NUMERICAL RESEARCH ON DYNAMICAL BEHAVIOUR OF ALLOY TITANIUM TC4

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RESUME :

On propose un modèle NAG-FRAG simplifié et implanté dans APHEMP, un hydrocode bidimensionnel élasto-plastique, pour simuler les résultats expérimentaux d'essais d'impact de plaques de tungstène contre des cibles en titane, à différentes vitesses d'impact. Après avoir brièvement décrit le modèle, on présente une comparaison entre les longueurs de bandes de cisaillement obtenues par l'expérience et par le calcul.

Abstract—A simplified NAG-FRAG model is proposed and added to APHEMP, two-dimensional elastic-plastic hydrodynamic code, to simulate experimental results of titanium plates impacted by tungsten projectiles with varied speeds. The simplifiers are described briefly and calculating lengths of shear bands are compared experimentally in this paper.

1.—Introduction.

A recent critical review of shear band deformation in metal at high strain rates has been provided by Timothy¹. Microstructural and fracture studies of impacted plates of Ti-6Al-4V alloy have been reported by many authors^{2,3,4}. Holt and others studied impact behaviour of Ti-6Al-4V alloy for the conditions in which the plug of disk material is pushed only partway through the disk⁵. Seaman, et al. developed and used a computational shear band model to predict the fragmentation response of explosively-loaded cylinders and munitions of HF-1 and Armco iron⁶.

Experimental results of alloy titanium TC4 plates impacted by tungsten projectiles with varied speed have been given by Wan, et al⁷. The results show the formation and development of shear bands. In 10mm target plates of titanium, shear bands begin to build up when speed of tungsten projectile is about 80 m / s, as projectile speed increases, the length of shear band grows until the band penetrates the target plate at a speed of about 240 m / s. Near this situation shear instability occurs.

APHEMP code is developed in our institute according to HEMP version. For calculating the shear bands, some simplifications on Nucleation and Growth and Fragmentation (NAG-FRAG) model are made to surmount the lack of microstructure parameters for alloy titanium. The simplified NAG-FRAG model is added to APHEMP code as a subroutine and used to calculate the impact problem.

2.—Simplified shear band model:

The NAG-FRAG model includes many physical processes. They are nucleation of shear bands, growth of the bands, coalescence to form fragments, and relaxation of the stresses in

accordance with the growth of damage. Each physical process contains some material parameters, which must be determined by experiments. To overcome the lack of experimental data, we made some simplifications on NAG-FRAG model as following. Only simplifications are described here, the details of the model can be found in reference 6.

1)Nucleation

Here a nucleation rate function is proposed to account for the observed relationships to plastic shear strain and yield strength and to provide an orientation dependence for the shear band process. The major assumption in the following proposed expression is that the nucleation rate in a direction is proportional to the plastic shear strain in the same direction.

$$\dot{N}_i = \frac{dN_i}{dt} = C_n F_i \left(\frac{1}{E_{cr}} \frac{dE^p}{dt} \right)^2 \frac{d\epsilon_i^{ps}}{dt} \quad (1)$$

where E_{cr} is a critical energy, taken to be the melt energy; dE^p / dt is the rate of increase of plastic strain energy; N_i is the number of shear bands per unit volume in the i direction; and ϵ_i^{ps} is the plastic shear strain in the same direction, here six directions are proposed. C_n is a coefficient with the unit of time squared, and F_i is a factor representing the fraction of solid angle assigned to the i direction. Shear bands may appear in many directions, we have chosen six specific directions to represent the continuum of all directions. The directions provided are for bands normal to three perpendicular coordinate directions and for three more at 45° to the first three. Each of the first three direction represents two points on the unit sphere whereas each of the 45° directions represent four points. Therefore, it seemed reasonable to weigh the six points with factors,

$F_i = 1/9, 1/9, 1/9, 2/9, 2/9, 2/9$ respectively.

For each orientation there is a set of size groups consisting of a number of bands per unit volume N_r with a radius R . Initially these size groups represent the continuous distribution

$$N_g = N_0 \exp(-R/R_n) \quad (2)$$

where N_g is the number of bands with a radius greater than R .

N_0 is the total number.

R_n is a nucleation size parameter.

Not using the set of size groups, we keep using continuous distribution, and define N_{ir} as the number of shear bands per unit volume in i direction with radiuses in unit R interval. According to equations (1) and (2) ΔN_{ir} , the increment of N_{ir} produced by nucleation in one time cycle Δt , can be expressed as

$$\begin{aligned} \Delta N_{ir} &= \frac{\dot{N}_i \Delta t}{R_n} \exp(-\frac{R}{R_n}) \\ &= \frac{\Delta N_i}{R_n} \exp(-\frac{R}{R_n}) \end{aligned} \quad (3)$$

where ΔN_i is the increment of the number of nucleation in timecycle Δt .

2)Growth

The growth of a shear band refers to the increase in radius R of the slipped area. Follow—

ing Seaman, et al⁶, we have assumed that the speed of the shear band is proportional both to its radius and to the plastic shear strain rate; that is

$$\frac{dR}{dt} = C \left[\frac{d\epsilon_i^{ps}}{dt} R \right] \quad (4)$$

Where C is a growth coefficient; $d\epsilon_i^{ps} / dt$ is the plastic shear strain rate in i-th direction; and R is the radius. Integrating equation(4), We get a relation between R and ϵ_i^{ps}

$$\frac{R}{R_0} = \exp[C(\epsilon_i^{ps} - \epsilon_{i0}^{ps})] \quad (5)$$

Where R_0 is the radius of shear band at the moment of nucleation; ϵ_{i0}^{ps} is the plastic shear strain at the same moment.

The total increment of N_{ir} should be added by the increment produced by growth process, then according to equation(5), the N_{ir} at n-th time cycle after first nucleation time is

$$\begin{aligned} N_{ir} = & \frac{\Delta N_{i,n}}{R_n} \exp\left(-\frac{R}{R_n}\right) + \frac{\Delta N_{i,n-1}}{R_n \exp[C(\epsilon_{i,n} - \epsilon_{i,n-1})]} \exp\left\{-\frac{R}{R_n \exp[C(\epsilon_{i,n} - \epsilon_{i,n-1})]}\right\} \\ & + \dots + \frac{\Delta N_{i,1}}{R_n \exp[C(\epsilon_{i,n} - \epsilon_{i,1})]} \exp\left\{-\frac{R}{R_n \exp[C(\epsilon_{i,n} - \epsilon_{i,1})]}\right\} \end{aligned} \quad (6)$$

for simplicity, we omitted the superscripts ps. Here subscript n express the values at different time cycles. For any constant A there is

$$\int_0^{\infty} \frac{1}{A} \exp\left(-\frac{R}{A}\right) dR = 1$$

The total number of shear bands is

$$N_i = \int_0^{\infty} N_{ir} dR = \sum_{n=1}^{\infty} \Delta N_{i,n} \quad (7)$$

That is the sum of the number of nucleation at each Δt .

3)Damage

Following Seaman, et al., the quantity $T_f \beta \gamma^3 \sum_i N_{ir} R_i^3$ is a measure of the damage associated with the bands in the i-th orientation. Here T_f is a dimensionless volume factor about $4\pi/3$, β is the ratio of numbers of fragments and numbers of shear bands. It is about $1/3 \sim 1/4$. The fragment sizes are related to the band sizes through the factor γ , γ is approximately equal to 1. Then

$$K = T_f \beta \gamma^3 = 0.33 \sim 0.44$$

If we use continuous contribution, damage D_i has

$$D_i = K \sum_r N_r R_r^3 = K \int_0^\infty N_r R^3 dR \quad (8)$$

By using formula (6) and

$$\begin{aligned} \int_0^\infty \frac{R^3}{A} \exp\left(-\frac{R}{A}\right) dR &= A^3 \int_0^\infty x^3 \exp(-x) dx \\ &= A^3 \Gamma(3+1) = (3!)A^3 = 6A^3 \end{aligned} \quad (9)$$

(here $x = \frac{R}{A}$) we get

$$\begin{aligned} D_{i,n} &= 6R_n^3 K \left\{ \Delta N_{i,n} + \Delta N_{i,n-1} \exp[3C(\varepsilon_{i,n} - \varepsilon_{i,n-1})] \right. \\ &\quad \left. + \dots + \Delta N_{i,1} \exp[3C(\varepsilon_{i,n} - \varepsilon_{i,1})] \right\} \end{aligned} \quad (10)$$

Equation (10) can be rewritten as following, more suitable to program

$$\begin{aligned} D_{i,1} &= 6R_n^3 K \Delta N_{i,1} = 6R_n^3 K M_1, \quad M_1 = \Delta N_{i,1} \\ D_{i,2} &= 6R_n^3 K \left\{ \Delta N_{i,2} + M_1 \exp[3C(\varepsilon_{i,2} - \varepsilon_{i,1})] \right\} \\ &= 6R_n^3 K M_2 \\ M_2 &= \Delta N_{i,2} + M_1 \exp[3C(\varepsilon_{i,2} - \varepsilon_{i,1})] \\ &\dots\dots \\ D_{i,n} &= 6R_n^3 K \left\{ \Delta N_{i,n} + M_{n-1} \exp[3C(\varepsilon_{i,n} - \varepsilon_{i,n-1})] \right\} \\ &= 6R_n^3 K M_n \\ M_n &= \Delta N_{i,n} + M_{n-1} \exp[3C(\varepsilon_{i,n} - \varepsilon_{i,n-1})] \end{aligned} \quad (11)$$

The total damage is the sum of damages in six directions

$$D_n = \sum_{i=1}^6 D_{i,n} \quad (12)$$

According to equations (1) and (3), $D_{i,n}$ have a factor $6C_n R_n^3 K \left(\frac{1}{E_{cr}}\right)^2$, which remains constant in calculation. We only calculate quantities $\frac{D_{i,n}}{6C_n R_n^3 K \left(\frac{1}{E_{cr}}\right)^2}$ and then

$\frac{D_n}{6C_n R_n^3 K \left(\frac{1}{E_{cr}}\right)^2}$. To simulate the shear band, slide line is used, when $D_n > D^*$, slide starts

along slide line. This slide line indicate the location of shear band.

3.-Results and Discussions.

Simplified NAG-FRAG model mentioned above is added to two-dimensional elastic-plastic hydrodynamic code APHEMP to simulate the shear bands in alloy titanium plate impacted by tungsten projectile. The titanium plate is 10mm in thickness. The tungsten projectile has a diameter of 8mm, a length of 8mm. The speed of projectiles range from 80m/s to over 300m/s. It is found that shear band penetrates the plate at a speed of about 240m/s and shear instability occurs. The thickness of shear bands is about 2–20 μ m. To simulate such a thin layer would be very difficult if ordinary elastic-plastic hydrodynamic code were used. So NAG-FRAG model is used, which consider microcrack properties as a similar of constitutive relation of material.

Table 1. The lengths of shear bands L and impact speeds of projectiles U

U(m/s)	150	200	230	240	250	270
L(mm)	0	2	7	9	10	10

Calculation results for 10mm thickness plate with different projectile speeds are shown in table 1. Figure 1. shows the comparison between experiments and calculations. A shear instability is found near the situation of $U = 240\text{m/s}$ both in experiment and in calculation. Experiments also show small shear bands (shorter than 1mm.) in the plates when projectile speeds range in 80m/s – 150m/s. Calculations did not give this results, for numerical mesh is comparative large (about 1mm). Figure 2. shows the increase of the depths of craters with the increase of projectile speeds. When speeds are less than 250m/s, simulation is satisfied, but the growth of crater is slower than that in experiments when speeds are greater than 250m/s

Figure 1. The lengths of shear bands. Comparison between Calculations and experiments.

Figure 2. The depths of craters. Comparison between Calculations and experiments.

Numerical simulation of high strain-rate shear band formation is a difficult and complicated problem. Here is only a preliminary approach, trying to use a simple model with fewer parameters to simulate the main properties. It can be predicted that some more details would be unsatisfied.

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