consistence conditions can be expressed as

$$\begin{cases} F(X_r, T) = \hat{F}(X_r, T), \dfrac{\partial}{\partial X}F(X_r, T) = \dfrac{\partial}{\partial X}\hat{F}(X_r, T), \dfrac{\partial^2}{\partial X^2}F(X_r, T) = \dfrac{\partial^2}{\partial X^2}\hat{F}(X_r, T), \\ \dfrac{\partial^3}{\partial X^3}F(X_r, T) + (\dfrac{\lambda}{\alpha} - 2\dfrac{\alpha}{\beta})\dfrac{\partial}{\partial X}F(X_r, T) = \dfrac{\partial^3}{\partial X^3}\hat{F}(X_r, T) + (\dfrac{\hat{\lambda}}{\alpha} - 2\dfrac{\alpha}{\beta})\dfrac{\partial}{\partial X}\hat{F}(X_r, T). \end{cases} \tag{13}$$

where $\hat{F}$ is a function in $X_r \le X \le 1$, $\hat{\lambda} = 2\lambda$. Eqs.(13) are

$$\mathbf{B}_4\,\mathbf{a}_1 = \mathbf{B}_5\,\mathbf{a}_2 \quad . \tag{14}$$

Eqs.(6), (10), (12) and (14) can be rewritten as

$$\mathbf{A}\,\mathbf{a} = \mathbf{0}; \quad \mathbf{A} = \begin{bmatrix} \mathbf{B}_1^{\mathrm{T}} & \mathbf{B}_4^{\mathrm{T}} & \mathbf{0} \\ \mathbf{0} & -\mathbf{B}_5^{\mathrm{T}} & \mathbf{B}_3^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \tag{15}$$

where $\mathbf{a} = (\mathbf{a}_1^{\mathrm{T}}, \mathbf{a}_2^{\mathrm{T}})^{\mathrm{T}}$. Before reflecting, Eq.(15) is hold too and it is taken $\mathbf{a} = \mathbf{a}_1 = \mathbf{a}_2$, $\hat{\lambda} = \lambda$ and $\mathbf{B}_3 = \mathbf{B}_2$. If $\mathbf{a} = \mathbf{0}$, $W \equiv 0$, namely the buckling does not appear. The condition that Eq.(6) has non-zero solution is transformed into Eq.(15), i.e., at the bifurcation point, the determinant of coefficients is zero:

$$|\,\mathbf{A}\,| = 0 \quad . \tag{16}$$

From the bifurcation condition (16) and Eqs.(6) and (15), the critical buckling load and mode corresponding to a specific time can be determined.

4. CRITICAL BUCKLING LOAD AND MODE

When the end of the shell is impacted by the step load, the critical buckling load and mode can be determined by Eqs. (15) and (16). Let $\beta = 0.001$ and $\alpha = 0.1$, we can get the relation curves between the critical load and time in Fig.1. We define that n is the order in Eq.(5), which shows the stage of the non-axisymmetric buckling. In fact, it can be known that n represents the number of corrugations about the circle buckling. For fixed n, the critical load is of multi-branches, we call them the first branch, the second branch and so on from the lowest one. The branch expresses the number of corrugations about the axial buckling. Figure 1 gives the first three orders and branches of critical load curves. From these curves, it can be known that the critical loads decrease with time T and tend to static one ($n = 0$) [13]; For each branches, the critical load is greater to higher order. This phenomenon shows that non-axisymmetric critical load is larger than axisymmetric one and is agreement with experiments; Besides, that $T = 1$ is the time that the wave reaches the other end of the shell and when $T > 1$, it shows the case after reflecting.

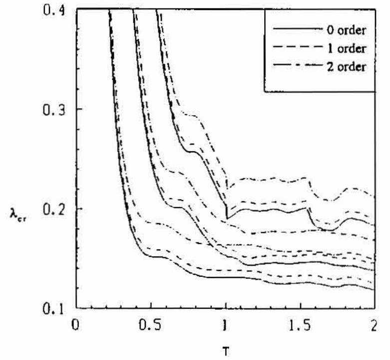
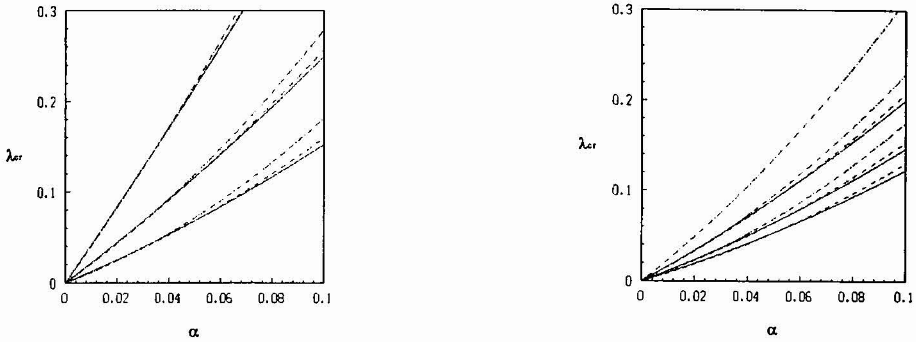


Figure 1 curves of $\lambda_{cr} - T$.

Figure 2 (n=0, 1, 2; a) for $T = 0.5$; b) for $T = 1.5$) shows two things: First, when α is larger, every order and branch critical loads is the bigger and the high order load is larger than the low order one; Then, the less the ratio of the wall thickness to the radius of the shells is, the easier the non-axisymmetric buckling

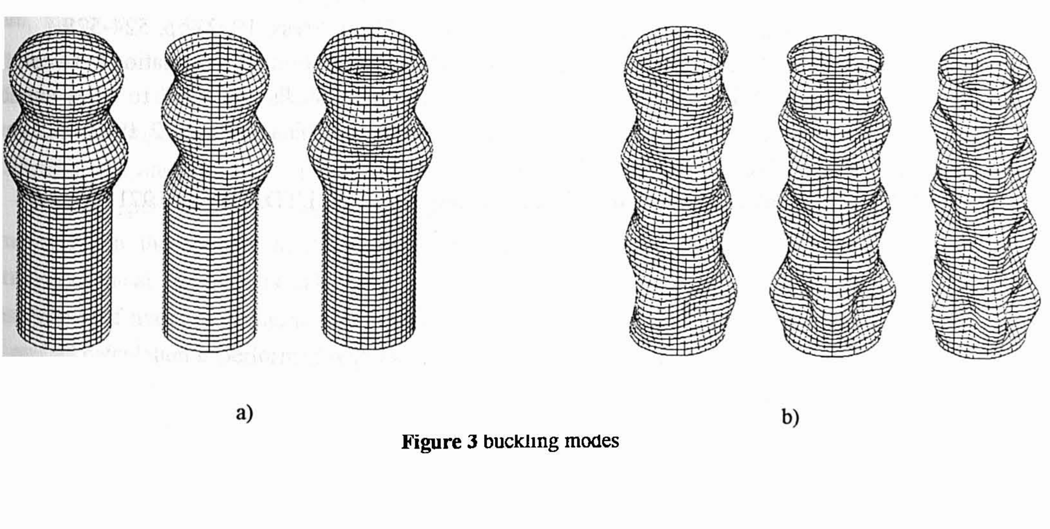


a) b)

Figure 2 curves of $\lambda_{cr} - \alpha$.

will take place, which is found in the experiment [12]. In this case, critical loads are concentrated considerably and when the time T takes a large value, these phenomena are more obvious after reflection.

According to the critical buckling loads, we can obtain the initial buckling modes at time T. Figure 3 a) gives three distributions of $W(X, \theta, T)$ in n=0, 1, 2 respectively before wave reflecting ($\beta = 0.01$; $\alpha = 0.1$; $T = 0.5$). In this Figure, first one is the form of axisymmetric buckling. After wave reflecting ($\beta = 0.01$; $\alpha = 0.1$; $T = 1.5$), buckling modes are shown for n=3, 4, 5 respectively in Fig.3 b). With order n increasing, circle buckling corrugations (θ -direction) raise, which can be showed in Eq.(4). The corrugations are more concentrated after wave reflecting and the degree of concentration depends on the order and branch of the critical buckling load.



a) b)

Figure 3 buckling modes

5. CONCLUSION

It is for the propagation and reflection of longitudinal wave that the buckling of the shell takes place and grows. The buckling can be axisymmetric and non-axisymmetric; The time of buckling relates to the impact load. In general, when the load is large, the buckling occurs in short time; The critical load is determined by the condition of bifurcation and is dependent on properties of the material, geometric parameters and time; With the wall thicknesses increasing, non-axisymmetric critical load is larger than axisymmetric one and with it decreasing, non-axisymmetric buckling can appear first; Axial and circle buckling forms are dependent on the load, wall thickness, and so on; When the order of the non-axisymmetric buckling increases, the circle buckling corrugations raise; Close the end impacted, axial buckling corrugations are the more concentrated than that near the elastic wave front or reflected end; Since the wave is reflected on the other end of the shell, the critical buckling load decreases further; The theory and the experiment are identical.

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