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► To cite this version:

O. Pade, A. Mandelis. Thermal-wave slice tomography using wave field reconstruction. Journal de Physique IV Proceedings, 1994, 04 (C7), pp.C7-99-C7-102. 10.1051/jp4:1994725 . jpa-00253253

HAL Id: jpa-00253253

<https://hal.science/jpa-00253253>

Submitted on 1 Jan 1994

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Thermal-wave slice tomography using wave field reconstruction

O. Pade⁽¹⁾ and A. Mandelis

Photothermal and Optoelectronic Diagnostics Laboratory, Department of Mechanical Engineering, University of Toronto, Toronto, Canada, M5S 1A4

A new matrix equation-based theoretical computation method for detection of subsurface defects by thermal-wave tomographic imaging is presented. The numerical calculations of the inverse problem were carried out using the Born approximation, however, the theoretical technique is capable of handling the general tomographic inverse case without recourse to the Born approximation.

I. INTRODUCTION

Thermal-wave Slice Diffraction Tomography (TSDT) was introduced as a photothermal imaging instrumentation technique [1-3] for the detection of sub-surface defects in solid material along cross-sectional planes perpendicular to the laser-beam scanned surface. The first TSDT instrument was based on contacting photopyroelectric tomographic detection [2,3] followed by ray-optic reconstruction of the cross-sectional thermal-wave image of the thermal diffusivity of the chosen slice [2]. The one-dimensional ray-optic based reconstruction technique was quite successful in illustrating the TSDT principle. However, using only ray-optic methods has many limitations, especially in highly dispersive wave fields, such as thermal waves. For this reason techniques familiar from X-ray cross-sectional tomography, such as the recovery of a 2-D image from an oversampled 1-D projection, cannot be applied to TSDT with satisfactory image contrast, spatial resolution and low distortion. We therefore present a novel Computational Slice Diffraction Tomography based on the matrix methodology of Ref. [4]. The physical methodology is described and the computational technique allowing slice reconstruction and imaging is given for experimental situations involving both back-propagation as well as forward (transmission) thermal-wave data numerically manufactured from the solution to the direct problem (Helmholtz pseudo-wave equation [5]).

2. MATHEMATICAL BACKGROUND

In the case of a harmonic photothermal excitation of a region of space, the temperature oscillation is found to obey the Helmholtz pseudo-wave equation [4-6]:

$$[\nabla^2 + \tilde{k}^2(r)] T(r) = 0 ; \tilde{k}(r) = (1 - i) \left[\frac{\omega}{2\alpha(r)} \right]^{1/2} \quad (1)$$

(1) On leave from Rafael, P.O. Box 2250, Haifa 31021, Israel

$\alpha(r)$ is the thermal diffusivity and ω is the angular frequency of the modulation of the laser beam intensity. $\tilde{k}(r)$ is the complex thermal wave number. Upon defining [5]

$$F(r) \equiv \begin{cases} k_0^2 [v^2(r) - 1] ; & r \in R \\ 0 & ; \quad r \notin R \end{cases} \quad (2)$$

the diffusion (Helmholtz pseudo-wave) equation takes the form [6]

$$(\nabla^2 + \tilde{k}_0^2) T(r) = -F(r)T(r) . \quad (3)$$

In equation (2) we defined R to be the object region and

$$v(r) \equiv [\alpha_0/\alpha(r)]^{1/2} ; \quad \tilde{k}_0 = (1 - i) \left(\frac{\omega}{2\alpha_0} \right)^{1/2} \equiv k_0 e^{-i\pi/4} . \quad (4)$$

α_0 is the diffusivity of the homogeneous (reference) region surrounding the object region R . The full solution of Eq. (3) satisfies in three dimensions,

$$T(r) = T_i(r) + \iiint_S G_0(r|\rho) F(\rho) T(\rho) d^3\rho - \iint_B [G_0(r|\rho) \frac{\partial T}{\partial n_r}(\rho) - T(\rho) \frac{\partial G_0}{\partial n_r}] d\rho \quad (5)$$

Fig. 1 Geometry for Thermal-Wave Slice Diffraction Tomography amenable to back-propagation (BP) and transmission (T) detection. For each position of the laser beam waist on the line $y = 0$, the photothermal detector measures the thermal-wave field characteristics (amplitude and phase) along the back surface ($y = l$ line; transmission) or along the front surface in the vicinity of the laser beam spot ($y = 0$ line; back-propagation). The region $0 < y < y_l$ is the object region S .

If the thermal-wave field $T(x, y = y_f)$ is measured (transmission mode), and if

$$T(r) = T_i(r) + T_s(r) , \quad (6)$$

then using Eq. (5) we obtain along the object region of the 2-D slice between $y = 0$ and $y = y_f$ and between $x = x_i$ and $x = x_f$:

$$T_s(x, y_f) = \int_{x_i}^{x_f} \int_0^{y_f} G_0[r(x, y_f) | \rho(\xi, \eta)] F[\rho(\xi, \eta)] T[\rho(\xi, \eta)] d\xi d\eta . \quad (7)$$

If the thermal-wave field $T(x, y = 0)$ is measured (back-propagated mode), then

$$T_s(x, 0) = \int_{x_i}^{x_f} \int_0^{y_f} G_0[r(x, 0) | \rho(\xi, \eta)] F[\rho(\xi, \eta)] T[\rho(\xi, \eta)] d\xi d\eta . \quad (8)$$

Note that the surface integral contribution to the photothermal signal has been neglected without proof. The adequacy of this approximation on the accuracy of reconstruction of the object field function will be examined via the various artificial situations to be described in what follows. Thus, it will be shown *a posteriori* that effective object field function reconstructions may be generated using only volume integral data in many situations corresponding to cases of practical interest and importance.

In Ref. [4] we show that Green's function for the two-dimensional thermal diffusion Helmholtz pseudo-wave Eq. (3) is

$$G_0(|r - \rho|) = \frac{i}{4} H_0^2(e^{i\pi/4} k_0 |r - \rho|) \quad (9)$$

where H_0^2 is the Hankel function of the second kind of order zero.

3. THE COMPUTATIONAL METHOD FOR THE INVERSE PROBLEM

The proposed technique is based on a special method of discretization of Eq. (5). Defining a matrix of coefficients A by $A_{km} = G_0(|r_k - \rho_m|)$, we have the following $n^2 \times n^2$ system of linear equations:

$$Af = t \quad (10)$$

where $f = FT$ and $t = T_s$ (T_s : scattered (measured) thermal-wave field). The main problem with this method, from the computational point of view, is that the matrix $G_0 T_i$ is in many cases almost singular. To overcome this problem we used the Tykhonov regularization method [7].

4. EXPERIMENTAL IMPLEMENTATION AND RESULTS

An aluminum sample the dimensions of which were 3.75 mm wide and 2 mm thick was illuminated on one side with a focused laser beam at various points and a photopyroelectric pin detector was scanned along the other side. Details of the experimental set-up are given elsewhere [2]. The sample has a machined hole in its middle with diameter of 1 mm. Fig. 2 describes the wave slice reconstruction by the current method, using only one laser position and 26 detector positions. We note the good location of the hole in both directions along with the realistic size of the defect. We also observe some edge effects (artifacts). Similar reconstructions using the ray optic approach resulted earlier [2] in imaging the defect,

but its location was inaccurate and its presence was much less noticeable. Using a finer computational grid has resulted in a well resolved shape contour of the edge of the hole on the side of the laser locations, Fig. 2b.

The new method yields superior reconstructions to those obtained by ray optic approximations, and is capable of solving the Helmholtz pseudo-wave equation and reconstructing a slice tomogram of the diffusivity field.

Fig. 2 a) Wave optic tomographic reconstruction of thermal diffusivity; hole at the center of the sample;
b) fine computational grid, laser at 2.4 mm from left edge.

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