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Résumé - Un modèle a été développé pour décrire la ruine sous chargement de tension uniaxiale. Il est basé sur des principes de la mécanique, de la rupture et les propriétés les plus représentatives du béton. Grâce à ce modèle la résistance peut être déterminée en fonction de la vitesse de mise en charge. Les résultats sont en très bon accord avec les données des essais de traction uniaxiale aux vitesses de chargement allant de 10^4 à 10^{12}Pa/s.

Abstract - On the basis of the principles of fracture mechanics and the most significant properties of concrete, a model has been developed which describes the failure under uniaxial tensile loading. Thanks to this model the strength can be determined as a function of the loading rate. The results are in very good agreement with the data of uniaxial tensile tests with loading rates of 10^4 to 10^{12}Pa/s.

I - INTRODUCTION

From recent investigations it appeared that in structures under impulse loading the force distribution differs a great deal from that under static loading. In the structure these appear to run high peaks of shear forces and moments /1/. Since the numerical values of the corresponding stresses are high, the chance of failure under these loading conditions is expected to increase.
From experiments, however, the resistance of structures against explosion and impact loading is known to be higher than is to be expected from theoretical force distribution.
Apparently the strength of concrete is greatly influenced by the loading rate. This influence of loading rate on the uniaxial tensile strength of concrete has been proven experimentally by several investigators. An increase in the loading rate from 10^4 to 10^9 Pa/s causes an increase in tensile strength of about 75%. The experiments of D.L. Birkimer have shown that the strength increases steeply at loading rates higher than 10^{11} Pa/s.
To enable the loading rate to be taken into account in response calculations a model has been developed which describes the influence of the loading rate on the strength.

II - TENSILE FRACTURE OF CONCRETE

Concrete is a composite material consisting of different sized aggregate particles embedded in cement paste. Because of the heterogeneity there are voids and cracks at all dimensional scale levels. Many of the microcracks are formed during the hardening process and therefore exist even before any load is applied. The heterogeneity, the existing microcracks and voids and the differences in stiffness govern the mechanical behaviour of concrete.
Under tensile loading the stress-strain curve is linear up to a stress level of 60% of the ultimate tensile strength (f_c). Above this level the bond microcracks start to grow but due to crack arrest crack propagations remains stable.
Beyond a stress level of about 0,75 f_c the microcracks in the mortar start to grow and bridging of the bond cracks occurs; macrocracks are formed; crack propagation becomes unstable and finally one major crack develops.

To model this mechanical behaviour fracture mechanics has been chosen as a tool because it provides the opportunity of describing the behaviour of materials with

internal damage or of heterogeneous composition. Application of fracture mechanics for a precise description of the behaviour of concrete until the ultimate strength is reached requires that the internal stress distribution and the energy demand for crack propagation are known at all dimensional scale levels.
The above review of the behaviour and the composition of concrete shows that the stress distribution and the energy demand cannot be described in detail, especially when the loading rate has to be taken into account.
Although the fracture phenomena under tension are quite well known /2/,/3/, modelling of the mechanical behaviour of concrete is only possible when it is focussed on the most significant features of the behaviour.

III - DESCRIPTION OF THE MODEL

Before loading voids and microcracks are already present in concrete. The degree of this internal damage can be expressed in a geometrical parameter: the ratio of the volumes of damaged and undamaged material. In the potential plane of failure, the critical situation is reached for the bond cracks that are present when the stress equals 0.6 f_c. At the stress level of 0.75 f_c the critical situation for the micro-cracks in the matrix is reached. These facts are used in the model.
The potential plane of failure with the cracks present is schematized as a plane with continuously distributed penny-shaped cracks (radius a_o) with an intermediate distance of 2b. The ratio $\left(\frac{a_o}{b}\right)$ is the parameter which characterizes the degree of internal damage. (Fig.1).

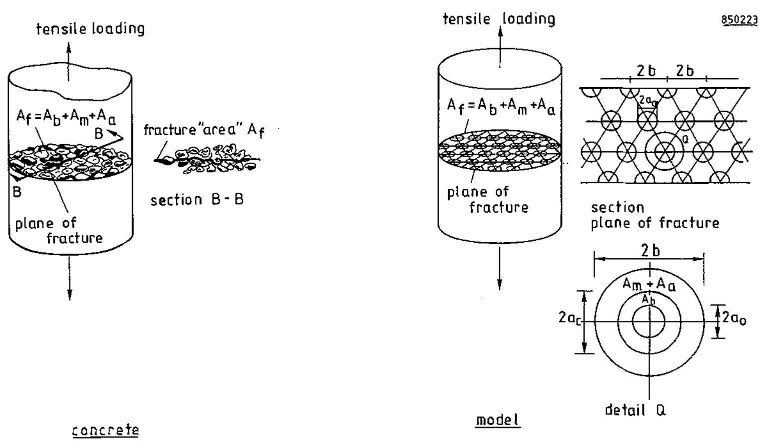


Figure 1 - The planes of fracture in concrete and model

Application of the strength criterion of fracture mechanics yields that the radius (a_o) of the penny-shaped cracks is the critical crack radius at $\sigma = 0.6\ f_c$. Crack propagation occurs when the release of strain energy equals or exceeds twice the specific surface energy (γ) of the new formed crack surfaces (energy criterion). During the loading trajectory $\sigma = 0.6 - 0.75\ f_c$ the crack radius increases to a_c, which is the critical crack size for crack propagation in the matrix at $\sigma = 0.75\ f_c$. During this trajectory the fracture energy (E_b) of the bond area (A_b) (see Fig. 1) is dissipated. When the stress increases from $\sigma = 0.75\ f_c$ to $1\ f_c$ the fracture energy of the matrix (E_m) and of the aggregate particle (E_a) is dissipated.
Crack extension is determined by the rate of energy supply which is coupled with the loading rate $\dot{\sigma}$, and by the difference between the strain energy release rate and the energy needed to form the new crack surfaces. The influence of the loading rate on

the behaviour of concrete has been investigated in several research programmes. It has been proven that with increasing loading rate crack extensions run increasingly through the aggregate particles rather than along the bond surfaces. Since the specific surface energy of the aggregate is more than the surface energy of the bond surface the energy demand for crack extension increases with increasing loading rate. The loading rate also influences the stress distribtion around the crack tip. The inertia effects become more important with increasing loading rate and multiple cracking will occur, which also means an increas in energy demand for crack extension.

In the model the energy release rate is determined with the Linear Elastic Fracture Mechanics (L.E.F.M.). Although concrete is not linear elastic the L.E.F.M. has been chosen because of its simplicity and its possibility to take the parameters $(\frac{a}{b})$, (γ), and $(\dot{\sigma})$ into account. Furthermore the purpose of the model is to describe the strength of concrete which is specially governed by the resistance of the material to crack propagation on small-scale levels while the behaviour of concrete becomes more linear when the dimensional scale level gets smaller.

The crack propagation in the model follows from the balance of energies. For the cracks in the model this balance is given by

$$d\,E_{kin} = d\,E_s - d\,E_f \tag{1}$$

which means that the difference between the release of strain energy ($d\,E_s$) and the energy needed for fracture ($d\,E_f$) equals the change in kinetic energy of the material around the crack tip /5/. By using the L.E.F.M. the change in E_{kin} (1) per unit length of the crack tip is given by

$$d\,E_{kin} = \left[\frac{4}{\pi}\,\frac{(1-\nu^2)}{E}\,\sigma^2 a f^2(\tfrac{a}{b}) + \frac{4(1-\nu^2)}{3\pi E}\left\{\frac{2\dot{\sigma}\sigma a^2 f^2(\frac{a}{b})}{\dot{a}} + \frac{a^2\sigma^2}{\dot{a}}\,\frac{df^2(\frac{a}{b})}{dt}\right\} - 2\,\gamma\right] da \tag{2}$$

where E = modulus of elasticity

ν = Poisson ratio

$f^2(\frac{a}{b})$ = function to bring the geometry into account

$\dot{a} = \frac{da}{dt}$ = velocity of crack extension

When the influence of dynamics on the strain energy release rate is ignored equation (2) changes into

$$d\,E_{kin} = \left[\frac{4}{\pi}\,\frac{(1-\nu^2)}{E}\,\sigma^2 a f^2(\tfrac{a}{b}) - 2\,\gamma\right] da \tag{3}$$

In the equations (2) and (3) is dE_f given by the term ($2\gamma da$).

The displacement field ($\underline{u}$) of the linear eleastic material around the crack tip is known, so the kinetic energy is also given by

$$E_{kin}\,(t) = \iint_A \tfrac{1}{2}\,\rho\,(\frac{\partial a}{\partial t})^2\,dA$$

$$E_{kin}\,(t) = (\frac{1}{C_s})^2\,\frac{1-\nu^2}{E}\,a^2 f^4(\tfrac{a}{b})\left\{af^2(\tfrac{a}{b}) + \sigma\frac{d}{dt}\,(af^2(\tfrac{a}{b}))\right\}^2 \tag{4}$$

In this equation C_s is a constant.
With the equations (2) and (4) or (3) and (4) a differential equation is derived which describes the crack extension in the model and hence the dissipation of energy during the failure process in concrete.

IV - COMPARISON OF THE THEORY WITH EXPERIMENTAL DATA

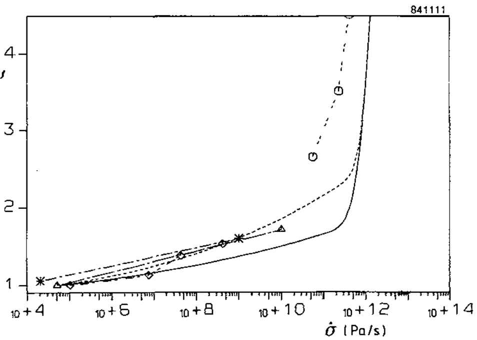
To show the possibilities of the way of modelling, as summarized in this paper, a simplification of the model will be worked out and the results will be compared with experimental data.

As a first approximation the influences of the loading rate on the energy demand for crack extension and the stress distribution, as mentioned in section III, have been compressed in a parameter α and the propagation velocity of the crack tip is given by

$$\dot{a}_d = \left(\frac{\dot{\sigma}_d}{\dot{\sigma}_s}\right) \dot{a}_s$$

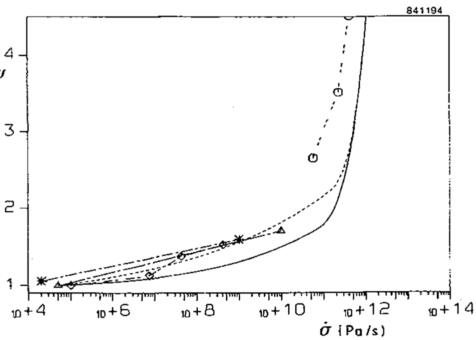
where index (d) stands for "dynamic" and (s) for "static". Parameter α will approach one because the average value of ($\dot{a}$) depends on the energy supply which is directly coupled to the loading rate ($\dot{\sigma}$). The average value of ($\dot{a}$) determines the time needed for the growth of the crack radius from (a_o) to (b), which interval is independent of the loading rate. As the strength increases with increasing loading rate, parameter α must not only be nearly one but also smaller than one.
A consequence of the approximation introduced above is that the strain energy release rate of the static case can be used (eq. 3). The difference in the energy release rate under dynamic and static loading has been investigated in /4/. From this investigation it follows that the difference in the calculated tensile strength will have the magnitude of $O(\Delta f_c) = \dot{\sigma}.10^{-5}$ Pa.

To simplify the calculation only one loading trajectory ($\sigma = 0{,}6 - 1\ f_c$) is taken into account and the fracture energy is given by an average value of the specific surface energy (γ). For the determination of the unknown parameters ($^{a_o}/_b$) and (γ) the behaviour of concrete under static loading and the composition of the concrete is used. The only parameter which cannot be determined without data of dynamic tensile tests is α, which almost equals one.
Figures 2 and 3 show experimental data of the relative tensile strength Ψ ($= \frac{f_{cd}}{f_{cs}}$) as a function of the loading rate ($\dot{\sigma}$) and the calculated relative strength.
In the calculation presented in Figure 2 the energy demand for crack extension is constant, while Figure 3 shows the result of the calculation in which the increasing energy demand for increasing loading rate has been taken into account. In both cases agreement with the experimental data is good and the unknown parameter α equals almost one indeed.



———	= 0.95
-----	= 0.93
γ	= 7.5 (Jm^{-2})
a/b	= 0.4
f_{cs}	= 5.10^6 (Pa)
⏀	= Birkimer
✳	= Takeda
◈	= Hatano
▲	= Univ. of Technology, Delft

Fig. 2 Relative tensile strength $\Psi(\frac{f_{cd}}{f_{cs}})$ as a function of loading rate ($\dot{\sigma}$)
- variation of α,
- constant γ



———	= 0.99
-----	= 0.95
γ (stat)	= 6.5 (Jm^{-2})
a/b	= 0.4
f_{cs}	= 5.10^6 (Pa)
⏀	= Birkimer
✳	= Takeda
◈	= Hatano
▲	= Univ. of Technology, Delft

Fig. 3 Relative tensile strength Ψ ($\frac{f_{cd}}{f_{cs}}$) as a function of loading rate ($\dot{\sigma}$)
- variation of α
- γ as a function of $\dot{\sigma}$

In spite of the simplifications and the approximations made in modelling and calculation the results show that the most relevant phenomena determining the influence of the loading rate on the failure process and the ultimate strength have been taken into account.

V - CONCLUSIONS

It can be concluded that the approach of determining the maximum tensile strength by describing the extension of characteristic cracks gives good results for loading rates from $\dot{\sigma} = 10^4$ to 10^{12} (Pas^{-1}). The extension is determined by the energy demand for crack propagation (γ) and by the rate of energy supply $\left(\frac{\dot{\sigma}_d}{\dot{\sigma}_s}\right)$.

Further the parameters used in the model can be determined from the behaviour and properties of concrete under static loading.

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