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# CONSTITUTIVE MODEL FOR GEOLOGICAL AND OTHER POROUS MATERIALS UNDER DYNAMIC LOADING

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Un modèle de comportement de matériaux poreux partiellement ou complètement saturés en eau est décrit. Ce modèle est utilisé dans des simulations numériques. La loi d'écoulement choisie pour la rupture en cisaillement montre une influence importante sur la propagation d'onde. Une loi d'écoulement entraînant une dilatation conduit à une atténuation inférieure à une loi créant une refermeture des vides en cisaillement, produit moins de liquéfaction et donne un niveau de contrainte plus élevé derrière le front d'onde.

**ABSTRACT** – An effective stress model is described for use in numerical calculations on porous materials which are partially or fully saturated with water. The flow rule chosen for the shear failure portion of the model is examined and shown to have a significant influence on wave propagation results. A flow rule which produces dilatancy results in less attenuation than a rule producing shear-enhanced void collapse. The dilatancy producing rule is less prone to producing liquefaction and results in significantly higher stress levels behind the wave front.

## INTRODUCTION

The mechanical influence of water partially or completely filling the pore space of a rock or soil on the deformation of that material has long been recognized. Fortunately, the mechanical properties of a large variety of geological materials have been shown to follow an effective pressure rule, where the effective stress tensor is defined as

$$S_{ij}^{eff} = S_{ij}^{tot} - \alpha P_p \delta_{ij} \qquad (1)$$

where $S_{ij}^{tot}$ is the total stress tensor, $\alpha$ is a constant and $P_p$ is the pore water pressure. The effective pressure rule says that the value of a given property, such as shear strength, is the same for all total pressure and pore water pressures which give the same effective pressure. Constructing models which follow the effective pressure law can be complicated since the deformation of the water, the pore space, and the solid component all need to be determined to find the total deformation. The work of Biot (1) and Biot and Willis (2) is important to the study of deformation and wave propagation in such materials under infinitesimal strain conditions. More recently, Carroll (3) and co- workers made important contributions developing models useful for finite strains and where spherical pores are an appropriate approximation. Computation of stress wave propagation near explosions requires additional complexities due to the non-linear response of the water and of the rock solids. Work by Garg, et al (4) and Swift and Burton (5) are important additions to this subject.

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In this paper, I describe an effective stress model being used in finite difference computations of explosions and the stress waves propagating away from them. The influence on stress wave attenuation from coupling between shear deformation and volumetric deformation caused by increases or decreases in pore volume due to shear failure is explored numerically with this model.

## EFFECTIVE STRESS MODEL

While most effective stress models used in finite difference codes are formulated according to an incremental approach, the model described here uses the integrated quantities. This approach, with the required relationships specified in tabular form, allows virtually direct use of experimental data to specify the model for a given rock or soil. The common assumption is made that the flow of pore water during the passage of a stress wave is negligible. The model then needs to solve the following set of equations.

The first equation is the obvious identity that the volume of the bulk material is the sum of the pore volume and the solids volume.

$$V_b(P, P_p, E) = V_p(P, P_p, E_w) + V_s(P, P_p, E_s) \tag{2}$$

Following Carroll (3) and introducing the porosity with the assumption that the porosity only depends on the effective pressure greatly simplifies the solution when the solids component is non-linear, particularly when phase transitions are present.

$$V_p(P, P_p, E_w) = \phi(P_{eff}) V_b(P, P_p, E) \tag{3}$$

where

$$P_{eff} = P - P_p \tag{4}$$

For unsaturated conditions:

$$P_p = 0 \tag{5}$$

while for saturated conditions a water equation of state is needed:

$$P_p = P_w(V_w, E_w) \tag{6}$$

A solids equation of state completes the set:

$$V_s(P, P_p, E_s) = V_s(P_s, E_s) \tag{7}$$

where

$$P_s = \frac{P_{eff}}{(1-\phi)} + P_p \tag{8}$$

The remaining terms have the following definitions:
$V_b$: specific volume of the bulk material,
$V_p$: pore volume,

$V_s$: solids volume,
$V_w$: pore water volume, $= V_p$ for saturated conditions,
$P$: mean total stress,
$P_p$: pore water pressure,
$P_s$: mean stress in solids component,
$\phi$: porosity,
$E$: specific internal energy of bulk material,
$E_s$: specific internal energy of solids,
$E_w$: specific internal energy of pore water.

During a finite difference computation, the above set of equations is solved simultaneously for $P$, $P_p$, and $V_p$ given that $V_b$, $E_s$ and $E_w$ are known initially. The equation of state of water is well known, as are the EOSs for the solids components commonly found in geological material. The porosity-effective pressure relationship is the most critical part of the model and can be generated directly from pressure- bulk volume measurements from drained experiments ($P_p$=0) by solving the relation:

$$(1-\phi)V(P) = V_s\left(\frac{P}{(1-\phi)}\right) \tag{9}$$

assuming that the solids $P-V$ relationship is known and where the solids pressure substitution from equation (8) has been made. For a linear solid, this expression reduces to a quadratic function of $(1-\phi)$.

## COUPLING BETWEEN SHEAR FAILURE AND VOLUMETRIC STRAIN

Laboratory experiments on the compressive strength of rocks show it to be a function of the effective pressure defined by equation 4 for a wide variety of rocks (Handin, et al (6)). In my numerical model I make this assumption and define the failure envelope as a function of the effective pressure. The failure envelope is allowed to expand or contract from an initial envelope to an ultimate envelope using the accumulated "plastic" work done in the computational cell as a hardening parameter. Figure 1 shows the initial and ultimate yield surfaces used in this study. For mean effective stresses less than 100 MPa, the ultimate failure surface lies below the initial yield surface and strain softening behavior will be produced under these stress conditions. Above 100 MPa, the ultimate failure surface lies above the initial and work hardening behavior will result. These yield surfaces are consistent with an observed brittle-ductile transition at 100 MPa mean effective stress in this material.

The effective pressure rule for the failure envelope provides one form of coupling between volumetric and shear behavior. The other form of coupling is controlled by the choice of flow rule. Since the failure envelope typically gives a larger shear strength at higher effective pressures, such as is shown in Fig. 1, an associated flow rule will lead to dilatancy, the increase of pore volume during shear failure, which is typically attributed to the opening of cracks subparallel to the maximum compression direction. Many rocks and soils, typically those with higher porosities, exhibit shear-enhanced void collapse during shear failure. Such behavior requires a non-associated flow rule to be properly modeled for yield surfaces as in Fig. 1.

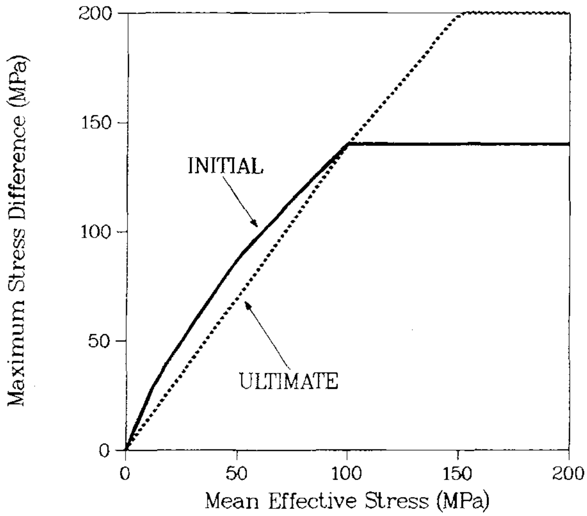


Figure. 1. Initial and ultimate yield surfaces for the limestone effective stress model.

Processes which lead to hysteresis in the stress-strain path during passage of a stress wave give rise to dissipation and further attenuation of the stress wave. Shear failure and pore crushing are two obvious examples. Even under fully saturated conditions, some pore crushing can occur, leading to dissipation and to a liquefaction state, with no shear strength, during unloading. These effects are discussed in work by Dey (7) and Dey and Brown (8). In this paper I look at the influence of the flow rule on wave propagation. The choice of the flow rule leads to different stress conditions and dissipation due to shear failure as well as to changes in the onset of liquefaction.

## FLOW RULE AND WAVE ATTENUATION

To investigate the influence of flow rule on wave propagation two calculations of a spherically symmetric explosion with energy equivalent to 1 ton of TNT were performed. The calculation models a point source explosion with the energy deposited in a high temperature gas of 0.2 m radius. The material is a homogeneous saturated limestone of 11% total porosity. The material model was developed from unpublished laboratory measurements on Indiana limestone samples done by S. Blouin of Applied Research Associates and J. Zelasko of Waterways Experiment Station. The model includes the influence of the calcite I-II and II-III phase transformations which occur within the 1-3 GPa pressure range. The liquid-steam phase transition and nonlinearity of water are included as well. The two calculations were performed with identical material models except for the choice of flow rule. One calculation used an associated flow rule in conjunction with the yield surfaces of Fig. 1. The other used a non-associated flow rule which was adjusted to fit the observed shear-enhanced void collapse which this limestone exhibits. The initial ratio of volumetric to deviatoric plastic shear strain is 0.3, with this ratio slowly reduced as the shear failure continues.

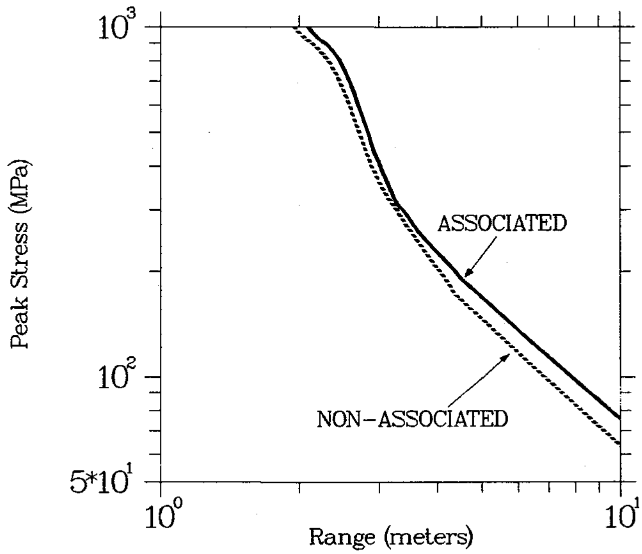


Figure. 2. Peak stress versus range for a 1 ton explosion in the modeled limestone for two choices of flow rule.

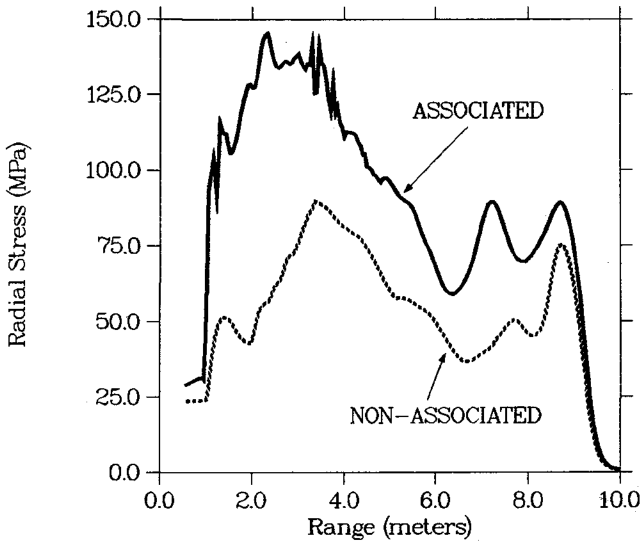


Figure. 3. Snapshot of total radial stress versus range at 2 ms for the two choices of flow rule.

Figure 2 shows results for the peak stress as a function of range at the completion of the two calculations. Note that the non-associated flow rule produces a greater rate of attenuation leading to a peak stress at 10 m range which is about 20% lower than with the associated flow rule. Figures 3 and 4 show differences in waveform resulting from the different flow rules. Figure 3 shows the total radial stress as a function of range at 2 ms. The non-associated flow rule shows a substantially lower amplitude everywhere. At this time in the calculation the explosion cavity gases have expanded to a little greater than 1 m radius with a pressure of about 25-30 MPa. Figure 4 shows the effective radial stress, the total stress minus the pore water pressure, at 2 ms. The non-associated flow rule develops a lower effective stress everywhere. At ranges less than about 3.5 m the effective stress is zero, with the high pore pressure assumed to cause a liquefaction condition with zero shear strength. In contrast, the associated flow rule gives high effective stresses everywhere except for a few zones near the cavity where liquefaction is obtained.

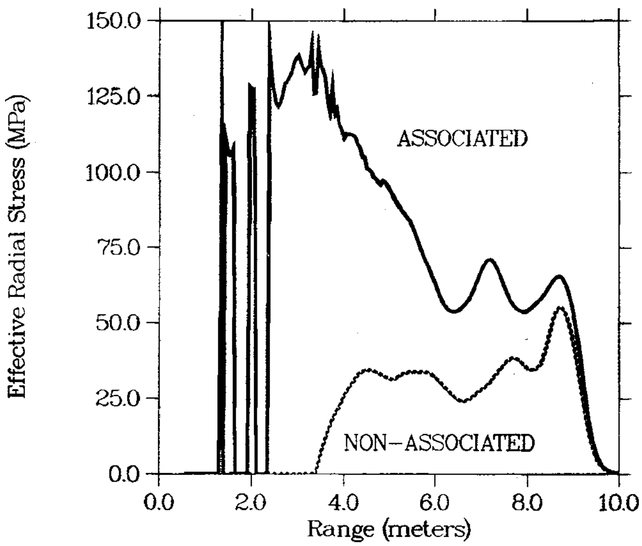


Figure 4. Snapshot of effective radial stress versus range at 2 ms for the two choices of flow rule.

## DISCUSSION

Under these explosive loading conditions the associated flow rule causes less attenuation and a different set of conditions behind the wave front than the non-associated flow rule. Figures 5 and 6 help explain this by showing the pressure-volume behavior of the material at 3 m range during the passage of the stress wave. The loading path to the peak pressure is very similar for the two flow rules. After the peak is passed, the material deformation is predominantly in shear, allowing the flow rule to have a greater influence than during the initial loading. Figure 5 shows results with the associated flow rule. The dilatant behavior of this flow rule, and consequent increase in pore volume, tends to lower the pore water pressure leaving the effective pressure relatively high. The higher effective pressure causes a higher shear strength to be retained. Note that at the later stages the volume increases to greater than the initial volume even though the pressure is about 50 MPa. Also, at these late stages, the pore pressure drops to zero making the total and the effective pressures equal. The net energy deposited by this $P-V$ curve is obviously small because of this late stage behavior.

Figure 6 shows results for the non-associated flow rule. In this model, pore space is being destroyed by both the hydrostatic and the deviatoric processes. The pore water pressure remains high causing the effective pressure to drop. Eventually zero effective pressure is reached and a liquefaction condition is assumed. The P-V loop clearly dissipates more energy for this model than the associated flow rule caused in Figure 5. The low or zero shear strength associated with the lower effective pressures in this model cannot support the late time stress field as well as the higher shear strengths from the associated flow rule (Figs. 3 and 4).

## CONCLUSIONS

Results for wave propagation calculations from explosions are sensitive to choice of the flow rule for the material model. A flow rule which causes shear-enhanced void collapse, which is the correct rule for this limestone, leads to greater attenuation and a substantially different waveform than a dilatancy producing flow rule. This issue is usually not addressed when setting up calculations on geological materials since it is already difficult to obtain the required data to set up the more basic parts of the models. Clearly, the flow rule issue needs to be addressed carefully to insure reliable results.

## ACKNOWLEDGEMENT

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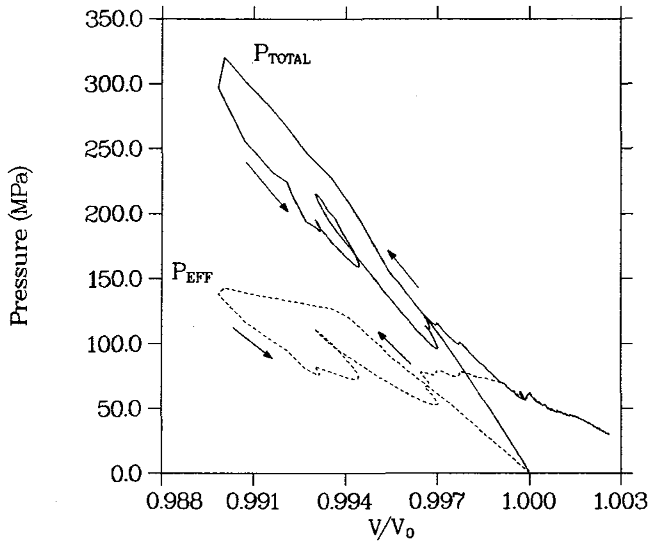


Figure. 5. $P-V$ behavior of material at 3 m range during passage of stress wave with the associated flow rule.

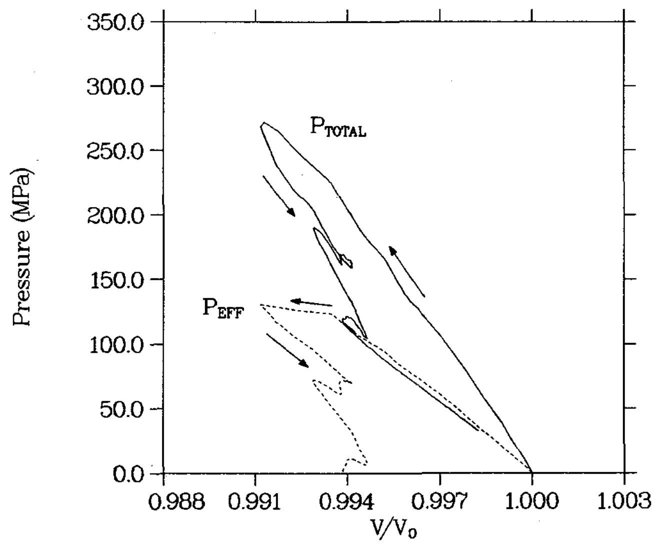


Figure. 6. $P-V$ behavior of material at 3 m range during passage of stress wave with the non-associated flow rule.