Direct extrusion of rate-sensitive materials through conical dies

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DIRECT EXTRUSION OF RATE-SENSITIVE MATERIALS THROUGH CONICAL DIES

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Résumé. — Il est présenté une analyse théorique pour estimer la force requise au filage d'un matériau sensible à la vitesse de déformation. Il est admis que le matériau suit la loi de comportement :

$$\sigma_e = \sigma_0 \sin h^{-1}(\dot{\varepsilon}_e/\dot{\varepsilon}_0)$$

et que les frottements sont du type Coulomb. Les expériences de filage menées sur l'alliage superplastique PbSn à la composition eutectique, sont en accord avec les prédictions théoriques dans un large domaine de rapports de filage et de vitesses du piston. La méthode théorique suivie qui tient compte de la sensibilité à la vitesse de déformation est comparée aux autres méthodes proposées dans la littérature. En conclusion, il est montré qu'il est important de ne pas négliger la sensibilité à la vitesse pour prédire correctement les pressions de filage.

Abstract. — A theoretical analysis to estimate the force required to extrude a rate-sensitive material is presented. The material is considered to behave according to the constitutive equation :

$$\sigma_e = \sigma_0 \sin h^{-1}(\dot{\varepsilon}_e/\dot{\varepsilon}_0)$$

and die friction is assumed to be of the Coulomb type. Extrusion experiments performed on superplastic Pb-Sn eutectic alloys agree well with the theoretical predictions over a wide range of extrusion ratios and ram speeds. The present theoretical method which accounts for rate sensitivity is compared with other methods available in the literature. It is concluded that consideration of rate sensitivity of the extruded material represents an important factor in predicting the extrusion pressure correctly.

NOMENCLATURE

σ_e Equivalent stress.
$\dot{\varepsilon}_e$ Equivalent strain-rate.
m Strain-rate sensitivity index.
$\dot{\varepsilon}_0$ Strain-rate constant in material law.
σ_0 Material parameter in material law.
D_0 Extrusion billet initial diameter.
D_f Extrusion final diameter.
V_0 Extrusion ram speed.
α Extrusion die semiangle.
R Extrusion ratio $R = (D_0/D_f)^2$.
σ_x Horizontal stress within the extrusion die.
q Die stress.
μ Coefficient of friction.
P Extrusion pressure.

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1. **Introduction.** — In estimating the force required for extrusion, attention is usually restricted to the four fundamental factors : reduction, frictional and geometrical boundary conditions, speed of operation and strain and/or strain-rate hardening characteristics of the material. It is hardly likely that an ideal theory of extrusion which considers all these processes variables will exist. For instance the slip-line field method, apart from the fact that it is strictly applicable to plane strain conditions, does not allow for the material strain and/or strain-rate hardening characteristics. Various approximate methods have been suggested to extend the applicability of the slip-line field solutions to the extrusion of rate-dependent materials [1], [2]. These methods are based on deriving an average value of the flow stress obtained from material test data over the appropriate range of strain and/or strain rate encountered during extrusion. However, the slip-line field solutions are only available up to extrusion ratios not exceeding 30 for a few die geometries and under special frictional conditions.

Jonas has calculated the force required to extrude a strain-rate sensitive material under conditions of frictionless homogeneous deformation by the uniform work method [3]. His analysis was supported by experimental results obtained from well-lubricated indirect extrusion of ice-billets.

In forward extrusion of metals, effective lubrication is very difficult to achieve and it has been concluded that lubricated extrusions are accompanied by coefficients of friction that are not small [4]. Hence it is sought here, to extend the homogeneous deformation method to include the extrusion of rate-sensitive materials through conical dies taking into account the friction losses inevitably present in the process.

2. **Material behaviour.** — The flow stress during metal forming operations is considered as a function of temperature, strain and strain-rate. In isothermal hot forming of metals at temperatures above the recrystallization temperature, the influence of strain upon flow stress is insignificant and the influence of strain-rate becomes increasingly important. Thus the flow stress can be expressed by

$$\sigma_e = f(\dot{\varepsilon}_e) . \qquad (1)$$

In micrograin superplasticity, it has been also shown that equation (1) is a good representation for the dependence of the flow stress on strain-rate during deformation. Figure 1 shows a typical stress-strain-rate diagram for superplastic alloys. The experimental data represented in this figure were derived from compression tests performed on the superplastic Pb-Sn alloy ats room temperature. It is seen from figure 1 that over a limited range of strain-rate, say two decades, equation (1) may be reduced to a power law of the form :

$$\sigma_e = \sigma_0(\dot{\varepsilon}_e/\dot{\varepsilon}_0)^m \qquad (2)$$

where m, the strain rate sensitivity index is taken to be constant over the strain-rate range of interest. However, over a wide range of strain-rates which may be more than four decades, equation (2) fails to describe the flow stress dependence on strain-rate. Another appropriate equation for the situation in which the strain-rate sensitivity decreases with increasing strain-rate is [5] :

$$\sigma_e = \sigma_0 \sin h^{-1}(\dot{\varepsilon}_e/\dot{\varepsilon}_0) . \qquad (3)$$

This empirical law originally suggested by Prandtl has been used in the phenomenological analysis of creep deformation [6]. It also appears that this law is rooted in the mechanism of viscous deformation of polycrystalline materials at elevated temperatures [7]. In fact a number of investigators [8] and [9] have expressed the deformation mechanism operating in micrograin superplasticity by stress strain-rate relationships similar to equation (3).

3. **Theoretical.** — The present analysis assumes that extrusion occurs under conditions of homogeneous deformation. Figure 2 represents the flow of a disc-shaped element of material through a conical die.

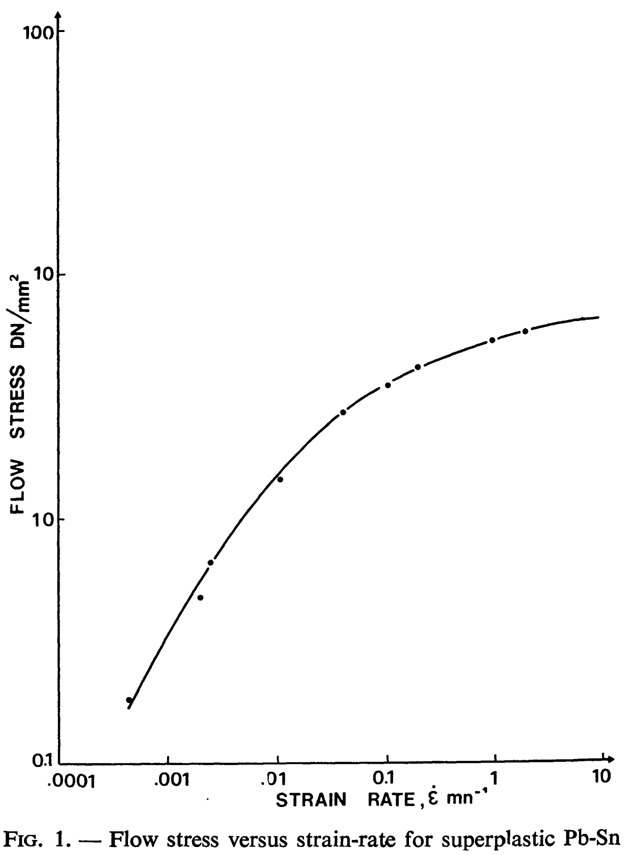

FIG. 1. — Flow stress versus strain-rate for superplastic Pb-Sn alloy at room temperature.

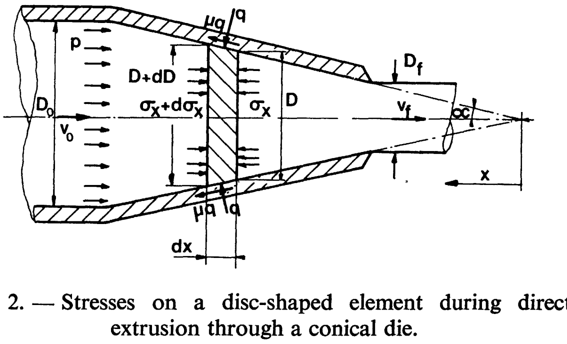

FIG. 2. — Stresses on a disc-shaped element during direct extrusion through a conical die.

3.1 STRAIN-RATE PROFILE IN EXTRUSION. — The local strain-rate at any diameter D within the die zone is easily shown to be given by [7] :

$$\dot{\varepsilon}_D = \frac{2\, V_0\, D_0^2 \tan \alpha}{D^3} . \qquad (4)$$

In view of the axial symmetry, the equivalent strain-rate can be expressed as :

$$\dot{\varepsilon}_e = 2\, \dot{\varepsilon}_D = \frac{4\, V_0\, D_0^2 \tan \alpha}{D^3} . \qquad (5)$$

Equation (5) establishes the strain-rate profile under conditions of homogeneous deformation in a conical extrusion die. Strain-rate profiles are shown in figure 3

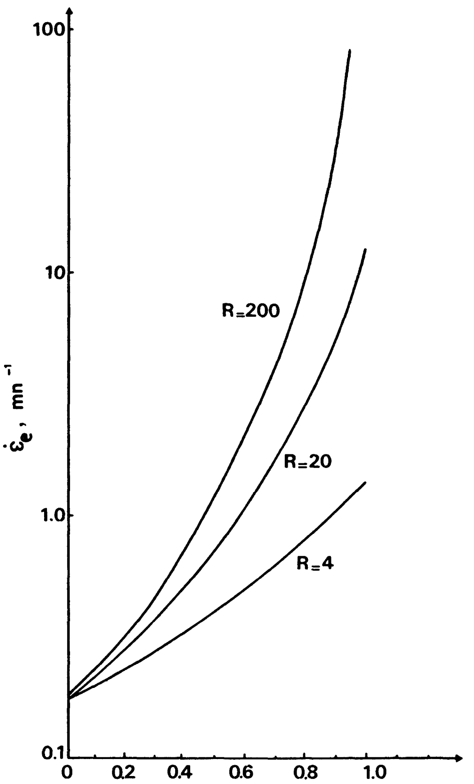

FIG. 3. — Local strain-rate as function of the fractional distance (axial distance measured from die entry/total axial length of the conical die) for various extrusion ratios. $V_0 = 1$ mm/min. ; $D_0 = 13.6$ mm and $\alpha = 30°$.

for various extrusion ratios ranging from 4 to 200 for a conical die of semi-angle 30°. It is seen from this figure that for a constant ram speed V_0, the local equivalent strain-rate varies, depending on the extrusion ratio, by three or about four orders of magnitude as the material progresses through the deformation zone.

A theoretical analysis for the extrusion problem which takes the rate sensitivity of the material into consideration should then employ a constitutive equation representing the flow stress data over a wide range of strain rates. A constitutive law of the form or equation (3) seems to be suitable for the case of extruding superplastic alloys at large extrusion ratios. This type of equation will be used in the theoretical analysis presented in the main text of this paper. Results of theoretical derivation giving the extrusion load based on a simple constitutive law, equation (2), are also given in the appendix.

3.2 EXPRESSION FOR THE EXTRUSION PRESSURE. — Considering the conditions of Coulomb friction along the die boundaries, the equation representing the horizontal equilibrium of forces acting on the element shown in figure 2, is

$$D\frac{d\sigma_x}{dD} + 2\,\sigma_x + 2\,q(1 + \mu \cot \alpha) = 0\,. \qquad (6)$$

Assuming small cone angles α so that the direction of the die pressure q is approximately normal to the direction of σ_x, and assuming that q and σ_x are principal stresses, then for cylindrical symmetry both Tresca and Von Mises yield criteria which lead to

$$\sigma_x + q = \sigma_e\,. \qquad (7)$$

Having established expressions for the equivalent stress and strain-rate as given by equations (5) and (7), equation (1) may be used to yield

$$\sigma_x + q = f(4\,V_0\,D_0^2 \tan \alpha / D^3)\,. \qquad (8)$$

Substituting for q from equation (8) into the equilibrium equation (6), the following differential equation is obtained :

$$D\frac{d\sigma_x}{dD} - 2\,\mu\sigma_x \cot \alpha = \\ = -\,2(1 + \mu \cot \alpha) f(4\,V_0\,D_0^2 \tan \alpha/D^3)\,. \qquad (9)$$

The successful analytical integration of equation (9) depends on the form of the function f. In the case where the material behaviour is represented by a hyperbolic sine law, equation (3), equation (9) reduces to

$$\frac{d\sigma_x}{dD} - 2\,\mu\frac{\cot\alpha}{D}\sigma_x = \\ = -\,2\frac{(1 + \mu \cot \alpha)}{D} \sinh^{-1}(4\,V_0\,D_0^2 \tan \alpha/\dot{\varepsilon}_0\,D^3)\,. \qquad (10)$$

Analytical solution of equation (10) is not available and a numerical integration scheme must be adopted in order to obtain the distribution of the horizontal stress σ_x within the conical die.

The theoretical extrusion pressure may be then calculated from the condition that :

$$P = [\sigma_x]_{D=D_0}\,. \qquad (11)$$

4. **Experimental.** — Cast ingots of 45 mm diameter of lead-tin eutectic were homogenised at 140 °C for 40 hours. They were then extruded to rods of 14 mm diameter which were finally rehomogenised at 140 °C for 3 hours. This treatment was carried out to develop a fine grain microstructure, a basic requirement for isothermal superplastic behaviour.

Compression tests were performed on the material to be extruded at different cross-head speeds on an Instron testing machine. The compression test-pieces were of 13 mm diameter and 10 mm long, this length having been determined as short enough to avoid tilting of the specimen during compression. Tiny grooves were machined in the ends of the specimens to facilitate the retention of lubricant used during compression.

The true stress-strain rate curve of the material to be extruded as calculated from the compression test data is shown in figure 1. It has been found that the dependence of the flow-stress on the value of strain during compression was negligible and hence a constitutive relation of the type given by equation (1) is satisfactory. A simple curve-fitting process has yielded the material law.

$$\sigma_e = 0.73 \sin h^{-1}(\dot{\varepsilon}_e/0.0018) .$$

Direct extrusion experiments of superplastic Pb-Sn billets of 13.6 mm diameter were performed at various constant ram speeds ranging from 0.01 to 10 mm/min. Extrusion ratios of 7.5, 20, 46 and 185 were realized by using different extrusion conical dies of different exit diameters with a semiangle of 30°. In order to maintain the maximum strain-rate during any extrusion experiment within the range of the available material data represented in figure 1, low extrusion ratios were coupled with high extrusion speeds and vice-versa. All extrusion experiments were lubricated by machine oil and conducted at room temperature.

Typical autographic diagrams as shown in figure 4 for the ram load versus ram travel were obtained for

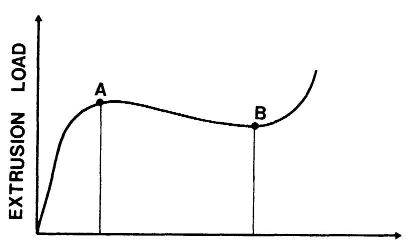

FIG. 4. — Extrusion load versus ram travel ; a typical curve for direct extrusion.

each extrusion speed. It has been explained that the rise in extrusion load from O to A is due to the adjustment of the material to the container and the die-cavity. After peak pressure has been reached, extrusion proceeds steadily. During this steady phase of extrusion the pressure gradually decreases because the frictional load between the billet and the container wall is decreasing as the billet gets shorter. At point B an unsteady phase begins and finally the extrusion pressure ceases to be meaningless [10].

It is the pressure represented by point B which was considered as the experimental extrusion pressure in this paper. Obviously this choice eliminates the need of considering the billet length in developing an expression for the extrusion pressure as function of the extrusion process variables.

5. **Comparison between theoretical and experimental results.** — The theoretical and experimental extrusion loads for Pb-Sn superplastic alloys are plotted versus the extrusion speed in figure 5 for various extrusion ratios. The solid lines shown in this figure represent theoretical predictions of the extrusion loads as determined from the numerical integration of equation (10) (i. e. employing a hyperbolic sine law to represent the material behaviour). The broken lines represent the extrusion load as determined from equation (A.3), in

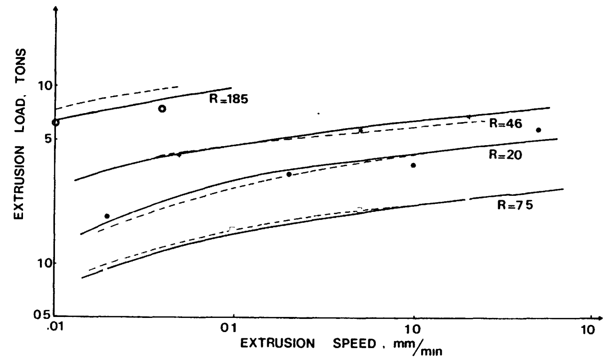

FIG. 5. — Extrusion load versus extrusion speed plotted logarithmically for superplastic Pb-Sn. $D_0 = 13.6$ mm ; $\alpha = 30°$. —— Theoretical line (hyperbolic sine material law). - - - - Theoretical line (simple power material law).

the appendix (i. e. employing a simple power law to represent the material behaviour).

It is seen that there is a good agreement betwen the theoretical loads and experiments. It is obvious however that the predictions of the theory employing a hyperbolic sine law are much closer to experiments especially at higher extrusion ratios. This is due to the fact that at these high extrusion ratios the strain-rate varies by three or four orders of magnitude within the die zone and hence it is not expected that the strain-rate sensitivity index remains constant as predicted by a simple power law. The use of a hyperbolic sine law implicitly takes into account the effect of the variation of the strain-rate sensitivity index of the material within the die zone.

The theoretical estimates of extrusion loads were based on a value for the coefficient of friction as 0.1. Although the extrusion experiments were lubricated, this assumed value of μ seemed to be reasonable as indicated by the agreement between theory and experiment. As mentioned before, it has been concluded by Johnson that lubricated extrusions are accompanied by coefficients of friction that are not small and hence

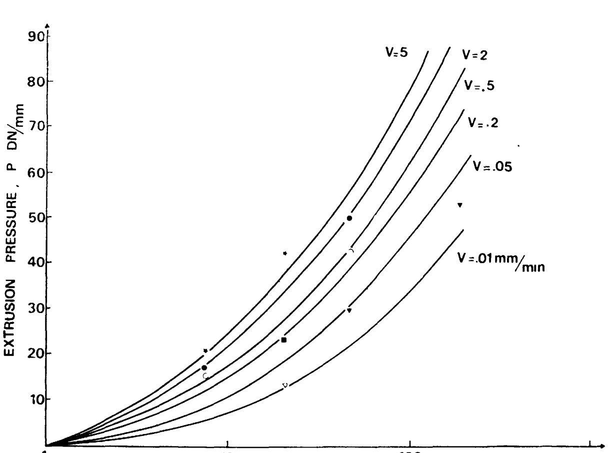

FIG. 6. — Theoretical extrusion pressures versus extrusion ratio for different ram speeds compared with experimental extrusion pressures for superplastic Pb-Sn.
$D_0 = 13.6$ mm and $\alpha = 30°$.

extrusion pressures can be reasonably well predicted if a relatively high value of μ is assumed [4].

Theoretical extrusion pressures as calculated from equation (10) are represented versus extrusion ratio for various extrusion speeds in figure 6. Once again consistency between the present theory and experiment is evident.

6. Discussion. — The object of this work is to explore the extension of the traditional slab method analysis to obtain load estimates for the extrusion of rate-sensitive materials. The material flow stress was considered to be dependent on the strain-rate only which represents the case of hot metal forming as well as micrograin superplastic deformation.

The strain-rate sensitivity of the material as it flows through conical extrusion dies is taken into consideration and a simple analysis yields reliable estimates for the extrusion load up to high extrusion ratios. The method takes into account the frictional losses present in the process but neglects those due to redundant deformation.

The theoretical load predictions are compared favourably with the results of direct extrusion experiments performed on superplastic lead-tin alloy for extrusion ratios 7.5, 20, 46 and 185. The analysis is well suited to give load estimates in the cases of direct and indirect extrusions. This has been confirmed, namely for indirect extrusion by the analysis and experiments presented by Jonas for the frictionless extrusion of ice billets [3].

The existing theories which may predict the pressures required to extrude rate-sensitive material are compared with the analysis presented here. The extrusion pressures as calculated using the homogeneous work method [11] and the upper bound technique due to Avitzur [12], as well as the present method are shown in figure 7. The pressures estimated using the first two methods are based on deriving an average value of the flow stress obtained from material test-data over the appropriate range of strain rate. The calculations were performed for an extrusion speed of 0.05 mm/min. and $\mu = 0.1$. It is evident from figure 7 that the predictions of the three methods are reasonably close at low extrusion ratios. At extrusion ratios greater than 10, the deviations among the predictions become significantly larger. The two experimental points are seen to correspond to the theoretical extrusion pressure curve as obtained from the analysis presented in this paper.

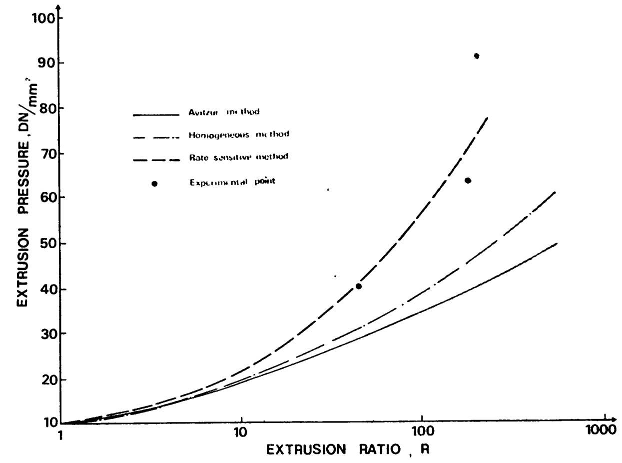

FIG. 7. — Theoretical extrusion pressures versus extrusion ratio as predicted by Avitzur's method, homogeneous work method and the present rate sensitive method. $V_0 = 0.05$ mm/min.; $D_0 = 13.6$ mm and $\alpha = 30°$. —— Avitzur method. —·—·— Homogeneous method. — — — Rate sensitive method. ● Experimental point.

Although the method due to Avitzur is based on suggesting a kinematically admissible velocity field, it has been shown that it does not constitute an upper bound in the case where Coulomb friction is assumed [12]. This explains the fact that the extrusion pressures calculated by this method are less than those given by the homogeneous work method for extrusion ratios greater than 6. Below this extrusion ratio Avitzur's method results in pressures slightly higher than those of homogeneous work method.

While Avitzur's method takes into account the contribution of redundant deformation to the extrusion pressure, the other two methods neglect its effect. It is confirmed by slip-line field analysis for a perfectly plastic material, that redundant deformation forms a significant part of the required extrusion pressure at low extrusion ratios. At large reductions the redundant work becomes proportionately much less marked [13]. It therefore seems acceptable to neglect the influence of redundant deformation on extrusion pressure at relatively high extrusion ratios, which are encountered in industry.

The consistency between the present method o analysis and experiment leads to the conclusion that the consideration of the rate sensitivity of the material as it flows through the extrusion die represents an important factor in predicting the extrusion pressure correctly.

Appendix

For a limited range of strain rate, oftenly the material behaviour in hot forming as well as in superplastic deformation is often represented by a simple power law of the form

$$\sigma_e = \sigma_0 \left(\frac{\dot{\varepsilon}_e}{\dot{\varepsilon}_0} \right)^m \tag{A.1}$$

where m, the strain rate sensitivity index is considered constant over the strain rate range of interest.

For a particular extrusion experiment, the material parameters σ_0, m are assumed constant and determined from figure 1. The maximum and minimum strain-rates within the die zone as calculated from equation (5) are used to define that portion of the stress-strain rate curve, figure 1, over which simple curve fitting is done.

Replacing the function f in equation (9), in the main text, by the right hand side of equation (A.1) yields

$$\frac{d\sigma_x}{dD} - \frac{2\,\mu \cot \alpha}{D}\sigma_x = 2\,\sigma_0(1 + \mu \cot \alpha)\,(4\,D_0^2\,V_0 \tan \alpha)^m\,D^{-3m-1}\,. \tag{A.2}$$

Integration of equation (A.2), together with the condition that $\sigma_x = 0$ at $D = D_f$ gives for the extrusion pressure

$$P = \frac{2\,\sigma_0(4\,D_0{}^2 V_0 \tan \alpha)^m (1 + \mu \cot \alpha)}{(3\,m + 2\,\mu \cot \alpha)} D_f^{2\mu \cot\alpha}\left\{ D_0^{-(3m+2\mu \cot\alpha)} - D_f^{-(3m+2\mu\cot\alpha)} \right\}. \tag{A.3}$$

Siebel *et al.* [14] have put forward a relationship between extrusion pressure and extrusion ratio based on the assumptions of frictionless homogeneous deformation. This relationship is

$$P = k \ln R \tag{A.4}$$

where k was defined as *the resistance to deformation... depends on temperature and... speed of extrusion.*

Following Jonas and setting equation (A.3) in a form similar to equation (A.4), the resistance to deformation during extrusion of rate sensitive material can be defined by

$$k = \frac{2\,\sigma_0(4\,D_0^2\,V_0 \tan \alpha)^m\,(1 + \mu \cot \alpha)}{(3\,m + 2\,\mu \cot \alpha) \ln R} D_f^{2\mu \cot\alpha}\left\{ D_0^{-(3m+2\mu \cot\alpha)} - D_f^{-(3m+2\mu\cot\alpha)} \right\}. \tag{A.5}$$

It is thus obvious that this resistance to deformation is a function of reduction, geometrical and frictional boundary conditions, speed of extrusion and the strain-rate hardening characteristics of the extruded material.

In view of the fact that the material behaviour is represented by the relation (A.1), a new mean strain-rate may be defined as :

$$\dot{\varepsilon}_{\text{mean}} = (4\,D_0^2\,V_0 \tan \alpha)\left\{ \frac{2(1 + \mu \cot \alpha)\,D_f^{2\mu \cot\alpha}}{(3\,m + 2\,\mu \cot \alpha) \ln R}\left[D_0^{-(3m+2\mu \cot\alpha)} - D_f^{-(3m+2\mu\cot\alpha)}\right] \right\}^{1/m}. \tag{A.6}$$

For the frictionless case, $\mu = 0$, this new mean strain-rate reduces to the root mean power strain-rate $\dot{\varepsilon}_{\text{RMP}}$ as defined previously by Jonas [3].

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