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ON THE GROUND STATE PROPERTIES OF THE $\pm J$ ISING MODEL

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Abstract. – In the combinatorial method or Grassmann algebra formalism the ground state properties of the $\pm J$ Ising model can be expressed in terms of the behaviour of the eigenvectors of a matrix. It is shown that a transition from localized to extended eigenvectors signals the breakdown of ferromagnetic rigidity.

A body of numerical work on the ground state properties of short-range Ising spin glasses has been established over the past ten years (for a review, see Binder and Young [1]). Various interpretations of the behaviour, including some based on percolation ideas, have been explored, but connections have proved elusive. A recent approach [2] to the ground state properties has been via the scaling of domain wall energies and the percolation of zero-energy domain walls as a signal of the breakdown of ferromagnetic rigidity. The authors find the absence of a single length scale which contrasts with familiar percolation ideas.

An alternative formalism that encompasses all ground state (absolute minimum energy) configurations in a single description was explored by the present authors [3, 4]. In two dimensions the partition function for an assembly of N Ising spins can be expressed as a non-interacting field theory [5, 6]

$$Z = 2^N \prod_{\langle ij \rangle} \cosh K_{ij} \int d\eta \exp \left(\sum_{\alpha\beta} \eta_\alpha A_{\alpha\beta} \eta_\beta \right) \quad (1)$$

$K_{ij} = J_{ij} / kT$ where J_{ij} is the bond strength for the pair of sites $\langle ij \rangle$ and the product is over all bonds of the lattice. The integral is over $4N$ Grassmann variables η_α (4 per lattice site) and the matrix A is a function of the set of $\tanh K_{ij}$. An equivalent expression of equation (1) is in the combinatorial method [7], where Z is written in terms of the determinant of the matrix A .

In studying the ground state properties of the $\pm J$ model, the eigenproperties of A in the low temperature limit have particular significance. For the perfect Ising ferromagnet, the eigenvalues of A lie in a band between $2 - \sqrt{2}$ and $2 + \sqrt{2}$. In the presence of frustration (due to a fraction x of negative bonds), new eigenvalues appear outside the band. They approach zero in the $T \rightarrow 0$ limit as

$$\epsilon = \frac{1}{2} X \exp(-2rJ / kT) \quad (2)$$

X is a constant and r is an integer ($r \geq 1$). The num-

ber of eigenstates of this form is equal to the number of frustrated plaquettes. The values of X and r for each eigenvalue depend on the distribution of frustration over the lattice. The ground state free energy and entropy can be expressed [4] in terms of these quantities

$$F = -2NJ + 2J \sum_{\lambda} r_{\lambda} \quad (3)$$

$$S = k \sum_{\lambda} \ln X_{\lambda} \quad (4)$$

where λ labels the eigenvalue.

It is possible to pursue this approach numerically by the use of degenerate state perturbation theory to determine exactly the X and r and also the spatial extent of the corresponding eigenvectors. In our earlier paper [4] we studied 20×20 samples and obtained strong indications that the breakdown of ferromagnetic rigidity (at x just below 0.15) is closely associated with the appearance of extended eigenvectors. The present paper develops that investigation for much larger samples to provide confirmation of the earlier observation.

For each eigenvalue of the form (2) there is an associated one at $-\epsilon$. A convenient (and physically significant) way to represent the spatial extent is by the distance between the "centres of mass" of the two members of the pair. This has been done for some 128×128 arrays of spins. The results are displayed in figures 1 and 2 for two configurations of negative bonds, one with $x = 0.1$ and the other with $x = 0.2$.

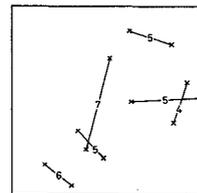


Fig. 1. – Spatial extent of eigenvectors in an $x = 0.1$ configuration on a 128×128 lattice.

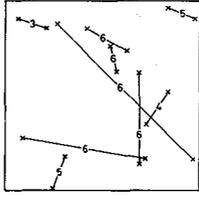


Fig. 2. - Spatial extent of eigenvectors in an $x = 0.2$ configuration on a 128×128 lattice.

The crosses represent "centre of mass" positions with associated pairs being denoted by a connecting line. The number on the line is the r value for that state. For the 10 % configuration there were 4752 frustrated plaquettes and the 20 % one had 7040. The number of eigenstates of the form (2) is the same, but only those with spatial extent greater than 20 lattice spacings are shown. It is clear from the diagrams that the eigenvectors at 20 % are more extended than those at 10 %. To make the definition of extension more precise we have examined a number of configurations on $32 \times L$ lattices with L ranging from 32 to 512. For four values of x we have calculated the spatial extent of the eigenvector with the greatest extension and then obtained the average value l of this over 10 configurations of negative bonds. The results are plotted against L in figure 3. It can be seen that, for 5 % and

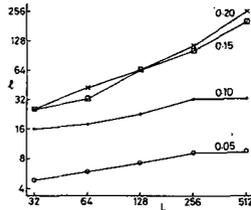


Fig. 3. - Mean (over 10 configurations) extent l of maximum eigenvectors on $32 \times L$ lattices. Values of x are shown on plots.

10 % concentrations of negative bonds, l tends to a finite value, but above the critical concentration for the breakdown of ferromagnetic rigidity l scales with the size of the sample.

We can understand the significance of these extended states as follows. Suppose, in a perfect lattice, we introduced a chain of negative parallel defects ($-J'$) of length l . A pair of states of the type we have described would be formed with $r = lJ'/J$. In our case then an eigenvector with spatial extent l has an effect similar to a chain of defects of length l with $J'/J = r/l$. From our calculations, the value of r for the most extended state shows negligible dependence on L , and, from figure 3, l itself scales with the size of the sample. In the $L \rightarrow \infty$ limit then, our extended states are equivalent to a chain of defects (with $J' \rightarrow 0$) of length $l \rightarrow \infty$ breaking up the lattice and destroying long range order.

These results, we believe, provide new insight into the nature of short-range frustrated systems and, in particular, the breakdown of ferromagnetic rigidity. We are currently attempting to improve the statistics on figure 3 by increasing both L and the number of configurations sampled. Consideration is also being given to 3D systems which can be approached by an interacting field theory [6].

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