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N. O'Dowd, J. Sumpter. Effect of Thermomechanical Loading on Near Tip Constraint. Journal de Physique IV Proceedings, 1996, 06 (C6), pp.C6-539-C6-548. 10.1051/jp4:1996654 . jpa-00254487

HAL Id: jpa-00254487

<https://hal.science/jpa-00254487>

Submitted on 1 Jan 1996

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Effect of Thermomechanical Loading on Near Tip Constraint

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Abstract. Two parameter approaches to elastic-plastic fracture mechanics were introduced to remove some of the conservatism inherent in the one parameter approach based on the J integral, ([1]) and to account for constraint effects on fracture toughness. It was shown in [2], [3] and [4] among others that much of the dependence of fracture toughness on specimen geometry could be explained by two parameter fracture theories based on Q or T . This paper examines the effect of residual or thermal stress fields on constraint in a mechanically loaded body. Finite element analyses of representative thermal loadings which give rise to high and low constraint stress fields (high and low T or Q) have been conducted. The structure is then subjected to mechanical loading, both tension and bending dominated to assess the effect on the near tip constraint. It is observed that the two parameter structure of the fields is maintained and values of Q stress have been obtained from the analyses. The thermal loading initially has a strong effect on the Q value but at higher loads when the mechanical loading dominates, this effect is much weaker. The ability of a two parameter approach using the T stress to account for constraint is assessed

1. INTRODUCTION

Thermal and residual stresses can often arise in a structural component and must be accounted for in a defect assessment of the component. If the body remains elastic the overall response may be obtained by simple superposition since the material response is linear. However, for non-linear elastic-plastic behaviour a finite element analysis of the full thermo-mechanical cycle must be carried out to predict the effect of residual stress on the overall response. When thermal strains are present a path independent J integral can still be defined and under small scale yielding J may be used to characterise the near tip stress and strain fields. Numerous schemes for estimating J under thermo-mechanical loading have been proposed, e.g. [5], [6]. The emphasis of this paper is on constraint effects, so these schemes will not be discussed in detail, however some discussion on the effect of thermal stress on J will be included.

The $J - Q$ approach was introduced in [7] and [8]. It was shown that a two parameter description using J and a constraint parameter Q fully characterises the near tip stress and strain states in a range of crack tip geometries. The parameter Q is determined from a finite element analysis and is the difference between the actual hoop stress and the reference field hoop stress. It was shown that Q is a measure of hydrostatic stress so that the near tip fields may be represented as

$$\sigma_{ij} = (\sigma_{ij})_{SSY} + Q\sigma_0\delta_{ij}, \quad \text{for } r > J/\sigma_0, \quad |\theta| \leq \pi/2. \quad (1)$$

The second parameter Q depends on the geometry and deformation and has been determined for a wide range of crack geometries under mechanical loading, [9]. For the cases examined, the value

of Q ranges from about -2 to about 0.5 ; low Q corresponding to low crack tip triaxiality, (low constraint) and positive Q corresponding to high triaxiality (high constraint). The elastic T stress, the second term in the Williams expansion of the elastic crack tip fields [10], has also been used to characterise constraint, e.g. [3], and under small scale yielding conditions the T and Q approaches are equivalent.

In this paper the effect of thermal loading on constraint, as measured by Q , is examined. Different temperature distributions giving rise to different near tip conditions were considered. These were not chosen for their physical relevance but to allow a systematic examination of different types of residual stress field on the resultant mechanical response. Bending and tensile loads are subsequently applied to the specimen and the effect of the residual stress on constraint examined.

2. MATERIAL PROPERTIES AND MODEL GEOMETRY

The material is assumed to be a linear elastic plastic power law hardening material and the stress strain law has the form in uniaxial tension:

$$\begin{aligned} \epsilon/\epsilon_0 &= \sigma/\sigma_0 & \sigma < \sigma_0 \\ &= (\sigma - \sigma_0)/\sigma_0 + (\sigma/\sigma_0)^n & \sigma > \sigma_0 \end{aligned} \quad (2)$$

where σ_0 and ϵ_0 are the yield stress and strain respectively. Results are presented for $n = 5$ only, but they are expected to be representative of any moderately hardening power law material. An edge crack bar under varying temperature fields is subjected to mechanical bending or tensile loading. An edge cracked bar was used rather than a center-cracked bar so that the same thermal loading could be applied to both bend and tension geometry. This allows a systematic examination of the effect of thermal stresses on constraint. Both shallow crack, $a/W = 0.1$, and deep crack bars, $a/W = 0.4$, are examined. The analysis was carried out using ABAQUS [11] and a typical mesh contains about 2000 four noded plane strain elements. To avoid numerical problems associated with plastic incompressibility reduced integration elements are used in the analysis.

3. ANALYSIS OF CRACKED GEOMETRIES

Figure 1: Schematic of tension and bend geometries

The two specimens examined are illustrated schematically in Figure 1. Note that for the tension geometry a uniform remote tension was not used when $a/W = 0.4$ as this led to a resultant bending moment on the ligament and a relatively high constraint geometry. By using a linearly varying stress a high negative T value was achieved, $T\sqrt{\pi a}/K \approx -0.8$. For the deeply cracked bend bar the T stress value is very close to zero. For the shallow crack tension geometry, a uniform remote tension was used giving $T\sqrt{\pi a}/K \approx -0.5$. For the shallow cracked bend bar, $T\sqrt{\pi a}/K \approx -0.4$. The Q values for the four geometries analysed are shown in Fig. 2. In each case the load, P , is normalised by the limit load per unit thickness, P_0 . The limit load was calculated from a finite element analysis of each geometry with the material response given by perfect plasticity.

It may be seen that the bend geometry gives a positive Q when $a/W = 0.4$, but a negative Q for $a/W = 0.1$. The tension geometry gives a negative Q value for both crack depths.

Figure 2: Q values plotted against normalised load for edge cracked tension (ect) and edge crack bend (ecb) geometries, (a) deep crack $a/W = 0.4$, (b) shallow crack, $a/W = 0.1$.

3.1 Thermal loading of deep crack specimen

In the analysis, the specimens are subject to both thermal and mechanical loading. We first examine the deep cracked specimen, $a/W = 0.4$. Two temperature distributions designated, T_a and T_b were examined. These are illustrated in Fig. 3, where x measures distance through the plate. It should again be emphasised that these distributions were chosen only for convenience and are not related to any physical thermal stress problem. Note that the temperature only varies through the plate thickness and does not depend on the y coordinate. The T stress associated with these temperature distributions was calculated and it was found that for the T_a distribution, $T\sqrt{\pi a}/K \approx -0.8$ and for the T_b distribution, $T\sqrt{\pi a}/K \approx 1.2$. Note that the normalised T value for the T_a distribution is identical to that for the plate loaded in tension and the consequence of this will be seen later. Both thermal distributions give rise to the same J level, $J/(a\sigma_0) = 4 \times 10^{-3}$ but very different near tip constraint conditions as illustrated in Fig. 4. Here the stress fields are plotted for the two temperature distributions at $J/(a\sigma_0) = 4 \times 10^{-3}$. Also included in Fig. 4 is the small scale yielding $T = 0$ reference field. It is seen that the T_a distribution gives rise to a low constraint field, $Q \approx -0.6$ and T_b a high constraint field, $Q \approx 0.4$.

Figure 3: Temperature distributions used in thermal analysis.

Figure 4: Stress field due to temperature distributions of Fig. 3.

Figure 4 shows the stress distribution when the temperature in the plate is given by distribution T_a and T_b . For each case the temperature is ramped up uniformly from room temperature and thus we can examine the variation in constraint as J increases.

Figure 5: Q plotted against normalised J for deeply cracked bar. T - Q estimate from Eqn (3) shown by symbols.

$$Q = 0.7639 \left(\frac{T}{\sigma_0} \right) - 0.32 \left(\frac{T}{\sigma_0} \right)^2 - 0.0906 \left(\frac{T}{\sigma_0} \right)^3 \quad (3)$$

The T - Q estimate is also shown in Fig. 5 and it is seen that there is good agreement between the T stress estimate and the numerical Q value for both temperature distributions.

3.2 Tensile loading of deep crack specimens

In what follows we consider the effects of these thermal stress fields, with the same J value but very different T and Q values, on the subsequent mechanical loading.

3.2.1 J results for thermomechanical loading

In Fig. 6 the J results for the bars with different thermal stress field subjected to remote tension is shown. Fig 6a gives the variation of normalised J with normalised load for the two thermal distributions. Note that the limit load P_0 is not adjusted for the thermal loading, i.e. the same reference load is used for all three curves. It is seen that, in this case, the total J values for the combined thermal and mechanical loading are very similar. Schemes for estimating the total J have been discussed in [5] and [6]. For small amounts of plasticity, simple superposition may be used. Note that since J varies with the square of the load, K values rather than J values should be added.

If we write

$$K_{\text{estimate}} = K_t + K_l \quad (4)$$

where K_t is the K value from thermal loading alone and K_l is the K value from mechanical loading alone, in terms of J we get

$$J_{\text{estimate}} = J_t + J_l + 2\sqrt{J_t J_l} \quad (5)$$

In this case $J_t = 4 \times 10^{-3} a \sigma_0$ and a comparison between Equation (5) and the actual J value is shown in Fig. 6b. Only the T_a distribution is shown as the J values are almost identical for both distributions. The good agreement between the J estimate and total J is evident up to $P/P_0 \approx 0.8$. For higher levels of load it was proposed in [5] that a modified reference load be used to account for the effect of the thermal loading on the total J , the effectiveness of such an approach was not investigated here.

Figure 6: (a) Normalised J due to thermo/mechanical loading and mechanical loading alone. (b) Comparison between J estimate and actual J values.

3.2.2 Constraint effects under thermomechanical loading

The near tip normalised stress distribution for the case of temperature distribution T_a and subsequent tension loading is shown in Fig. 7. The stress due to thermal loading alone is shown by the dash line and that due to the subsequent mechanical loading in the solid line. These distributions are very similar to those presented in [7] and [8], i.e. the normalised opening stress, σ_{22} depends strongly dependent on loading, but the shear stress $\sigma_{r\theta}$ much less so. Similar results are observed for the other temperature distributions and mechanical loadings, i.e. the structure of the $J-Q$ field appear to be maintained for combined thermo-mechanical loading. In all cases the loading remains nearly proportional ahead of the crack, i.e. deviatoric stress components increases monotonically and in fixed proportion to one another.

Figure 7: (a) Opening stress ahead of the crack for thermo-mechanical loading. (b) Shear stress at $r/(J/\sigma_0) = 2$ for thermo-mechanical loading

In Fig. 8 the Q values are shown for the two temperature distributions under tension loading. Q is plotted against normalised J in Fig. 8a and against normalised load in Fig. 8b. Both the thermal and mechanical $J-Q$ history is shown in Fig. 8a. The $J-Q$ history for the edge crack tension specimen in the absence of thermal loading is shown by the solid line. Note that initially the T_a curve follows this line very closely. This is because as stated earlier the T stress values for these two loadings are almost identical.

For both temperature distributions, the mechanical loading is applied when $J/(a\sigma_0) = 4 \times 10^{-3}$. The temperature then remains constant and the load is increased incrementally. For the T_a distribution it may be seen that after the mechanical load is applied the Q value remains almost

constant until $J/(a\sigma_0) = 0.01$. (This is also reflected in the stress distribution shown in Fig. 7a. The fact that Q is constant does not imply that the stress is constant since J continues to increase.)

Figure 8: Q values for edge crack tension geometry with different thermal stress distributions under thermomechanical loading, (a) Q plotted against normalised J (b) Q plotted against normalised load.

Clearly the thermal stress has a strong effect on the constraint. For both temperature distributions at intermediate levels of deformation, the Q value is higher than for mechanical loading alone at the same overall J level. However, it is seen that as the levels of plasticity increase the result for the thermo-mechanical loading approaches that due to the mechanical load alone.

It is perhaps more enlightening to consider the effect on constraint at the same applied load, rather than the same J , since J is made up of a thermal and mechanical part. This is shown in Fig. 8b. Here the initial Q value at $P/P_0 = 0$, is that due to the thermal loading. It may be seen that the low constraint temperature field, T_a , tends to reduce the constraint, at the same applied load and the high constraint temperature field, T_b , tends to increase the constraint. Thus it may be noted that the edge crack tension geometry, which is nominally a low constraint geometry, can behave as a high constraint geometry in the presence of a high constraint thermal stress field. For loads $P/P_0 > 1.5$ the effect of the thermal loading on constraint is rather small.

Figure 9: Comparison with T stress predictions for edge crack tension geometry.

3.3 Deep crack specimen under bending load

The effect of the same thermal stress fields on a high constraint bending field is next examined. As before, we first examine the effect of the combined thermal and mechanical loading on J . This

It was shown that the T stress provides a good estimate for Q under thermal loading. Under combined thermomechanical loading an attractive approach would be to use the total T stress, due to thermal and mechanical loading, to obtain an estimate for Q . This is illustrated in Fig. 9, the actual Q value given by the lines and the T estimate by the symbols. It may be seen that for the T_b distribution good agreement is obtained between the T stress prediction and the actual Q value up to $P/P_0 = 1$. However, for the low constraint temperature field, T_a , the agreement is considerably poorer and in fact the T stress does not provide a reliable prediction even for very low mechanical loadings.

is illustrated in Fig. 10. The solid line shows the J value in the absence of a thermal load and the dashed lines show the J value from the different thermal loadings. Note that in this case, even though the J due to the thermal loading is the same for temperature distribution T_a and T_b the response to the combined loading is somewhat different even at low loads. This suggests that the linear superposition approach used earlier will not be successful in predicting J for both geometries. The J estimate is also included on Fig. 10 and it is seen that this estimation agrees more closely with the T_b distribution than the T_a distribution.

Figure 10: J values versus normalised load for edge crack bend geometry. (b) Comparison between J estimate and actual J values.

In Fig. 11 the Q values are plotted against normalised J and normalised load for the edge crack bend geometry. Note that for the T_a distribution, the Q value first decreases under the thermal loading and then increases due to the superimposed bending field. Again it is seen that at high loads, $J/(a\sigma_0) > 0.1$, $P/P_0 > 2.5$ the effect of the thermal loading on constraint is negligible.

For this geometry under bending alone, $T/\sigma_0 \approx 0$, so $Q \approx 0$ for $J/(a\sigma_0) < 0.01$. Using a combined T stress approximation would thus predict that for the thermomechanical loading at low loads, the Q value will remain constant at 0.4 for the T_b distribution and at -0.6 for the T_a distribution.

Figure 11: (a) Q plotted against normalised J (b) Q plotted against normalised load

While this is a good approximation for T_b it is clear that it will not provide a good estimate for the T_a distribution.

It has been seen that for both the bend and tension geometry, a T estimate scheme works poorly for the T_a distribution. The poor agreement may be due to the greater extent of plasticity for the latter case. Although the J levels are the same for both distributions, the plastic zone is considerably larger for the T_a distribution than for the T_b distribution suggesting that an estimate based on the elastic T stress will be less successful for this case. It should be noted that both

temperature distributions lie outside small scale yielding conditions—the plastic zone is about $1/3$ of the crack length. Further work needs to be done to establish confidence levels for estimating constraint using T under thermomechanical loading.

3.4 Results for shallow crack specimens, $a/W=0.1$

A similar analysis was carried out for a shallow crack specimen. It was not found possible to identify a temperature distribution of the form $T(x)$ which gave a positive T stress in this case. The cases examined give $T\sqrt{\pi a}/K$ of -0.5 and -0.1 respectively. The temperature distributions and resultant stress fields are shown in Fig. 12. The distributions are again designated T_a and T_b though they are not the same as the temperature distributions of the previous section. In both cases the temperature distribution was again chosen to give a final J value of $J/(a\sigma_0) = 4 \times 10^{-3}$. The corresponding Q values may be seen from Fig. 12b to be -0.4 and -0.1 .

Figure 12: (a) Temperature fields for shallow crack specimens (b) Stress fields due to temperature distribution of Fig. 12a

Figure 13: Q values for $a/W=0.1$ under thermal loading.

In Fig. 13 the values of Q obtained from the thermal analysis are plotted. Also included is the T stress estimate using Equation (3). It may be seen that the T stress overestimates the loss of constraint somewhat in both cases.

In Fig. 14 the values of Q obtained from the bending and tensile loading are plotted versus normalised load. Note that for the shallow crack, both geometries are low constraint fields as was seen in Fig. 2 and the effects of the thermal stress fields on constraint are similar in both cases. Again it is seen that initially the Q value is strongly affected by the thermal stress field but after some deformation agrees well with the solution for mechanical loading only.

The value of Q for the T_b temperature distribution is rather low and it is seen that the subsequent effect on constraint is weak. This is clearly not the case for the T_a distribution. Although not shown the T stress approach again does not provide a good estimate of the constraint for the low temperature T_a distribution though it works well for the T_b distribution.

Figure 14: (a) Q for edge crack tension specimen (b) Q for edge crack bend specimen

4. DISCUSSION

Thermo-mechanical analyses have been carried out for a number of geometries subjected to different residual stress fields and mechanical loads. For the cases examined the two parameter description of the fields appears to be maintained under combined thermal and mechanical loading. It is shown that the thermal stress can have a considerable effect on the total crack tip constraint. For instance a configuration which has low crack tip constraint under purely mechanical loading can be transformed to high constraint by a disadvantageous thermal stress field. In some of the cases examined it was found that a reasonable approximation of the constraint was obtained by adding the T stress values due to the thermal and mechanical loading and using the small scale yielding Q - T relationship derived for mechanical loading. However, this does not hold true in general and in particular for low constraint temperature loadings and/or when the amount of plasticity due to the thermal loading is large a non-conservative estimate of Q may be obtained by using a T stress approach. In all cases after fully plastic conditions are reached the effect of the thermal stress on constraint is negligible and can be ignored. This holds for both the low and high constraint geometries.

5. ACKNOWLEDGEMENT

The assistance of Dr. A. Sprock, now at SMS Schloemann-Siemag AG, with the numerical aspects of the work is acknowledged.

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