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P. Delobelle, C. Lexcellent. A Phenomenological three Dimensional Model for Pseudoelastic Behavior of Shape Memory Alloys. Journal de Physique IV Proceedings, 1996, 06 (C1), pp.C1-293-C1-300. 10.1051/jp4:1996128 . jpa-00254160

HAL Id: jpa-00254160

<https://hal.science/jpa-00254160>

Submitted on 1 Jan 1996

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A Phenomenological three Dimensional Model for Pseudoelastic Behavior of Shape Memory Alloys

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Abstract. An unified viscoplastic model with internal kinematic variables is used to improve the description of the uniaxial and proportional loadings performed in the pseudoelastic range of shape memory alloys. There is a good agreement between experimental data and simulations. Moreover, this constitutive set of equations is efficient for the description of internal loops and the isothermal cyclic loading which constitutes an important training process. In the future, some progress must be performed to take into account non proportional paths and anisothermal thermomechanical loadings.

1. INTRODUCTION

It is known that at temperatures $T \geq A_F^\circ$ (A_F° being the conventional austenite-finish temperature at the stress-free state) in the neighbourhood of A_F° , the behaviour of shape memory alloys (SMA) is pseudoelastic, provided that the intensity of the applied stress is smaller than the conventional yield limit of the weaker phase. Pseudoelasticity is associated with ongoing forward and reverse martensitic phase transformations. The observed hysteresis effects offer evidence that the processes are energy-dissipative.

In the fields of physics and material sciences, a huge amount of detailed researches on thermoelastic martensitic transformation have been done and a quite complete theoretical system including the theory of the crystallographic transformation and thermodynamics has been established (see Delaey et al. [1], Christian [2]). With the increasing number of applications of shape memory alloys, intelligent material systems and structures, the research on the constitutive relations of these materials presents a strong interest and much work to be done, including :

- 1) Application of the Landau-Devonshire phenomenological theory to shape memory alloys, and the derivation of one dimensional stress-strain relation at different temperatures (see Falk [3], Muller and Xu [4]).
- 2) Phenomenological thermodynamics models for transformation plasticity (see Raniecki and Lexcelent [5], Tanaka et al. [6], Graesser and Cozzarelli [7], Manach and Favier [8]).
- 3) Application of the continuum theory of thermoelasticity to the study of the constitutive laws and of the material instability during phase transition (see Abeyaratne and Knowles [9]).
- 4) Micromechanics constitutive theory of transformation plasticity (see Patoor et al. [10,11], Sun and Huang [12], Sun et al. [13]).

Moreover, Delobelle [14] built a very efficient model to simulate the elastoviscoplastic behavior of a large variety of materials over a vast temperature range under uniaxial and biaxial loadings. Written in the frame of the thermodynamics of irreversible processes, this unified model with internal variables is based essentially on a kinematic mind. The aim of this paper is to "apply" this viscoplastic formulation to the pseudoelastic one inherent to SMA behavior.

First of all, we give the three dimensional model frame (part 2). Then we present the one dimensional identification (simple tension, cyclic tensile compressive tests, isothermal training and external loops) (part 3) and finally in the part 4 the three dimensional identification (proportional and non proportional tensile-torsional loadings).

2. THREE DIMENSIONAL CONSTITUTIVE MODEL

In a classical way, we split the total macroscopic deformation rate $\dot{\underline{\varepsilon}}^T$ in three components : the classical elastic one $\dot{\underline{\varepsilon}}^e$, the pseudoelastic one $\dot{\underline{\varepsilon}}^{pe}$ and the viscoplastic one $\dot{\underline{\varepsilon}}^{vp}$ (the thermal component is neglected). More over, the three dimensional formulation is considered on the assumption that pseudoelastic deformation is proportional to stress deviator [5]. Hence, we establish the model frame (equations 1 to 9) with the introduction of the von Mises equivalent stress and strain :

$$\dot{\underline{\varepsilon}}^T = \dot{\underline{\varepsilon}}^e + \dot{\underline{\varepsilon}}^{pe} + \dot{\underline{\varepsilon}}^{vp} \quad (1),$$

with :

$$\dot{\underline{\varepsilon}}^e = \frac{1+\nu}{E} \dot{\underline{\sigma}} - \frac{\nu}{E} (\text{tr } \underline{\sigma}) \underline{\underline{I}} \quad (2),$$

and two state equations for $\dot{\underline{\varepsilon}}^{pe}$ and $\dot{\underline{\varepsilon}}^{vp}$:

$$\left\{ \begin{array}{l} \dot{\underline{\varepsilon}}^{pe} = \dot{\varepsilon}_0^{pe} \left(\frac{\overline{\underline{\sigma} - \underline{\alpha}}}{N_1} \right)^{n_1} \frac{\text{dev}(\underline{\sigma} - \underline{\alpha})}{\overline{\underline{\sigma} - \underline{\alpha}}} \end{array} \right. \quad (3),$$

$$\left\{ \begin{array}{l} \dot{\underline{\varepsilon}}^{vp} = \dot{\varepsilon}_0^{vp} H(\overline{\underline{\varepsilon}}^{pe} - \varepsilon_0) \left(\frac{\overline{\underline{\sigma} - \underline{\alpha}}}{N_2} \right)^{n_2} \frac{\text{dev}(\underline{\sigma} - \underline{\alpha})}{\overline{\underline{\sigma} - \underline{\alpha}}} \end{array} \right. \quad (4),$$

$$\text{with } H(x) \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$$

$$\text{dev } \underline{\sigma} = \underline{\sigma} - \left(\frac{1}{3} \text{tr } \underline{\sigma} \right) \underline{\underline{I}}, \quad \text{dev } \underline{\alpha} = \underline{\alpha} - \left(\frac{1}{3} \text{tr } \underline{\alpha} \right) \underline{\underline{I}}$$

$$\overline{\underline{\sigma} - \underline{\alpha}} = \left(\frac{3}{2} \text{dev}(\underline{\sigma} - \underline{\alpha}) : \text{dev}(\underline{\sigma} - \underline{\alpha}) \right)^{1/2}$$

The global kinematic variable $\underline{\alpha}$ can be splitted into four internal variables $\underline{\alpha}^{(n)}$:

$$\underline{\alpha} = \sum_{n=1}^4 \underline{\alpha}^{(n)}, \quad \underline{\alpha} = \underline{\alpha}^{(1)} + \underline{\alpha}^{(2)} + \underline{\alpha}^{(3)} + \underline{\alpha}^{(4)} \quad (5),$$

with their own kinetic laws :

$$\dot{\underline{\alpha}}^{(1)} = p_1 \left(Y^{(1)} - \overline{\underline{\alpha}}^{(1)} \right) \dot{\underline{\varepsilon}}^{pe} \quad (6a),$$

$$\dot{\underline{\alpha}}^{(2)} = p_2 \left(Y^{(2)} \dot{\underline{\varepsilon}}^{pe} - \overline{\underline{\alpha}}^{(2)} \dot{\underline{\varepsilon}}^{pe} \right) \quad (6b),$$

$$\dot{\underline{\alpha}}^{(3)} = p_3 Y^{(3)} \dot{\underline{\varepsilon}}^{pe} \quad (6c),$$

and

$$\dot{\underline{\alpha}}^{(4)} = p_4 \left(Y^{(4)} \dot{\underline{\varepsilon}}^{vp} - \overline{\underline{\alpha}}^{(4)} \dot{\underline{\varepsilon}}^{vp} \right) \quad (7),$$

with :

$$\bar{\alpha}^{(n)} = \left(\frac{3}{2} \text{dev } \underline{\alpha}^{(n)} : \text{dev } \underline{\alpha}^{(n)} \right)^{1/2}, \quad \bar{\varepsilon}^{pe} = \left(\frac{2}{3} \underline{\dot{\varepsilon}}^{pe} : \underline{\dot{\varepsilon}}^{pe} \right)^{1/2}, \quad \bar{\varepsilon}^{vp} = \left(\frac{2}{3} \underline{\dot{\varepsilon}}^{vp} : \underline{\dot{\varepsilon}}^{vp} \right)^{1/2}$$

The pseudoelastic behavior is then governed by the three internal variables $\underline{\alpha}^{(1)}$, $\underline{\alpha}^{(2)}$, $\underline{\alpha}^{(3)}$ and the plasticity is linked to $\underline{\alpha}^{(4)}$.

The following comments can be made on equations (6) and (7).

(i) $\underline{\alpha}^{(1)}$ and $\underline{\alpha}^{(3)}$ are two kinematic hardening variables with a reversible evolution. Note that $\underline{\alpha}^{(1)}$ is non linear and $\underline{\alpha}^{(3)}$ linear. These two variables describe the non linear reversible pseudoelastic behaviour linked to the phase transition (Austenite A $\leftrightarrow$ Martensite M).

(ii) $\underline{\alpha}^{(2)}$ is a non linear kinematic variable with a non reversible evolution which describes the progressive opening of the loops when the temperature T increases towards A_F° . $\underline{\alpha}^{(2)}$ can be associated with the energy dissipation created by the interface dislocations allowing for the accommodation between the parent phase A and the product phase M.

(iii) The asymptotic values of $\underline{\alpha}^{(1)}$ and $\underline{\alpha}^{(2)}$ are respectively $Y^{(1)}$ and $Y^{(2)}$ which can take constant values or evolving ones. The cyclic loading is characterized by a decrease in yield stress for direct and forward phase transition associated with an increase in residual strain at stress free state.

In order to extend the model to the cyclic behaviour, $Y^{(1)}$ and $Y^{(2)}$, instead being constant are taken as functions of the accumulated pseudoelastic strain.

$$\begin{cases} Y^{(1)} = Y_{01}(T) + Y^{(1*)} \\ Y^{(2)} = Y_{02}(T) + Y^{(2*)} \end{cases} \quad \text{with} \quad \begin{cases} \dot{Y}^{(1*)} = b_1(Y_\infty^{(1)} - Y^{(1*)})\bar{\varepsilon}^{pe} \\ \dot{Y}^{(2*)} = b_2(Y_\infty^{(2)} - Y^{(2*)})\bar{\varepsilon}^{pe} \end{cases} \quad (8),$$

and for isothermal conditions :

$$\begin{cases} Y_{01} = C_1(T - M_s^\circ) \\ Y_{02} = C_2(T - M_s^\circ) \end{cases} \quad (9),$$

with M_s° the Martensitic start transformation temperature at stress free state.

(iv) Even with a very weak value, the choice of a linear kinematic variable $\underline{\alpha}^{(3)}$ allows to reach, in a systematic way, the stress associated with the critical deformation ε_o (eq. 4). At last, we have to point out that the viscoplastic deformation is taken into account by the variable $\underline{\alpha}^{(4)}$ when the pseudoelastic deformation $\bar{\varepsilon}^{pe}$ becomes higher than ε_o .

3. ONE DIMENSIONAL IDENTIFICATION

We present some simulations of typical cases of mechanical behaviours obtained with different SMA (Cu Zn Al, Cu Al Ni, Ni Ti).

Figure 1 shows the modeling of a classical tensile test ($\dot{\sigma} = 1 \text{ MPas}^{-1}$) in the pseudoelastic range ($T = 97^\circ\text{C}$, $T - A_F^\circ = 45^\circ\text{C}$) of a Cu Zn Al polycrystal [15]. Without plasticity effect ($|\varepsilon^{pe}| < \varepsilon_o$) the identified parameters are :

$$\begin{aligned} \dot{\varepsilon}_o^{pe} &= 10^{-4} \text{ s}^{-1}, \quad N_1 = 2 \text{ MPa}, \quad n_1 = 5, \quad p_1 = 1000, \\ Y^{(1)} &= 125.2 \text{ MPa}, \quad p_2 = 95, \quad Y^{(2)} = 102.4 \text{ MPa}, \quad p_3 = 0. \end{aligned}$$

On figure 2, (Cu Al Ni polycrystal [16] ($M_F^\circ = 54^\circ\text{C}$, $M_s^\circ = 64^\circ\text{C}$, $A_s^\circ = 75^\circ\text{C}$ and $A_F^\circ = 85^\circ\text{C}$)), we simulate the pseudoplastic behaviour of martensite associated with the reorientation of its platelets under the external applied stress ($T = 60^\circ\text{C} < A_s^\circ$: Fig. 2a), an intermediate loop obtained for $A_s^\circ < T = 80^\circ\text{C} < A_F^\circ$ (Fig. 2b) and the pseudoelastic range corresponding to the phase transition $A \leftrightarrow M$ under applied stress ($T = 100^\circ\text{C} > A_F^\circ$: Fig. 2c).

For these cyclic curves, as $|\dot{\epsilon}^{pe}| < \dot{\epsilon}_0$, the viscoplastic strain remains equal to zero (equation 4). The parameters are :

$$\dot{\epsilon}_0^{pe} = 10^{-4} \text{ s}^{-1}, N_1 = 2 \text{ MPa}, n_1 = 5, p_1 = 750, p_2 = 350, P_3 Y^{(3)} = 2000 \text{ MPa}.$$

T°C	$Y^{(1)}$ (MPa)	$Y^{(2)}$ (MPa)
60	15.6	140.4
80	43.2	52.8
100	124.8	31.2

The cycles for different temperatures are fairly well described with increasing $Y^{(1)}$ and decreasing $Y^{(2)}$ with respect to temperature. Nevertheless, the dissymetry between stress level in tension and compression is not taken into account.

Figure 3 represents four pseudoelastic curves on a Cu Zn Al single crystal ($M_s^\circ = -9^\circ\text{C}$) [4]. The simulation of these curves is very efficient with the choice of $Y^{(1)}$ and $Y^{(2)}$ proportional to $(T - M_s^\circ)$, i.e the yield stress for direct phase transformation $A \rightarrow M$. ($Y^{(1*)} = Y^{(2*)} = 0$ in the equation 8).

Hence, the parameters are :

$$\begin{aligned} \dot{\epsilon}_0^{pe} &= 10^{-4} \text{ s}^{-1}, N_1 = 2 \text{ MPa}, n_1 = 5, p_1 = 230, p_2 = 40, p_3 Y^{(3)} = 70 \text{ MPa}, \\ \text{and } Y^{(1)} &= C_1(T - M_s^\circ), Y^{(2)} = C_2(T - M_s^\circ), C_1 = 1.62, C_2 = 0.18, M_s^\circ = -9^\circ\text{C}. \end{aligned}$$

In the case of the pseudoelastic behaviour of Ti Ni polycrystal ($T = 80^\circ\text{C} > A_F^\circ = 48^\circ\text{C}$) [17], the modeling of external loops for complete phase transformation and internal loops for partial transition constitutes nowadays an open problem. With our constitutive equations, the simulation of the internal and external loops is solved by the use of a non linear irreversible kinematic hardening variable whose mean value is adjusted to the value of the reversible kinematic part ($\alpha^{(1)} + \alpha^{(3)}$) for the given deformation (fig. 4).

In this case, the maximum imposed strain (8.10^{-2}) is such that the phase transformation $A \leftrightarrow M$ is completed for $\dot{\epsilon}_0 = 6.8.10^{-2}$. In these conditions, all the model parameters are involved in the simulation :

$$\begin{aligned} \dot{\epsilon}_0^{pe} &= \dot{\epsilon}_0^{vp} = 10^{-4} \text{ s}^{-1}, N_1 = N_2 = 2 \text{ MPa}, n_1 = n_2 = 5, \\ \dot{\epsilon}_0 &= 6.8.10^{-2}, p_1 = 600, Y^{(1)} = 455 \text{ MPa}, p_2 = 700, \\ Y^{(2)} &= 290 \text{ MPa}, p_3 Y^{(3)} = 0, p_4 = 350, Y^{(4)} = 300 \text{ MPa}. \end{aligned}$$

Fig. 1 : Modeling of the pseudoelastic behaviour of Cu Zn Al. (polycrystal) ($T = 97^{\circ}\text{C}$).

Fig. 3 : Modeling of the pseudoelastic behaviour of Cu Zn Al (single crystal) with respect to temperature.

Fig. 2 : Modeling of the pseudoelasticity of the martensite for an Cu Al Ni alloy.
(a) $T = 60^{\circ}\text{C}$, (b) $T = 80^{\circ}\text{C}$, (c) $T = 100^{\circ}\text{C}$.

Fig. 4 : Modeling of the external loops obtained for a Ti Ni alloy ($T = 80^{\circ}\text{C}$).
(a) strain history, (b) stress-strain loop.

We have to point out that the loop stress width is a function of the viscoplastic amplitude reached after the completion of the phase transformation.

Let us now consider the cyclic loading between a stress free state ($\sigma = 0$) and a prescribed total strain amplitude ε^T which constitutes a very interesting training process. As we say before, modeling experimental cyclic tests performed on Ni Ti alloys at 100°C by Tobushi et al. [18] requires the complete formulation of $Y^{(1)}$ and $Y^{(2)}$ given in equations (8) and (9).

During a cyclic process, a decreasing $Y^{(2)}$ induces a stress decrease and a slight increase of $Y^{(1)}$ produces an increment of residual deformation at stress free state.

Hence, we can adjust separately the cyclic softening and the residual strain obtained for each cycle and at zero stress. A simulation is given in figure 5. The parameters for $T = 100^\circ\text{C}$, are :

$$\dot{\varepsilon}_0^{pe} = \dot{\varepsilon}_0^{vp} = 10^{-4} \text{ s}^{-1}, N_1 = N_2 = 2 \text{ MPa}, n_1 = n_2 = 5, \varepsilon_0 = 6,7 \cdot 10^{-2},$$

$$p_1 = 250, Y_{01} = 585 \text{ MPa}, b_1 = 0.1, Y_\infty^{(1)} = 2.2 \cdot 10^{-2} \text{ MPa},$$

$$p_2 = 400, Y_{02} = 195 \text{ MPa}, b_2 = 1, Y_\infty^{(2)} = -230 \text{ MPa},$$

$$p_3 Y^{(3)} = 1500 \text{ MPa}, p_4 = 350 \text{ and } Y^{(4)} = 300 \text{ MPa}.$$

Fig. 4 : Modeling of the internal loops obtained for a Ti Ni alloy ($T = 80^\circ\text{C}$)
(c) strain history, (d) stress-strain loop.

Fig. 5 : Modeling of a cyclic loading between stress free state and a fixed total strain, performed on a TiNi alloy ($T = 100^\circ\text{C}$).

4. TWO DIMENSIONAL IDENTIFICATION

We examine tension-torsion pseudoelastic tests performed at room temperature on thin tubes Cu Zn Al polycrystal by Rogueda [19].

The model parameters are identified on the tensile curve (fig. 6a). Then, the pure torsional curve (fig. 6b) and a proportional tensile-torsional loading (fig. 7) are simulated. If we consider the rate stress control, the obtained results can be considered as fairly good, but a more precise analysis of the radial

loadings reveals a slight anisotropy of these tubes. The introduction of this anisotropy in the frame of constitutive equations cannot be considered as a priori difficult (see Nouailhas and Freed [20], Delobelle and Robinet [21]). However, this model must be improved to described the non proportional loadings. It can be show that the first shown simulations of the non proportional tension-torsion tests (square paths) are not in agreement with the real behaviour (fig. 8). The reorientation of martensitic phase with respect to the loading path must be taken into account with the introduction of a new parameter affecting the $\bar{\varepsilon}^{pe}$ term to obtain a reversible behaviour.

Fig. 6 : Modeling of the pseudoelastic behavior of a Cu Zn Al alloy, $T = 20^\circ\text{C}$.
(a) tension, (b) torsion.

Fig. 7 : Modeling of the pseudoelastic behavior of a Cu Zn Al alloy under proportional loading.
(a) stress-strain curves ; (b) evolution of the ratio $\varepsilon_{z\theta}^T / \varepsilon_{zz}^T$.

Fig. 8 : Modeling of the pseudoelastic behavior of Cu Zn Al under a non proportional loading.

5. CONCLUSION

In the case of pseudoelasticity, the model with kinematic internal variables permits good simulation of uniaxial or radial loadings. This formulation is also efficient for isothermal cycling loading which constitutes an important training process.

More over, simulation of internal loops which is nowadays an open question, does not pose any problem. But, some progress must be done to take into account non proportional loadings.

Thus, we have to link more precisely the internal variables with some phase transformation physical parameters. For instance, some micromechanical approaches [12,13] make the essential distinction between the two mechanisms of creation and reorientation of martensites platelets.

At last, we have to introduce in a fair way the temperature dependency in order to predict the anisothermal behaviour under thermomechanical loading. For technical applications the simulation of a very efficient training process which consists in thermal cycling under constant stress or strain will be very useful.

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