ONE POSSIBILITY OF SOLVING THE « IMPEDANCE PROBLEM » FOR JOSEPHSON JUNCTIONS

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Résumé. — On résout le problème relatif à la traversée d'un écran semi-transparent par un rayonnement issu d'un réseau de jonctions Josephson. On en déduit une condition pour transférer à l'espace libre, l'énergie haute fréquence émise par des jonctions Josephson. La condition peut toujours être satisfaite moyennant un choix judicieux des caractéristiques de réflexion et de la position de l'écran semi-transparent. On applique cette étude au couplage à un guide d'onde et à une cavité résonnante.

Abstract. — The problem of electromagnetic radiation from a system of tunnel junctions through a partially transparent screen is solved. A condition is derived for matching to free space the high-frequency energy generated by the Josephson junctions. The condition can always be satisfied by appropriate choice of reflective characteristics and position for the semitransparent screen. Examples are given of the use of this approach in coupling to a waveguide and to a resonant cavity.

It is known that the phenomena of weak superconductivity provide possibilities for designing quantum units for the microwave range which possess unique characteristics. Besides magnetometers, these first of all embrace generators and electromagnetic wave detectors [1].

From the radio engineering point of view, one of the important problems encountered in designing such units is connected with the difficulty of achieving matching for tunnel junctions with free space or a transmission line (the so called « impedance problem »). This fact, which is due to the smallness of the wave resistance of the tunnel junction ($\rho \sim 10^{-3}\ \Omega$), leads to practically complete reflection to the electromagnetic wave from the junction edges. This causes essential deterioration of cryogenic quantum units.

This paper deals with the problem of electromagnetic energy radiation from the tunnel junction system through a partially transparent screen. The analysis of the solution obtained assisted in finding the condition for a complete transmission of super-high-frequency power generated by the Josephson junctions into free space.

We shall consider the system of identical superconducting films periodical along the OX axis (the period is $2\,L$) and separated by the dielectric layers of $2\,l \sim 10^{-7}$ Å (Fig. 1). The system size along OY is sufficiently large, so that the y-dependence for fields and currents may be neglected. The depth of the field penetration into the superconductor is $\Lambda \ll L$.

A homogeneous partially reflecting screen is positioned at the distance h from the system in the $Z = 0$ plane. Its integral characteristics are given by the

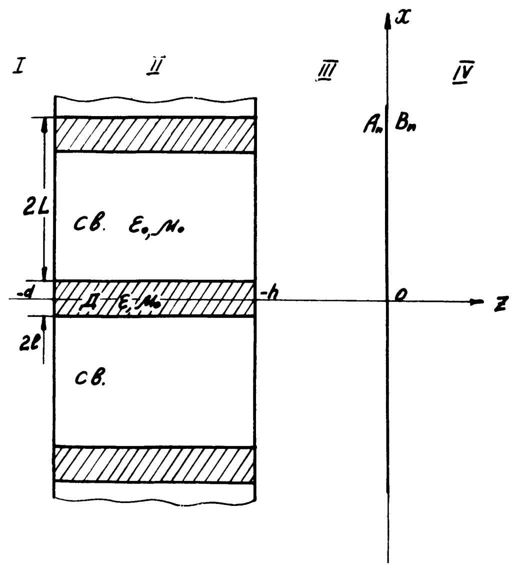

FIG. 1.

complex reflection coefficients A_n and transmission coefficients B_n.

If each of the tunnel junctions is supplied a constant potential difference V and the whole system is in the external magnetic field H parallel to the Y axis, then the Josephson current flowing in each dielectric barrier is represented in the linear approximation by the following relation [2] :

$$\mathfrak{J}_g = \mathfrak{J}_c \sin(\omega t - kz) \; ; \; \omega = \frac{2\,eV}{\hbar} \; ; \; k = \frac{4\,e(l + \Lambda)\,H}{\hbar c} . \tag{1}$$

From the Maxwell equations

$$\operatorname{rot}\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t}$$
$$\operatorname{rot}\mathbf{H} = \frac{\varepsilon}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c}\mathfrak{J} \tag{2}$$

taking account of (1) we obtain the set of equations for finding the fields excited by the Josephson current in the junction

$$\begin{cases} \dfrac{\partial^2 H_y}{\partial z^2} + \varkappa\left[\dfrac{\partial^2 H_y}{\partial x^2} + \dfrac{\omega^2\varepsilon}{c^2}H_y\right] = -\dfrac{4\pi}{c}\dfrac{\partial \mathfrak{J}_g}{\partial z}; \\ E_x = \dfrac{ic}{\omega\varepsilon\varkappa}\left(\dfrac{\partial H_y}{\partial z} + \dfrac{4\pi}{c}\mathfrak{J}_x\right); \\ E_z = -\dfrac{ic}{\omega\varepsilon}\dfrac{\partial H_y}{\partial x}; \qquad \varkappa = 1 - i\dfrac{4\pi\sigma}{\omega\varepsilon} \end{cases} \tag{3}$$

$$\begin{cases} \dfrac{\partial^2 H_y}{\partial z^2} + \dfrac{\partial^2 H_y}{\partial x^2} - \alpha^2 H_y = 0; \\ E_x = -\dfrac{i\omega}{c}\dfrac{1}{\alpha^2}\dfrac{\partial H_y}{\partial z}; \\ E_z = \dfrac{i\omega}{c}\dfrac{1}{\alpha^2}\dfrac{\partial H_y}{\partial x}; \quad \alpha^2 = \Lambda^{-2} + \dfrac{4\pi i\omega\sigma_n}{c^2} - \dfrac{\omega^2}{c^2}. \end{cases} \tag{4}$$

In (4) the current in the superconductor was written according to the Gorter-Casimir-London theory [3] :

$$\mathfrak{J} = \mathfrak{J}_n + \mathfrak{J}_s; \quad \mathfrak{J}_n = \sigma_n\mathbf{E}; \quad \frac{\partial \mathfrak{J}_s}{\partial t} = \frac{c^2}{4\pi\Lambda^2}\mathbf{E}.$$

The solution of wave eq. (3) taking into account the inequality $l \ll \lambda$ has the form

$$H_y = (i\kappa H_0\,e^{-i\kappa z} + v_1\,e^{iaz} + v_2\,e^{-iaz})\,e^{-\delta|x|}; \tag{5}$$

where

$$\delta^2 = \frac{a^2}{\varkappa^2} - \frac{\omega^2\varepsilon}{c^2}; \qquad H_0 = \frac{4\pi}{c}\frac{\mathfrak{J}_c}{a^2 - k^2};$$

In (5) the wave having the amplitude H_0 is excited by the Josephson current, and the waves of unknown amplitudes v_1 and v_2 result from re-reflection of the primary wave by the junction edges.

From (4) the field penetrating the superconductor is

$$H_y = (i\kappa H_0\,e^{-i\kappa z} + v_1\,e^{iaz} + v_2\,e^{-iaz})\,e^{-\alpha|x|};$$

The propagation constant a is found from the dispersion equation

$$\operatorname{th} 2\delta l = \frac{2R}{1 + R^2}; \qquad R = \frac{a^2 - \delta^2}{\alpha\delta}$$

which holds for superconductor-dielectric-superconductor systems. This expression is a particular case of the eq. in [4].

The structure periodicity and screen homogeneity permit the fields out of the system to be represented in the form of a Fourier series. Employing the Maxwell equations and the condition for radiation at infinity for the regions $z < -d$, $-h < z < 0$, $z > 0$, we obtain respectively

$$H_y = \sum_{n=-\infty}^{\infty} d_n \exp(i\gamma_n z)\exp\left(i\frac{\pi n}{L}x\right) \tag{6}$$

$$H_y = \sum_{n=-\infty}^{\infty} \left[a_n \exp(-i\gamma_n z) + b_n \exp(i\gamma_n z)\right] \times \exp\left(i\frac{\pi n}{L}x\right) \tag{7}$$

$$H_y = \sum_{n=-\infty}^{\infty} c_n \exp(-i\gamma_n z)\exp\left(i\frac{\pi n}{L}x\right) \tag{8}$$

where

$$\gamma_n = \sqrt{\kappa_0^2 - \left(\frac{\pi n}{L}\right)^2}; \qquad \kappa_0 = \frac{\omega}{c}.$$

Due to the effect of partially reflecting screen the amplitudes of the fields in front of and behind the screen are related as follows :

$$b_n = \mathcal{A}_n a_n; \qquad c_n = \mathcal{B}_n a_n.$$

To find the unknown amplitudes $v_1, v_2, a_n, b_n, c_n, d_n$, we use the following boundary conditions :

$$\begin{aligned} z = -h \quad & \left.\begin{aligned} H_y^{\text{II}} = H_y^{\text{III}};\ E_x^{\text{II}} = E_x^{\text{III}} \\ H_y^{\text{I}} = H_y^{\text{II}};\ E_x^{\text{I}} = E_x^{\text{II}} \end{aligned}\right\} |x| < l \\ z = -d \quad & \left.\begin{aligned} E_x^{\text{II}} = E_x^{\text{III}} \\ E_x^{\text{I}} = E_x^{\text{II}} \end{aligned}\right\} l \leqslant |x| \leqslant L. \end{aligned} \tag{9}$$

The set of functional equations obtained when (9) is obeyed and $L \ll \lambda$ taken into account has the following analytical solution [5] (*) :

$$\tilde{a}_0 = \frac{H_0\,i\kappa\,e^{i\kappa h}}{e^{i\kappa_0 h} - \mathcal{A}_0\,e^{-i\kappa_0 h}}\frac{ca}{\omega\varepsilon}\frac{l}{L} \times \left\{\frac{a}{\kappa} + \left[i\sin a(d-h)\cdot\frac{ca}{\omega\varepsilon}\varphi + \cos a(d-h)\right]N\right\}$$

$$\tilde{d}_0 = -i\kappa H_0\frac{ca}{\omega\varepsilon}\frac{l}{L}e^{i(\kappa_0 d + \kappa h)}.N$$

$$\tilde{v}_1 = -\frac{H_0}{2}i\kappa\left(1 + \frac{ca}{\omega\varepsilon}\frac{l}{L}\right)e^{i(\kappa h + ad)}N$$

$$\tilde{v}_2 = \frac{H_0}{2}i\kappa\left(1 - \frac{ca}{\omega\varepsilon}\frac{l}{L}\right)e^{i(\kappa h - ad)}.N \tag{10}$$

(*) To make the results more obvious, eq. (10) has been written down without taking into account primary wave reflections from the junction edge at $z = -d$.

where

$$N = \frac{e^{i\kappa_0 h}\left(1 - \frac{a}{\kappa}\frac{ca}{\omega\varepsilon}\frac{l}{L}\right) - \mathcal{A}_0\, e^{-i\kappa_0 h}\left(1 + \frac{a}{\kappa}\frac{ca}{\omega\varepsilon}\frac{l}{L}\right)}{i \sin a(d-h) + 2\frac{ca}{\varepsilon\omega}\frac{l}{L}\cos a(d-h)}.$$

The analysis of the above expressions shows that there is a condition

$$\mathcal{A}_0 = \frac{1 - \frac{a}{\kappa}\frac{ca}{\omega\varepsilon}\frac{l}{L}}{1 + \frac{a}{\kappa}\frac{ca}{\omega\varepsilon}\frac{l}{L}}\, e^{i2\kappa_0 h}; \qquad (11)$$

When it is obeyed, the field amplitudes $\tilde{v}_1$, $\tilde{v}_2$, $\tilde{d}_0$ turn to zero.

Thus, for tunnel junction system under consideration and the emitted electromagnetic wavelength it is always possible, according to (11), to choose such reflective characteristics and the position for a semi-transparent screen that the superhigh-frequency energy generated by the Josephson current inside the tunnel junctions is completely emitted into the free space.

One should note that, as follows from (10), the complete radiation of electromagnetic energy is possible only in the direction of the screen. At any choice of the screen reflective characteristics and position the ratio of the power emitted towards $z = -d$ to the power generated in the junction does not exceed the ratio for the junction wave resistances and the free space.

The practical usage of the scheme proposed above may be illustrated by solving the problem of superhigh-frequency energy radiation by the system of tunnel or point junctions into the waveguide in the presence of the partially reflecting screen (Fig. 2) (Fig. 3).

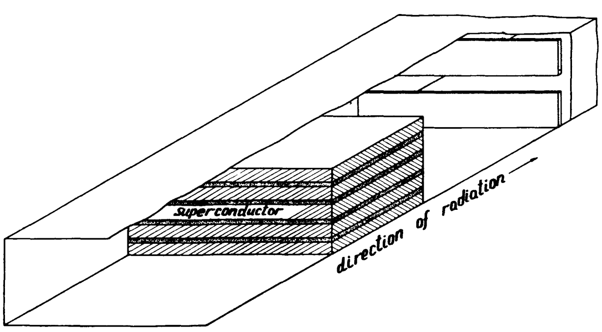

FIG. 2.

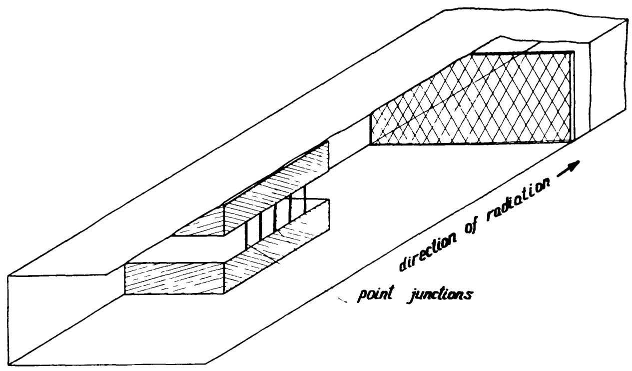

FIG. 3.

The solution analysis shows that in the case when the reflection coefficient and the matching element position obey the relation

$$R_{01} = \frac{1-\beta}{1+\beta}\, e^{i(\Gamma_{01}h - \operatorname{arctg}\beta)}$$

(where $\beta \sim 10^{-5}$ is the parameter characterizing the relation between the wave resistances in the junction-and-waveguide system

$$\beta = 2N\frac{l . a^2 . \Gamma_{01} . c^2}{L\kappa\varepsilon\omega^2}$$

N is the number of junctions, l the dielectric thickness, L the superconducting film thickness,

$$\Gamma_{01} = \sqrt{\frac{\omega^2}{c^2} - \left(\frac{\pi}{2b}\right)^2}$$

the propagation constant of the wave TE_{01} in the waveguide of thickness b) then the superhigh-frequency power generated by the Josephson current in the junction would be emitted completely into the waveguide.

It is easy to show that the dependence on the ohmic losses in the resonance cavity η for the ratio of the power transmitted to the waveguide P_{rad} and the power generated in the junction P_{max} is determined by the following expression

$$\frac{P_{rad}}{P_{max}} \approx (1 + \eta\beta)^{-2}.$$

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