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► To cite this version:

J.H. Rose, D. Hsu, L. Adler. ULTRASONIC CHARACTERIZATION OF POROSITY USING THE KRAMERS-KRONIG RELATIONS. Journal de Physique Colloques, 1985, 46 (C10), pp.C10-787-C10-790. 10.1051/jphyscol:198510172 . jpa-00225381

HAL Id: jpa-00225381

<https://hal.science/jpa-00225381>

Submitted on 1 Jan 1985

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ULTRASONIC CHARACTERIZATION OF POROSITY USING THE KRAMERS-KRONIG RELATIONS

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Abstract - A new algorithm is proposed to determine the volume fraction of pores in solids using the frequency dependent ultrasonic attenuation. The algorithm was developed by examining the Kramers-Kronig relation between the porosity induced ultrasonic attenuation and the change in sound velocity. The method is tested using data measured for several porous aluminum samples.

I - INTRODUCTION

This paper is structured as follows. First, a model is given which relates the porosity induced attenuation and the shift in the sound speed. Algorithms for the volume fraction, c , and the average pore size, $\bar{a}$, are then derived. Finally, these algorithms are tested and found to be satisfactory using experimental attenuations measured in porous cast aluminum samples.

Porosity is modeled as a uniform random distribution of spherical voids of various radii in an otherwise homogeneous and isotropic elastic solid. The volume fraction is assumed small ($c \rightarrow 0$, practically $c < 5\%$). Following recent treatments of this model (in particular Ref. 1), we obtain

$$\alpha(k) = \frac{2\pi}{k} \int_0^\infty da \, n(a) \operatorname{Im} A(k, a), \quad (1)$$

and

$$\frac{\Delta V(k)}{V_0} = - \frac{2\pi}{k^2} \int_0^\infty da \, n(a) \operatorname{Re} A(k, a) \quad (2)$$

Here α , ΔV and V_0 are respectively the attenuation, the velocity shift and, the velocity of longitudinal sound in pore free material, for a longitudinally polarized displacement field. Also $n(a)da$ denotes the number of pores per unit volume with radii between a and $a+da$. The wave vector is denoted by k . $A(k, a)$ denotes the longitudinal to longitudinal forward scattering amplitude.

The expression of causality in the frequency domain (Kramers-Kronig relations)² implies

$$\frac{\text{Re } A(k)}{k^2} = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\text{Im } A(k') dk'}{k'^2 (k' - k)}, \quad (3)$$

and a similar relation for $\text{Im } A(k)/k^2$. The slash indicates the principal part of the integral. Substitution of (3) in (2) yields

$$\frac{\Delta V(k)}{V_0} = -\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\alpha(k') dk'}{k'^2 (k' - k)}, \quad (4)$$

an expression familiar from other contexts; see for example Ref. 3.

The long wavelength limit of the velocity shift [Eq. (4)] leads to our basic result. As $k \rightarrow 0$, Eq. (4) becomes

$$\frac{\Delta V(k=0)}{V_0} = -\frac{2}{\pi} \int_0^{\infty} \frac{\alpha(k') dk'}{k'^2}. \quad (5)$$

On intuitive grounds $\Delta V(k \rightarrow 0)$ is (1) expected to depend linearly on c as $c \rightarrow 0$ and (2) to be independent of the pore size distribution. Consequently Eq. (5) can be converted into an algorithm for determining c from $\alpha(k)$.

We proceed by evaluating Eq. (2) directly in the long wavelength limit. As $k \rightarrow 0$, $\text{Re } A(k, a) = a(A_2 k^2 a^2 + 0(k^4 a^4))$. Here A_2 is a dimensionless expansion coefficient which depends only on the Poisson ratio of the host material. An analytic expression for A_2 can be obtained from Ref. 4. Substitution of the $k \rightarrow 0$ form of $\text{Re } A(k)$ in Eq. (2) yields

$$c = \frac{4}{3\pi A_2} \int_0^{\infty} \frac{\alpha(k')}{k'^2} dk'. \quad (6)$$

This is our basic algorithm. It determines c from $\alpha(k)$ in a way which does not depend explicitly on the pore size distribution.

Once c has been determined from Eq. (6), a rough estimate of the average pore size can be obtained. At high frequencies ($ka \gg 1$) the attenuation can be computed from an acoustic ray picture. It becomes $\alpha(k \rightarrow \infty) = N \pi \langle a^2 \rangle$. Here N is the total number of pores per unit volume. The average cross-sectional area per pore is given by $\pi \langle a^2 \rangle$; where $\langle \dots \rangle$ denotes the expectation value over the size distribution. On the other hand $c = 4 \pi N \langle a^3 \rangle / 3$. Combining those results yields the estimate $\bar{a} = \langle a^3 \rangle / \langle a^2 \rangle = 3C / (4 \alpha(k \rightarrow \infty))$. We expect $\bar{a}$ to provide a reasonable estimate for the radius, if the distribution is sharply peaked about a mean value.

EXPERIMENTAL RESULTS

A number of samples of A357 cast aluminum alloy with varying degrees of porosity (1-5%) were produced at the OSU foundry. The flat samples were immersed in water and a wide-band ultrasonic pulse was scattered from them at normal incidence. The ratio of the back-surface to the front-surface echo allowed the estimation of $\alpha(k)$. Diffraction effects were accounted for. Details are given in Ref. 5.

Fig. 1A. Shows experimental (•••) and theoretical attenuation (—) of Sample #1510.

Fig. 1B. Shows integrand in Eq. (6). Vertical dashed lines indicate minimum and maximum usable frequencies (#1510).

Table 1. Gives sample #, actual c (density measurement), $\bar{c}$ (ultrasonic experiment), and radius estimate, $\bar{a}$.

#	c	$\bar{c}$	$\bar{a}(\mu\text{m})$
1510	.021	.019	190
1810	.009	.014	120
1820	.011	.016	130
1830	.012	.017	140
1920	.039	.047	170

Figure (1A) shows the experimental attenuation measured for sample 1510. Figure (1B) shows a plot of $\alpha(k)/k^2$; the integrand in Eq. (6). The evaluation of the integral from 0 to ∞ requires that the data be extended using prior information. The known asymptotic form of $\alpha(k)$ leads us (1) to set $\alpha(k) = \alpha(k_{\max})$ for wavevectors $k > (k_{\max})$; and (2) to let $\alpha(k) \sim k^4$ for k less than the smallest usable wavevector. These extensions typically account for 20% of the total integral. Finally $A_2 = .57$ for A357 aluminum alloy.

Results are listed for a variety of samples in Table 1. The trends indicate relatively good qualitative agreement between theory and experiment. The origin of the discrepancies are uncertain. However the assumptions that (1) the pores are spherical and (2) distributed uniformly in space are only approximate. The removal of these assumptions is expected to improve the currently good qualitative agreement.

ACKNOWLEDGEMENT

This work was sponsored by the Center for Advanced Nondestructive Evaluation, operated by the Ames Laboratory, USDOE, for the Air Force Wright Aeronautical Laboratories/Materials Laboratory under Contract No. W-7405-ENG-82 with Iowa State University.

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