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Influence of acoustical field on small particles

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Abstract: The paper concerns the problem of the drift of small particles suspended in acoustic field. It is shown that the drift occurs in acoustic fields in which the amplitude of medium velocity depends on the position of the particle. In contrast with other mechanisms used to explain the drift of small particles, the effect can be derived even in acoustic field of infinitely small amplitude. This type of drift, called the asymmetry drift, is discussed for the plane progressive wave, for the plane standing wave, and for the spherical progressive wave. Some remarks are made on the two-dimensional standing wave problem.

1. INTRODUCTION

The problem of motion of small solid or liquid particles suspended in gaseous medium (aerosols and fumes) has been studied intensively starting from the forties of our century in connection with technical application of acoustic coagulation for precipitation of gases. The technique has reached only limited commercial application, although it seems to be indispensable on some special cases, when chemically active or explosive aerosols are involved [1]. Changes in concentration of suspension of particles in strong acoustic fields can be used also for measurements of suspension density and dispersion, e.g. in paper making industry [2]. Along with its technical applications, the problem of motion of small particles in acoustic field is interesting by itself as one of fundamental physical effects of sound. Among others, the problem of the drift, i.e. steady motion of average position of a particle oscillating together with the gaseous medium, is one of the most interesting.

The first mechanism of the drift has been presented by King [3] and is called the radiation drift. It is connected with the radiation pressure acting on the particle as the result of momentum carried out by acoustic wave diffracted on the particle. This type of drift is important for rather large particles.

For small particles of radii of order of microns, many authors introduced different mechanisms and considered different special cases (see [4] for review of literature and comparison of amplitudes of drift forces). The most important models concern:

— the effect of variations of viscosity in the wave field caused by local changes of the temperature during periodical compressions and decompressions of the medium [5];

- the effect of presence of higher harmonicals in the wave of finite amplitude [6]; WEST 11
- the effect of dependence of the medium velocity amplitude on instantaneous position occupied by a suspended particle, in which case the oscillatory motion of the particle is asymmetrical and results in steady drift towards regions of less amplitude of medium oscillations.

Only in the last case the drift is predicted also in an acoustic wave described by linear wave equation (the wave of infinitely small amplitude). Therefore the so called asymmetry drift is the most natural one and its mechanism is physically the most fundamental. Moreover, this type of the drift becomes dominant in the region of particles radii less then one micron [4]. We will make some remarks on this very type of drift.

2. THE EQUATION OF MOTION

In the following we assume that the motion of the particle (its parameters are denoted by index p) is caused by the Stokes drag force connected with motion of the surrounding gas (the respective quantities are denoted by index g). Using standard physical notation we may therefore write

$$m_p \dot{u}_p = 6\pi\eta r_p (u_g - u_p). \quad (1)$$

To consider the effect of interest, we have to take into account that the function u_g in the above equation depends on the position of the particle x_p . We may assume in the following that $x_p = x_0 + \xi_p$, $u_p = \dot{x}_0 + \dot{\xi}_p$, where x_0 is the instantaneous equilibrium position of the particle and is a slow-varying function of time, while ξ_p represents the fast oscillatory motion.

Now, the problem can be solved in two steps. First, we may neglect in Eq. (1) the velocity of steady motion $\dot{x}_0$ with respect to the velocity of oscillations and assume that the displacement ξ_p is small enough such that u_g depends only on the mean position x_0 . Therefore we obtain (for harmonic oscillations of the gas)

$$m_p \ddot{\xi}_p + 6\pi\eta r_p \dot{\xi}_p = 6\pi\eta r_p U_g(x_0) \cos \omega t. \quad (2)$$

In the following step, if necessary, we may take into account that for small ξ_p

$$U_g(x_0 + \xi_p) \approx U_g(x_0) + U'_g(x_0)\xi_p. \quad (3)$$

For ξ_p we may take its approximate value calculated from Eq. (2). Finally, to determine motion of the particle's mean position we may come back to Eq. (1) and to neglect all oscillatory terms. Therefore we obtain

$$m_p \ddot{x}_0 + 6\pi\eta r_p \dot{x}_0 = 6\pi\eta r_p \overline{U'_g(x_0)\xi_p \cos \omega t} \equiv F_{DRIFT}. \quad (4)$$

The above equation describes motion of a particle of mass m_p under influence of the Stokes drag force (l.h.s) and some external force (r.h.s). We call this external force the asymmetry drift force.

3. THE PLANE PROGRESSIVE WAVE

It is obvious that the particle suspended in the linear (small amplitude) plane progressive wave will perform oscillations of the same frequency as the particles of the surrounding gaseous medium. If the diffraction effect is neglected, it is easy to see that the direction of the wave motion is also of no importance.

In Eq. (2) we put simply $U_g(x_0) = U_{0g}$. If we denote for brevity $\tau = m_p/(6\pi\eta r_p)$ (the so called relaxation time), we get

$$\tau \ddot{\xi}_p + \dot{\xi}_p = U_{0g} \cos \omega t. \quad (5)$$

This is an elementary equation. Its periodical solution is [7]

$$\dot{\xi}_p = U_{0p} \cos(\omega t - \varphi) = \mu_p U_{0g} \cos(\omega t - \varphi), \quad (6)$$

where

$$\mu_p = \frac{1}{\sqrt{1 + \omega^2 \tau^2}} \equiv \cos \varphi \quad (7)$$

is called the grab coefficient of the particle and is the proportionality coefficient between velocity amplitudes of the gas and of the particle. φ represents the phase delay of the particle motion with respect to the gas.

From Eq. (6) we obtain

$$\xi_p = \omega^{-1} \mu_p U_{0g} \sin(\omega t - \varphi). \quad (8)$$

Obviously there is no drift in the plane progressive wave, at least in this approximation. Anyhow, the above basic solution will be useful in asymmetric problems, as it represents the first step approximation needed to determine the drift force from Eq. (4).

4. THE PLANE STANDING WAVE

In the plane standing wave, we have

$$u_g = U_g(x_0) \cos \omega t = U_{0g} \sin kx_0 \cos \omega t. \quad (9)$$

In the first approximation we have then

$$\xi_p = \omega^{-1} \mu_p U_{0g} \sin kx_0 \sin(\omega t - \varphi). \quad (10)$$

After some algebraic transformations we obtain from Eqs. (4), (7), (9), and (10)

$$F_{DRIFT} = -\frac{1}{4} m_p k U_{0g}^2 \mu_p^2 \sin 2kx_0. \quad (11)$$

It can be easily seen that the force represented by the above formula makes particles to move towards nodes of the velocity in the standing wave.

5. THE PROGRESSIVE SPHERICAL WAVE

It is well known [8] that the velocity of the medium in the free field of pulsating spherical source of volume output a is given by

$$U_g(r) = \frac{k^2 a}{4\pi} \left[\frac{\cos(kr - \omega t)}{(kr)^2} + \frac{\sin(kr - \omega t)}{kr} \right]. \quad (12)$$

The direct calculation of the drift force in general case is rather complicated and not very instructive. For simplicity, we may consider separately the near field case (neglecting the second term) and the far field case (neglecting the first term). To consider both cases at the same time, we assume that

$$U_g = \frac{k^2 a}{4\pi(kr)^\alpha} \cos \omega t. \quad (12)$$

Therefore we have $\alpha = 2$ in the near field and $\alpha = 1$ in the far field, with intermediate values corresponding to the transition region. It is easy to show that in this case

$$F_{DRIFT} = \frac{\alpha k^5 a^2 m_p \mu_p^2}{32\pi^2 (kr)^{2\alpha+1}}. \quad (13)$$

Therefore in the near field the drift force is proportional to r^{-5} , while in the far field it decreases as r^{-3} . The particles are driven out from the source.

6. THE TWO-DIMENSIONAL STANDING WAVE PROBLEM

For theoretical and practical reason, the two-dimensional standing wave configuration seems to be of special interest. As shown in [9], in the case when the standing wave nodal and antinodal surfaces are curved, the particles move along some curves and, as we expect, gather in some particular points. This case could be probably applied with success in technical installation for coagulation of aerosols.

In this case the equations of motion should be given in vector form and Eq. (3) replaced by

$$\vec{U}_g(\vec{r}_0 + \vec{\xi}_p) \approx \vec{U}_g(\vec{r}_0) + \text{grad} \vec{U}_g(\vec{r}_0) \cdot \vec{\xi}_p. \quad (14)$$

Some examples of this type will be presented during the conference and solutions for cases of special interest will be published elsewhere [10].

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