The coercive force and rotational hysteresis of elongated ferromagnetic particles

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THE COERCIVE FORCE AND ROTATIONAL HYSTERESIS OF ELONGATED FERROMAGNETIC PARTICLES

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Résumé. — Le champ coercitif d'un cylindre ferromagnétique de longueur infinie est calculé en fonction de son rayon et de l'inclinaison de l'axe par rapport au champ appliqué. Pour ce calcul on ne tient compte que du « curling » et de la rotation à l'unisson et l'on suppose que lorsque le « curling » est associé avec une discontinuité, l'aimantation est amenée au stade de plus faible énergie donné par Stoner et Wohlfarth. En admettant les mêmes hypothèses, les pertes d'hystérésis rotationnelle et l'intégrale d'hystérésis rotationnelle sont calculées à la fois pour un assemblage de cylindres alignés ou répartis au hasard. Les résultats sont sensiblement concordants avec les mesures de Jacobs et Luborsky sur les particules magnétiques allongées.

Abstract. — The coercive force of an infinite ferromagnetic cylinder is calculated as a function of the radius and of the inclination of the axis to the applied field. For this calculation it is assumed that only curling and rotation in unison take place ; and that whenever the curling is associated with a discontinuous jump, the magnetization is brought to the lower energy state given by Stoner and Wohlfarth. Using the same assumptions, the rotational hysteresis loss and integral are calculated both for an aligned and for a random assembly of cylinders. The results are found to be in fair agreement with the measurements of Jacobs and Luborsky on elongated ferromagnetic particles.

Recently, a new theory of magnetization changes in ferromagnetic particles was developed [1], [2], [3] and was used to calculate the hysteresis curve of an infinite ferromagnetic cylinder whose axis is parallel to the applied field [4], [5]. In the following the former calculations are extended to the case in which the cylinder is not parallel to the field. The dependence of the coercive force on the angle between the cylinder and the applied field and the rotational hysteresis loss are evaluated and found to be in rough agreement with results of experiments made by Jacobs and Luborsky [6] on elongated ferromagnetic fine particles.

The nucleation field. — Consider the descending hysteresis loop of an infinite cylinder inclined at an angle Ω to the applied field. Up to a certain negative field — the nucleation field — the magnetization changes reversibly by rotation in unison in accordance with the calculations of Stoner and Wohlfarth [7]. Following Brown [2] it can be shown that, neglecting the magnetocrystalline anisotropy, this nucleation field, H_{tn}, is the least eigenvalue H_t of the set of equations

$$- (2A/I_s)\, \nabla^2 \alpha_x + \cos\, \omega \frac{\partial U}{\partial x} - \sin\, \omega \frac{\partial U}{\partial z} + H_t\, \alpha_x = 0$$

$$- (2A/I_s)\, \nabla^2 \alpha_y + \frac{\partial U}{\partial y} + H_t\, \alpha_y = 0 \qquad (1)$$

$$\nabla^2 U = 4\pi I_s \left(\frac{\partial \alpha_x}{\partial x} \cos\, \omega - \frac{\partial \alpha_x}{\partial z} \sin\, \omega + \frac{\partial \alpha_y}{\partial y} \right)$$

for

$$\bar{x}^2 + \bar{y}^2 \leqslant R^2$$

and

$$\nabla^2 U = 0 \text{ for } \bar{x}^2 + \bar{y}^2 \geqslant R^2 \qquad (2)$$

and the following boundary conditions at $\bar{x}^2 + \bar{y}^2 = R^2$

$$\frac{\partial \alpha_x}{\partial n} = \frac{\partial \alpha_y}{\partial n} = 0$$

$$U_{\text{in}} = U_{\text{out}} \qquad (3)$$

$$- \partial U_{\text{in}}/\partial n + 4\pi \bar{I}_n = - \partial U_{\text{out}}/\partial n$$

provided that

$$\pi I_s^2 \sin 2\omega - HI_s \sin (\Omega - \omega) = 0 \qquad (4)$$

which is the equilibrium condition before nucleation.

In the above, H_t is the internal magnetic field before nucleation comprising the external and demagnetizing fields :

$$H_t = - 2\pi I_s \sin^2\, \omega + H \cos (\Omega - \omega), \qquad (5)$$

U is the change in the magnetostatic potential and $\bar{I}_n$ the change in the component of the magnetization perpendicular to the surface of the cylinder which accompanies nucleation. I_s and A are the saturation magnetization and the exchange constant respectively, n is the normal to the cylinder and α_x, α_y, α_z the direction cosines of the spin with respect to the (x, y, z) coordinate system. The other notations are shown in figure 1, which gives the equilibrium state before nucleation (when all the spins are aligned).

To determine the whole spectrum of eigenvalues

of the above equations is difficult. However, it can be shown by substitution that in a cylindrical coordinate system $(r, \varphi, \bar{z})$

$$\alpha_x = -\frac{B}{\cos \omega} \sin \varphi \, J_1[1.841 \, r/R]$$

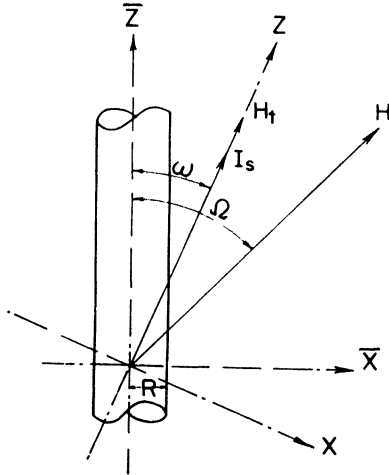$$\alpha_y = B \cos \varphi \, J_1[1.841 \, r/R] \tag{6}$$

$$U = 0$$

FIG. 1. — Geometry of the equilibrium state before nucleation with all spins aligned. H is the applied field while H_t is the total internal field comprising the applied field and the demagnetization one.

is a solution of equations (1) (2) and (3) provided that

$$\frac{H_t}{2\pi I_s} = -1.08 \, S^{-2} \tag{7}$$

Here

$$S = R/R_0 \tag{8}$$

the characteristic radius R_0 is given by

$$R_0 = A^{1/2}/I_s \tag{9}$$

and J_1 is the Bessel function of the first kind and order. The above mode of nucleation is, in fact, an extension of the magnetization curling [3]. Eq. (7) shows that the internal nucleation field is independent of inclination. The dependence of the external nucleation field H_n on inclination for the curling mode can be obtained by eliminating ω and H_t from eq. (4). (5) and (7)

$$h_n = \frac{H_n}{2\pi I_s}$$

$$= \frac{-1.08 \, S^{-2}(1 - 1.08 \, S^{-2})}{[(1 - 1.08 \, S^2)^2 - \sin^2 \Omega(1 - 2.16 \, S^{-2})]^{1/2}}. \tag{10}$$

The other important mode of nucleation is the one of rotation in unison. The nucleation fields for this mode are the critical fields given by Stoner and Wohlfarth [7]. In view of results obtained

for the applied field parallel to the cylinder [5], it is assumed here that for any size of the cylinder and any inclination the easier mode of nucleation of the above two modes will yield practically the lowest nucleation field.

The coercive force and rotational hysteresis.
— In order to calculate the magnetization curve the behavior after nucleation must be studied. It can be shown that at nucleation the magnetization changes discontinuously, for practically every radius and inclination. It is assumed that the discontinuous jump brings the magnetization to the lower energy equilibrium given by Stoner and Wohlfarth [7]. It follows that the coercive force is identical with the nucleation field when nucleation occurs for positive values of the magnetization. The calculated dependence of the coercive
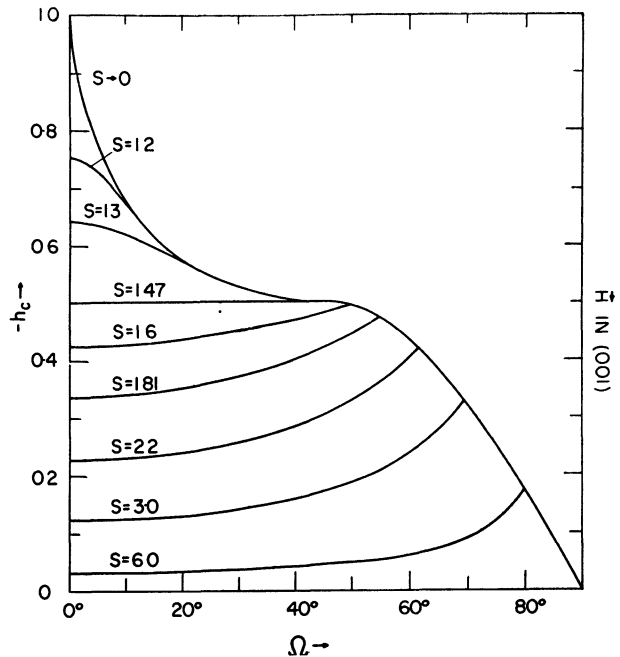
FIG. 2. — Theoretical dependance of the reduced coercive force h_c on the angle Ω to the field with the reduced radius S as a parameter.

force on inclination with size as a parameter is given in figure 2.

Using the above and tables of the equilibrium state tabulated by Stoner and Wohlfarth [7], the rotational hysteresis of an aligned assembly is calculated and the results plotted in figure 3 versus the applied field with the radius of the cylinder as a parameter. Figure 4 shows the same for an assembly with random orientation of the cylinders.

The rotational hysteresis integral P as defined by [6]

$$P = \int \frac{W_r}{I_s} \, \mathrm{d}(1/H) \tag{11}$$

was calculated for an aligned and random assembly of cylinders from figures 3 and 4 respectively and is shown in figure 5 as a function of S. As $S \to \infty$ the integral asymptotically approaches
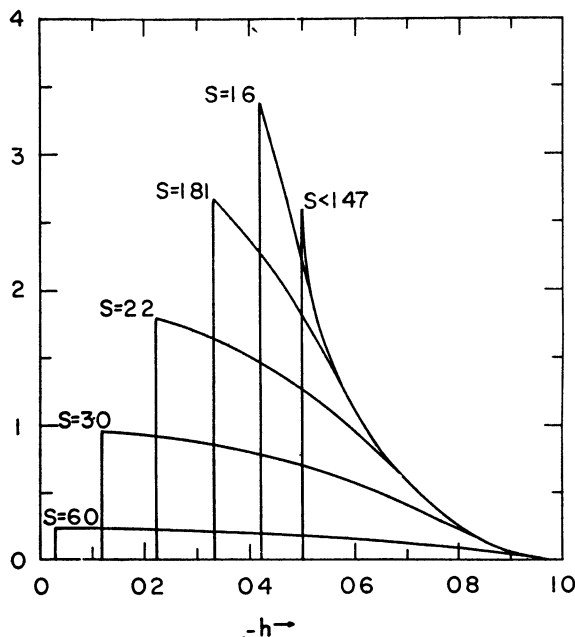

FIG. 3. — Theoretical plot of the dependence of the rotational hysteresis loss W_r of an aligned assembly of infinite cylinders on the reduced applied field h with the reduced size S as a parameter. I_s is the saturation magnetization.
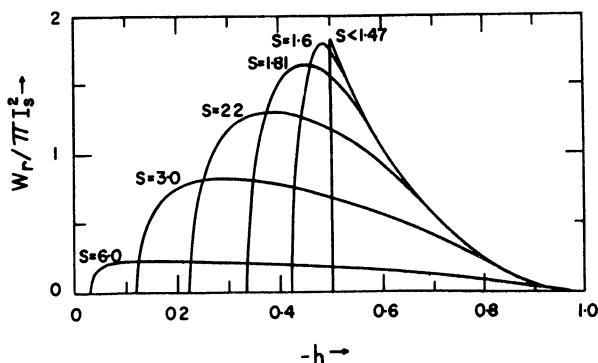

FIG. 4. — Theoretical plot of the rotational hysteresis W_r of an assembly of infinite cylinders with randon orientation on the reduced applied field h with the reduced size S as a parameter. I_s is the saturation magnetization.

the value 4 for the aligned assembly and the value π for the random one. The ratio of h_m, the maximum coercive force, to h_0, the coercive force for $\Omega = 0$, and the angle Ω for $h = h_m$ are also plotted in figure 5. The infinite cylinder considered above is a rough approximation to the elon-

gated particles measured by Jacobs and Luborsky [6]. Figure 5 is used to evaluate the size of the particles from the appropriate experimental values measured by Jacobs and Luborsky. The values thus obtained and the directly measured one are given in table 1. In view of the idealized model used, the agreement between the theoretical and the experimental values seems satisfactory. Furthermore for Alnico 5 for which [8] $S < 1$ Jacobs and Luborsky find [6] a monotonic decrease of the coercive force with inclination in agreement with figure 2.
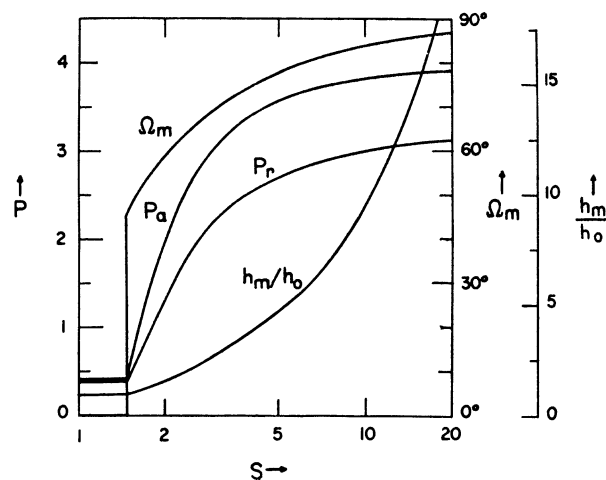

FIG. 5. — Theoretical plot of P_e and P_r, the rotational hysteresis integral, for an aligned and randon assembly of cylinders, h_m/h_0 the ratio of the maximum coercive force to the coercive force for $\Omega = 0$ and the angle Ω_m for $h = h_m$ versus the reduced radius S.

TABLE I

MATERIAL	DIRECTLY MEASURED SIZE (A°)	SIZE (A°) CALCULATED FROM :				
		$\dfrac{h_m}{h_0}$	Ω_m	P_a	P_r	h_c
—	—	—	—	—	—	—
Fe	150 ± 25	180	190	220	240	320
FeCo	200 ± 35	180	240	220	240	310

Comparison of the calculated size of elongated ferromagnetic particles, which were measured by Jacobs and Luborsky [6], using magnetic data, with the directly measured one. The first four calculated columns were obtained using figure 5 and the last one from the formula for the coercive force evaluated previously [5] for and infinite cylinder parallel to the applied field. The characteristic radius was taken as before [3] to be $R_0 = 60°$ Å.

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DISCUSSION

Mr. Kondorskij (Remark). — It seems that in the case of the long cylinder " nucleation field " is not a suitable expression : we have uniform or non-uniform rotation throughout the sample, and not the local nucleation of an inverse domain.

Mr. Brown. — I should like to point out the close relation between the calculation of the nucleation field and certain calculations in ferromagnetic resonance. If a specimen is subjected to a large constant field and to a small alternating field, the alternating field causes the magnetization at any point to precess about the constant field. If there are no losses, then for certain values of the frequency at given constant field, or for certain values of the constant field at given frequency, the precession can be maintained by an alternating field of zero amplitude ; this is the resonance condition. Now if the resonance calculation is done completely, the algebraically largest resonance field at zero frequency is identically the static nucleation field. This theorem follows directly from the complete partial differential equations and boundary conditions of the resonance problem. But the calculation has never been done completely. Workers in ferromagnetic resonance have tried to avoid mathematical complexities by introducing only such terms as seemed immediately important, by assuming uniform magnetization as long as possible, and so on. Consequently, the relation between their dynamic problem and our static problem has been so completely overlooked that in the present conference, these two topics alone have been selected for simultaneous discussion in different rooms. I hope that the next conference will find them happily united.

Mr. Kaczer. — Every magnetization reversal is a dynamic process and eddy current have to be considered especially for stability considerations. I think that without taking into account this fact, the stability conditions are not quite correct.

Mr. Shtrikman. — I think this is right in principle. However the experiment shows us that for low enough frequencies, the magnetization curves are to a good approximation independent of frequency. This indicates that one can then neglect the effect of eddy currents.

Mr. Hoselitz. — The present paper and similar treatments consider cylindrical bodies. There exists evidence presented at Boston in 1957 that in the case of changes of magnetization by rotation in thin films, this rotation is not always coherent. It appears to me, that processes of changes of magnetization in thin films are of great importance and I wonder whether calculations of critical fields of magnetization reversal by non-ordered rotation in thin films could be made or are being considered.

Mr. Shtrikman. — I would like to point out that according to our calculations the magnetization changes in thin films by rotation in unison because of the negligible demagnetization energies involved. No domains should exist in ideal thin films ; probably the introduction of defects might account for their experimental appearance.

Mr. Bates. — In dealing with thin films I would suggests that the authors consider the Bitter figures obtained by Šur on very thin films of silicon-iron, which show that Néel spikes persist in an extraordinary way.

Mr. Shtrikman. — I am aware of that, but, as I said, we unfortunately cannot get any domain walls in our calculations for ideal films. They might be obtained when the theory is extended to films with defects. We have started some work along these lines, but the calculations are quite involved.