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An Elastic-Visco-Plastic Damage Model: From Theory to Application

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Abstract. An energy-based two-variable damage theory is applied to Bodner's model. It gives an elastic-visco-plastic damage model. Some theoretical details are described in this paper. The parameters identification procedure is discussed and a complete set of parameters for an aluminium is presented. Numerical modelling of the laboratory tests are used to validate the model. An industrial aeronautic rod fabrication process is simulated and some numerical results are presented in this paper.

1. INTRODUCTION

Bodner's elastic-visco-plastic theory represents one of the first unified constitutive models formulated explicitly without a yield criterion [1]. It has been widely used since its appearance because of its ability to represent various material properties and to describe material response for steady and varying strain rate over a wide range of conditions.

In order to predict the local fracture initiation and its further evolution in a material, Bodner has extended this elastic-visco-plastic model to include damage using an one-scalar-variable damage theory. In this model, the damage does not influence the elastic answer but reduces the hardening [2,3].

A different approach to include damage in an elasto-plastic constitutive law was used in [4]. Its principal points are as follows: 1. using two scalar damage parameters d and δ (deviatoric and volumetric components); 2. using the energy equivalence between damage and virgin material states. The application of this approach to Bodner's model may be realised within the same framework [5]. As Bodner's model assumes no yield surface, an explicit equation is used to describe directly the damage evolution.

In this paper, we give a theoretical description about this elastic-visco-plastic damage model. The parameters identification procedure is discussed and the material parameters identified for an industrial aluminium are presented. For checking the ability of the developed constitutive model to represent the material behaviour up to the failure, various 2D or 3D fracture tests are carried out in laboratory and then simulated numerically. The comparison between the numerical and the experimental results is shown.

The calibrated damage model is used in the 2D/3D finite element (F.E.) modelling of an aeronautic rod fabrication process. Satisfactory results, including the exact shape of forged pieces, the distribution of the stresses and the plastic deformations and possible initiation of fracture during the forging process, are obtained. They are helpful to optimise the industrial fabrication process and to control the products quality.

2. BODNER'S ELASTIC-VISCO-PLASTIC DAMAGE MODEL

2.1. Bodner's constitutive law

The general formulation of Bodner's model [1] is based on the decomposition of the total strain rate into elastic and inelastic components which are both non-zero for all loading and unloading conditions:

$$\dot{\underline{\epsilon}} = \dot{\underline{\epsilon}}^e + \dot{\underline{\epsilon}}^p \quad (2.1)$$

where $\underline{\dot{\epsilon}}$ is the symmetric part of the velocity gradient tensor. The generalised Prandtl-Reuss flow law is used together with the volume conservation to establish a tensorial relation between the deviatoric flow stress $\underline{S}$ and inelastic strain rate $\underline{\dot{\epsilon}}^p$:

$$\underline{\dot{\epsilon}}^p = \frac{D_0}{\sqrt{J_2}} \exp\left(-\frac{1}{2} \left(\frac{Z^2}{3J_2}\right)^n\right) \underline{S} \quad (2.2)$$

where D_0 is an assumed limit value of plastic strain rate in shear for large J_2 ; n is the rate sensitivity coefficient; Z is a total scalar hardening variable, it is the sum of the isotropic and directional parts:

$$Z = K + Z_D \quad (2.3)$$

The directional component Z_D is given by:

$$Z_D = \underline{\beta} : \underline{u} \quad (2.4)$$

where $\underline{\beta}$ is a symmetric second order tensor and

$$\underline{u} = \frac{\underline{\sigma}}{\sqrt{\underline{\sigma} : \underline{\sigma}}} \quad (2.5)$$

is the directional index for hardening, assumed to be the current stress direction.

The isotropic and directional hardening are both supposed to occur under the action of two simultaneously competing mechanism: a hardening process proceeding with deformation and a softening or recovery process evolving with time:

$$\dot{K} = m_1(K_1 - K)\dot{W}^p - A_1 K_1 \left(\frac{K - K_2}{K_1}\right)^{r_1} \quad (2.6)$$

$$\dot{\underline{\beta}} = m_2(D_1 \underline{u} - \underline{\beta})\dot{W}^p - A_2 K_1 \left(\frac{\sqrt{\underline{\beta} : \underline{\beta}}}{K_1}\right)^{r_2} \frac{\underline{\beta}}{\sqrt{\underline{\beta} : \underline{\beta}}} + (\underline{\Omega} \underline{\beta} - \underline{\beta} \underline{\Omega})$$

where $\dot{W}^p$ is the plastic work rate. The initial value of K at zero inelastic strain is K_0 ; K_1 is the maximum value of K ; K_2 is the minimum (or stable) value of K at a given temperature at which creep is occurring. The initial and minimum (or stable) values of $\underline{\beta}$ are zero corresponding to the isotropic state. The last term of second equation of (2.6) is due to Jaumann objective correction used for large strain, where $\underline{\Omega}$ is the antisymmetric part of the velocity gradient tensor.

There are totally 14 material parameters in Bodner's model. They are: E , ν , D_0 , K_0 , K_1 , K_2 , D_1 , m_1 , m_2 , A_1 , r_1 , A_2 , r_2 and n . If the terms related to the thermal recovery are omitted (this is the case in the following discussion), this number may be reduced to 10. The physical meanings of these parameters are given in Table 3.1.

2.2. Damage theory

Two scalar damage variables, the deviatoric component d and the volumetric component δ , are used to represent the average material degradation which reflects the various types of damage at the micro-scale due to nucleation, growth and coalescence of voids, to micro-cracks and to other microscopic defects. According to Ladevèze [6], these two variables are necessary to modify the elastic moduli and the Poisson's ratio with damage growth. The stress relations between damage and virgin states are defined as:

$$\frac{\underline{\hat{\sigma}}}{(1-d)} = \underline{\bar{\sigma}} \quad \frac{\bar{\sigma}_m}{(1-\delta)} = \underline{\bar{\sigma}}_m \quad (2.7)$$

where $\underline{\hat{\sigma}}$ is the deviatoric stress tensor and $\bar{\sigma}_m$ is the mean stress. They are defined for the damage state. $\underline{\bar{\sigma}}$ and $\bar{\sigma}_m$ are their effective values, defined for the virgin state.

For continuum damage models, various equivalence hypotheses have been proposed in order to transform the damage state into virgin state, such as strain equivalence [7], stress equivalence [8], elastic energy equivalence [9]. From the viewpoint of energy conservation, the energy equivalence may be of more physical significance. In this case, we have:

$$\bar{\underline{\varepsilon}}^e = (1-d)\hat{\underline{\varepsilon}}^e \quad \bar{\varepsilon}_m^e = (1-\delta)\varepsilon_m^e \quad (2.8)$$

Here, $\hat{\underline{\varepsilon}}$ is the deviatoric strain tensor and ε_m the mean strain. As to the inelastic components between the damage and the virgin states, the conservation of the plastic work dissipation gives the following relation:

$$\bar{\underline{\varepsilon}}^p = \underline{\varepsilon}^p (1-d) \quad (2.9)$$

As the Bodner's model assumes no yield surface, this characteristic is kept for the damage growth: it is described directly by an explicit expression. This approach is classical [10,11,12]. One possible choice proposed for an one-damage-variable (D) model is given in [13] for the uniaxial case:

$$\dot{D} = \left(\frac{\langle \sigma - \sigma_D \rangle}{A(1-D)} \right)^s |\dot{\varepsilon}^p|^r \quad (2.10)$$

It is adapted to our two-damage-variable model and rewritten in triaxial case as follows:

$$\dot{d} = \frac{1}{2(1-d)} \left(\frac{\langle \chi(\underline{\sigma}) - \sigma_D \rangle}{A(1-d)} \right)^s \cdot (\dot{\varepsilon}_{eq}^p)^r \quad (2.11)$$

with $\dot{\varepsilon}_{eq}^p$ the equivalent plastic strain rate and

$$\langle \chi(\underline{\sigma}) - \sigma_D \rangle = \begin{cases} \chi(\underline{\sigma}) - \sigma_D & \text{if } (\chi(\underline{\sigma}) - \sigma_D) \geq 0 \\ 0 & \text{if } (\chi(\underline{\sigma}) - \sigma_D) < 0 \end{cases} \quad (2.12)$$

where $\chi(\underline{\sigma})$ is the triaxiality function used to extend the uniaxial stress state to triaxial one. We use [13]:

$$\chi(\underline{\sigma}) = (1-\alpha)\sqrt{3J_2} + 3\alpha\sigma_m \quad (2.13)$$

The principal advantage of this definition is that only one parameter α is introduced to distinguish tension from compression or from other stress states.

The evolution of volumetric damage variable is supposed to be in dependence on the deviatoric one:

$$\dot{\delta} = \langle \tau \rangle \dot{d} \quad (2.14)$$

in which $\langle \tau \rangle = \frac{\delta}{d}$ for tension and $\langle \tau \rangle = 0$ for compression. τ is usually constant. This can be verified experimentally.

We have finally 6 damage parameters to be determined. They are listed in Table 3.1.

2.3. Bodner's model with damage

According to the coupled approach of damage theory, the effective stresses and strains follow the chosen behaviour law. The application of above damage theory to Bodner's model gives the following relations:

$$\bar{\underline{\varepsilon}}^p = \lambda \bar{\underline{\sigma}} \quad \text{and} \quad \lambda = \frac{D_o}{\sqrt{J_2}} \exp \left(-\frac{1}{2} \left(\frac{Z^2}{3J_2} \right)^n \right) \quad (2.15)$$

where $\bar{J}_2 = \frac{1}{2} (\bar{\underline{\sigma}} : \bar{\underline{\sigma}})$. Or with the conception of the "true" stress and strain rate:

$$\underline{\varepsilon}^p = \lambda \hat{\underline{\sigma}} \quad \text{and} \quad \lambda = \frac{D_o}{\sqrt{J_2(1-d)}} \exp \left(-\frac{1}{2} \left(\frac{Z^2(1-d)^2}{3J_2} \right)^n \right) \quad (2.16)$$

3. IDENTIFICATION OF THE PARAMETERS

3.1. Parameters identification procedure

The parameters identification procedure is summarised in Fig.3.1. Bodner's parameters and damage parameters are identified separately. The first ones are obtained with a classical procedure used in [14], except that the identification is not performed over the original but the effective strain-stress curves. For getting this effective modification, one should realise the so-called damage tests.

Fig. 1. Material parameters identification procedure

One of these tests is loading-unloading uniaxial tests. For each cycle of loading-unloading, we measure the current elastic moduli $\bar{E}$, $\bar{G}$ and $\bar{\chi}$. The damage variables are calculated by:

$$d = 1 - \sqrt{\frac{\bar{G}}{G_0}}, \quad \delta = 1 - \sqrt{\frac{\bar{\chi}}{\chi_0}} \quad \text{and} \quad \tau = \frac{\delta}{d} \quad (3.1)$$

As it is not certain that damage grows identically for different stress states, the loading-unloading tests are carried out in tension and in compression. We get so two damage evolution curves, one for tension and another for compression. They are generally different one from another.

In the triaxiality function (2.13), the parameter α describes such difference. For uniaxial tension and compression cases, this function becomes respectively:

$$\chi(\underline{\sigma}) = \begin{cases} \sigma & \text{for uniaxial tension} \\ (1-2\alpha)\sigma & \text{for uniaxial compression} \end{cases} \quad (3.2)$$

In uniaxial tension, α disappears. In the other stress states, α plays an important role in triaxiality function, and so in damage evolution. In uniaxial compression, $\alpha = 0$ means that damage grows in the same manner as in tension. $\alpha \geq 0.5$ means no damage evolution. So, a normal value of α is between 0 and 0.5.

The parameters σ_D , S and A are obtained from experimental damage curve in tension with the so-called "inverse method" (see e.g. [15]) by matching the theoretical model (2.11) to the measured curve with an iteration procedure. Fixing them, we can identify α from experimental damage curve in compression. The final parameter τ is obtained directly from the ratio of δ to d of the tension test curves.

3.2. An industrial aluminium

Modelling of the industrial process of the aeronautic rod fabrication is our final objective. So we calibrate the Bodner's damage model for the material used in one industry. It is in fact a common commercial

aluminium submitted to a special thermal treatment for improving its mechanical behaviour. The material parameters obtained by the described procedure are given in Table 3.1.

Table 3.1. Material parameters

parameters		physical meanings	values	units
Bodner's parameters	E	Young's elastic modulus	72505	Mpa
	ν	Poisson's ratio	0.31	-
	D_0	assumed limiting plastic-shear strain rate	10×10^8	s^{-1}
	K_0	initial isotropic hardness	159.514	Mpa
	K_1	maximum or limiting isotropic hardness	529.267	Mpa
	K_2	minimum or stable isotropic hardness	159.514	Mpa
	D_1	maximum directional hardness	311.371	Mpa
	m_1	hardening coefficient of isotropic hardness	0.025	Mpa^{-1}
	m_2	hardening coefficient of directional hardness	0.500	Mpa^{-1}
	n	strain rate sensitivity coefficient	2.436	-
damage parameters	S	stress function exponent in damage model	3.669	-
	A	stress function constant in damage model	247.53	Mpa
	σ_D	damage threshold stress	0	Mpa
	r	exponent of equivalent plastic strain-rate in damage model	1	-
	τ	ratio between volumetric and deviatoric damage	2.500	-
	β	parameter in triaxiality function	0.180	-

4. MODEL VALIDATION

In order to check its ability to represent the material behaviour up to the failure, the developed model has to be validated. We have performed a series of tests in laboratory. They are chosen to represent different load and fracture modes. As the material is only available in a tube shape, prepared for the aeronautic rod fabrication, the validation tests are designed according to this fact. These tests are then simulated with our non-linear F.E. code LAGAMINE. Some numerical results and their comparison with the experimental measured values are shown in this section.

4.1. Notch test

The first example is a notch test, see Fig. 4.1. The specimen is discretised in axisymmetric 4-node mixed finite elements. The force versus relative displacement between A and A' is given in Fig. 4.2.

4.2. Tensile test of holed specimen

The second validation test is shown in Fig. 4.3. The specimen is cut from the tube wall and is holed. This is a three-dimensional problem. The specimen is discretised in 8-node 3D mixed finite elements (Fig. 4.4). The comparison between the numerical and experimental result is presented in Fig. 4.5.

4.3. Shear test of holed tube

The third example is a shear test. The end of the tube is holed as shown in Fig. 4.6 for getting the shear-dominated load. It is discretised in 8-node mixed finite elements (Fig. 4.7). The numerical result in comparison with experimental one is given in Fig. 4.8.

4.4. Discussion

The numerical results of the validation tests presented in this section are generally satisfactory. Their

details are in course of examination. Especially, the comparison of numerical fracture initiation and growth with the experimental video records will refine our conclusions and modify if necessary the numerical model for improving its prediction of the crack growth. Some techniques of mesh refinement should be added after fracture initiation.

Fig. 4.1. Notch test: the dimension of the specimen and its F.E. mesh (one quarter is simulated)

Fig. 4.2. Notch test: force versus relative displacement (A-A')

Fig. 4.3. Tensile test of holed specimen

Fig. 4.4. Tensile test of holed specimen: 3D F.E. mesh (right half is simulated)

Fig. 4.5. Tensile test of holed specimen: force versus relative displacement (A-A')

Fig. 4.6. Shear test of holed tube: the dimension of the specimen

Fig. 4.7. Shear test of holed tube: 3D F.E. mesh (one quarter is simulated)

Fig. 4.8.: Shear test of holed tube: punch force versus its vertical displacement

5. APPLICATION

The F.E. modelling of the aeronautic rod fabrication process (see [16]) is realised using the elastic-visco-plastic damage law described in the previous sections. This process is composed of several stages of rotary forging, see Fig. 5.1 and Fig. 5.2. An automatic 2D/3D simulation procedure has been developed. It includes 2D meshing, axisymmetric modelling, 3D remeshing, 2D/3D information transfer, several times of replacement of hammer system, etc. They are executed one by one automatically. Final numerical results are obtained directly from the introduction of the necessary industrial data and the material parameters.

In Fig. 5.3 and Fig 5.4, we give the deformed shapes and the damage distribution after each fabrication stage.

6. CONCLUSIONS

An energy-based two-variable damage theory is introduced in Bodner's constitutive law. This elastic-visco-plastic damage model is used to simulate the aeronautic rod fabrication process. The main objective is to get a better representation of material mechanical behaviour up to the failure for the industrial

problems. The actual numerical results show that we are in the good direction. The modelling of the industrial problem gives the satisfactory results. Among others, exact final shape of rod and the dangerous region where the fracture is more likely to appear are well predicted. Based on these results, the initial tube shape, the machine system or the fabrication process can be modified. This modification will be essential for improving the products quality and optimising the industrial fabrication process.

Fig. 5.1. Industrial aeronautic rod fabrication process

Fig. 5.2 Hammer system of a forging machine

Fig. 5.3. Forged tube after different fabrication stages

Fig. 5.4. Damage distribution after different fabrication stages

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