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Résumé - On présente une dérivation de la dynamique fluide nucléaire pour les systèmes finis de Fermi, qui se base sur le principe variationnel dépendant du temps. La fonction variationnelle considérée est un déterminant Slater $|\phi\rangle = \exp(i\hat{F}/\hbar)|\phi\rangle$ où $|\phi_f\rangle$ est un déterminant Slater qui comprend des déformations locales d'équilibre et où F est un générateur hermitien dépendant du temps de la forme $F = \chi + \bar{s}.\bar{p}. + \frac{1}{2} p_\alpha p_\beta \Phi_{\alpha\beta}$. Ce schéma de dynamique des fluides fournit une description simultanée des états de basse énergie et des résonances géantes dans les noyaux. Sous l'hypothèse de forces δ dépendant de la densité on montre, à la limite classique, la densité de l'état normal est de $\rho_o(\bar{r}) = \rho_o(0)\ \theta(R-r)$ où $\rho_o(0)$ est la densité d'équilibre de la matière nucléaire.

Abstract - A derivation of nuclear fluid dynamics for finite Fermi systems based on the time dependent variational principle is presented. As variational function a Slater determinant $|\phi\rangle = \exp(i\hat{F}/\hbar)|\phi_f\rangle$ is considered where $|\phi_f\rangle$ is a Slater determinant which includes local equilibrium deformations and F is an hermitian time dependent generator of the form $F = \chi + \bar{s}.\bar{p} + \frac{1}{2} p_\alpha p_\beta \Phi_{\alpha\beta}$. This fluid dynamical scheme provides a simultaneous description of low lying states and giant resonances in atomic nuclei. Assuming density dependent δ forces it is shown that in the classical limit the ground state density is $\rho_o(\bar{r}) = \rho_o(0)\ \theta(R-r)$ where $\rho_o(0)$ is the nuclear matter equilibrium density.

1 - INTRODUCTION

Since the last decade when many Giant Resonances were found there has been renewed interest for macroscopic models /1-11/ describing nuclear collective motion.

On the microscopic level the random phase approximation provides a very detailed but necessarily numerical description of collective vibrations. It is therefore important to have simpler pictures which preserve the physical content of the microscopic theory. Macroscopic approaches to the nuclear motion give us the possibility of describing general features of collective states in a systematic way using a limited set of parameters which characterize global properties of nuclei.

Though fluid dynamical models admit a completely classical interpretation they can be derived starting from a microscopic point of view with the explicit use of a quantum energy functional. In particular we will derive a fluid dynamical scheme starting from the quantum mechanical lagrangian and taking Slater determinants as variational functions.

2 - THE STATIC CASE

In the first sound case /1-3/ it is assumed that the sphericity of the Fermi surface in momentum space is preserved when nuclear collective motion takes place (Thomas-Fermi approximation). In particular for a time even deformation we have the following distribution function

$$f_f = \theta\left(\lambda - \frac{p^2}{2m} - U_0(\bar{r}) - w(\bar{r},t)\right), \tag{1}$$

where $U_0(\bar{r})$ is the Hartree-Fock ground state potential and $w(\bar{r},t)$ accounts for local equilibrium distortions. The field $w(\bar{r},t)$ is a variational field and is also responsible for the interplay between first sound and zero sound.

The energy functional associated to a distribution function f is

$$W[f] = \int d\Gamma \frac{p^2}{2m} f + \frac{1}{2!} \iint d\Gamma_1 d\Gamma_2\, f(1)f(2)\, v_{12} + \frac{1}{3!} \iiint d\Gamma_1 d\Gamma_2 d\Gamma_3\, f(1)f(2)f(3)\, v_{123} + \ldots \quad (2)$$

where $d\Gamma = g d^3r\, d^3p/(2\pi\hbar)^3$, v_{12} stands for the two body interaction, v_{123} stands for the three body interaction and so on. Using δ forces ($v_{12} = t_0\delta(\bar{r}_1-\bar{r}_2)$, $v_{123} = t_3\,\delta(\bar{r}_1-\bar{r}_2)\,\delta(\bar{r}_2-\bar{r}_3),\ldots$) the potential part of the energy functional is

$$W_V[f] = \int d^3r \sum_\sigma a_\sigma \rho^\sigma \quad (3)$$

where ρ is the density corresponding to the distribution function f

$$\rho = g \int \frac{d^3p}{(2\pi\hbar)^3} f \quad . \quad (4)$$

With respect to the kinetic part we will have, for the particular case of a distribution function of the type of f_f, that the kinetic part of the energy functional is also a functional of the density

$$W_K[f_f] = \int d^3r \frac{3\hbar^2}{10m} \left(\frac{6\pi^2}{g}\right)^{2/3} \rho^{5/3} \quad . \quad (5)$$

Therefore for a distribution function of the type f_f, which describes a system instantaneously at rest, the energy of a time even deformation may always be written as a functional of the density

$$E[\rho] = \int d^3r \left[\frac{3\hbar^2}{10m}\left(\frac{6\pi^2}{g}\right)^{2/3} \rho^{5/3} + \sum_\sigma a_\sigma \rho^\sigma\right] \quad . \quad (6)$$

Obviously this functional should also be valid for the ground state. Therefore minimizing the energy and taking into account as a subsidiary condition that the particle number A remains constant we obtain the ground state density $\rho_0(\bar{r})$.

$$\begin{aligned}\delta(E-\lambda A) = &\int_D d^3r\,\delta\rho \left[\frac{\hbar^2}{2m}\left(\frac{6\pi^2}{g}\right)^{2/3} \rho^{2/3} + \sum_\sigma a_\sigma\, \sigma\rho^{\sigma-1} + \lambda\right] + \\ &+\oint_S dS(\delta\bar{R}.\hat{n})\left[\frac{3}{10m}\hbar^2\left(\frac{6\pi^2}{g}\right)^{2/3}\rho^{5/3} + \sum_\sigma a_\sigma \rho^\sigma + \lambda\rho\right] = 0 \quad .\end{aligned} \quad (7)$$

The domain D is the region where $\rho \neq 0$, S is the boundary of the domain D, $\delta\bar{R}$ denotes the displacement of the boundary and $\hat{n}$ is the outwards normal. The ground-state density $\rho_0(\bar{r})$ is a solution of the following set of equations

$$\frac{\hbar^2}{2m}\left(\frac{6\pi^2}{g}\right)^{2/3} \rho_0^{2/3} + \sum_\sigma a_\sigma\, \sigma\, \rho_0^{\sigma-1} + \lambda = 0 \qquad r \leqslant R \quad , \quad (8)$$

$$\frac{3}{10m}\hbar^2\left(\frac{6\pi^2}{g}\right)^{2/3} \rho_0^{5/3} + \sum_\sigma a_\sigma \rho_0^\sigma + \lambda\rho_0 \Big|_{r=R} = 0 \quad . \quad (9)$$

Equation (8) implies that ρ_0 is independent of $\bar{r}$ and combining (8) and (9) it follows that the value of ρ_0 is obtained by minimizing the total energy.

$$\frac{d}{d\rho}\left\{\frac{1}{\rho}\left(\frac{3}{10m}\hbar^2\left(\frac{6\pi^2}{g}\right)^{2/3} \rho^{5/3} + \sum_\sigma a_\sigma \rho^\sigma\right)\right\}_{\rho=\rho_0} = 0 \qquad r \leqslant R \quad . \quad (10)$$

From now on we will therefore consider the ground state density

$$\rho_0(\bar{r}) = \rho_0(0)\; \theta\,(R-r) \quad (11)$$

where, according to equation (10), $\rho_0(0)$ is the nuclear matter equilibrium density and the nuclear radius R is fixed by $\rho_0(0)$ and A.

3 - THE GENERALIZED SCALING MODEL

A typical quantum effect exhibited by Fermi systems is the deformation of the Fermi sphere in momentum space while the nuclear collective motion takes place. The generalized scaling approach implies such an effect and permits to describe a new class of phenomena the most famous of which is the Giant Quadrupole Resonance.

Let's consider a Slater determinant $|\psi\rangle$ which is related to the Hartree-Fock groundstate $|\phi_0\rangle$ by means of the hermitian time dependent generators Q and P

$$|\psi\rangle = \exp\left(\frac{i}{\hbar}(\hat{Q}-\hat{P})\right)|\phi_0\rangle \tag{12}$$

where Q is time even and P is time odd. To derive the equations of motion in the generalized scaling approach one looks for approximate solutions of the TDHF equations belonging to the family of Slater determinants where P is restricted to the first term in an expansion in powers of the momentum

$$P = -\bar{p}\cdot\bar{s}(\bar{r},t) \quad . \tag{13}$$

The scaling field $\bar{s}$ changes the density of the ground state $|\phi_0\rangle$ by the amount $\delta\rho = \bar{\nabla}.(\rho_0\bar{s})$. In fact the density is obtained integrating the distribution function in momentum space

$$\rho = g\int\frac{d^3p}{(2\pi\hbar)^3}\left(f_0+\{P,f_0\}\right) = \rho_0 + \bar{\nabla}.(\rho_0\bar{s}) \quad . \tag{14}$$

The generator Q reproduces time odd components in the wave function without changing its density,

$$Q = \chi(\bar{r},t) + \frac{1}{2}p_\alpha p_\beta \Phi_{\alpha\beta}(\bar{r},t) \quad . \tag{15}$$

In order to allow for vorticity we go beyond the local χ approximation considering also the term $\frac{1}{2}p_\alpha p_\beta\Phi_{\alpha\beta}$. The importance of non local components of this kind is suggested for instance by the analysis of low lying vibrations.

4 - THE INTERPLAY BETWEEN FIRST SOUND AND ZERO SOUND

Fluid dynamics as based on the generalized scaling approach seems unable to reproduce low-lying states which we would expect in a microscopic theory.

The truncation of expansions for Q and P imposes a restriction on the trial wave functions. This restriction may be compensated in part by allowing the canonical transformation generated by Q and P to act not on the groundstate $|\phi_0\rangle$ but on a deformed state $|\phi_f\rangle$ which includes local equilibrium distortions and is described by the distribution function f_f. It is clear that the extra freedom we gain dy deforming $|\phi_f\rangle$ instead of $|\phi_0\rangle$ is redundant if Q and P are the most general generators of a canonical transformation. Any further parameters would then be necessarily redundant. This is not the case if Q and P are not the most general generators.

The equations of motion as well as the boundary conditions are obtained by minimizing the quantum mechanical action integral. We start from the quantum mechanical lagrangian L

$$L = i\hbar\langle\psi|\dot{\psi}\rangle - \langle\psi|H|\psi\rangle \tag{16}$$

where $|\psi\rangle$ is the Slater determinant

$$|\psi\rangle = \exp\left(\frac{i}{\hbar}(\hat{Q}-\hat{P})\right)|\phi_f\rangle \tag{17}$$

and the generators P and Q are defined by equations (13) and (15). Afterwards we allow for arbitrary variations within the family of the Slater determinants $|\psi\rangle$.

The ground state density ρ_0 as well as the density ρ_f are restricted to the same domain D such that $r \leqslant R$. It is convenient to introduce the field $\rho_f^{(1)} = \rho_f - \rho_0$. Since ρ_0 and ρ_f have the same boundary S it is clear that $\rho_f^{(1)}$ has no singularities at the surface of the nucleus.

We have now an additional contribution to the transition density $\delta\rho$ because we are deforming a state $|\phi_f\rangle$ to which corresponds a density ρ_f .

$$\delta\rho = \rho_f^{(1)} + \bar{\nabla}.(\rho_0 \bar{s}) \tag{18}$$

The current $\bar{j}$ is obtained from its definition.

$$j_\alpha = g \int \frac{d^3p}{(2\pi\hbar)^3} \frac{p_\alpha}{m} \{f_0, Q\} = \frac{\rho_0}{m} \{\partial_\alpha \chi + \frac{p_F^2}{5} (\frac{1}{2}\partial_\alpha \Phi_{\gamma\gamma} + \partial_\gamma \Phi_{\alpha\gamma})\} + \frac{1}{5m} \Phi_{\alpha\gamma} \partial_\gamma (\rho_0 p_F^2) \tag{19}$$

Here we encounter the difficulty that the last term in the expression for the current leads to a pure surface current unless we impose the boundary condition

$$x_\gamma \Phi_{\alpha\gamma} \Big|_{r=R} = 0 \quad . \tag{20}$$

Another reason for imposing this boundary condition is the appearance in $<\psi|V|\psi>$ of a surface integral

$$\oint_S dS\, (\sum_\sigma a_\sigma \rho_0^{\sigma-1} x_\alpha \Phi_{\alpha\beta} j_\beta) \tag{21}$$

which will diverge unless we require at the surface the condition (20). We will therefore impose (20) as a boundary condition on the variational field $\Phi_{\alpha\beta}$.

Since we are only considering small amplitude oscillations we will take into account in the lagrangian terms up to second order in the variational fields $\chi, \bar{s}, \Phi_{\alpha\beta}$ and $\rho_f^{(1)}$. The energy functional $E^{(2)}[\rho_f^{(1)}, \bar{s}]$ contains all $\rho_f^{(1)}$ and $\bar{s}$ dependent terms of $<\psi|H|\psi>$.

$$E^{(2)}[\rho_f^{(1)}, \bar{s}] = \int d^3r\, \{\frac{p_F^2}{3m} \rho_f^{(1)} (\frac{1}{2} \frac{\rho_f^{(1)}}{\rho_0} + \bar{\nabla}.\bar{s}) + \frac{\rho_0 p_F^2}{10m} [(\nabla.\bar{s})^2 + \frac{1}{2}(\partial_\alpha s_\beta + \partial_\beta s_\alpha)^2] +$$
$$+ \sum_\sigma a_\sigma \frac{\sigma(\sigma-1)\rho_0^{\sigma-2}}{2} (\rho_f^{(1)} + \rho_0 \bar{\nabla}.\bar{s})^2\} \tag{22}$$

Replacing the variational function (17) in the quantum mechanical lagrangian (16) and taking into account the boundary condition (20) we obtain the fluid dynamical lagrangian $L^{(2)}$

$$L^{(2)} = \int d^3r\, \{-\rho_f^{(1)} (\dot{\chi} + \frac{p_F^2}{6} \dot{\Phi}_{\alpha\alpha}) + \rho_0 s_\alpha [\partial_\alpha \dot{\chi} + \frac{p_F^2}{5} (\frac{1}{2} \partial_\alpha \dot{\Phi}_{\gamma\gamma} + \partial_\gamma \dot{\Phi}_{\alpha\gamma})] -$$
$$- \frac{\rho_0}{2m} [(\bar{\nabla}\chi).(\bar{\nabla}\chi) + \frac{2}{5} p_F^2 (\partial_\alpha \chi)(\frac{1}{2} \partial_\alpha \Phi_{\gamma\gamma} + \partial_\gamma \Phi_{\alpha\gamma}) +$$
$$+ \frac{p_F^4}{35} ((\frac{1}{2}\partial_\alpha \Phi_{\gamma\gamma} + \partial_\gamma \Phi_{\alpha\gamma})^2 + \frac{1}{6} (\partial_\alpha \Phi_{\beta\gamma} + \partial_\beta \Phi_{\gamma\alpha} \quad \partial_\gamma \Phi_{\alpha\beta})^2)]\} - E^{(2)}[\rho_f^{(1)}, \bar{s}] \quad . \tag{23}$$

The equations of motion and boundary conditions are now obtained by minimizing the action integral with respect to arbitrary variations of $\chi, \bar{s}, \Phi_{\alpha\beta}$ and $\rho_f^{(1)}$

$$\delta \int dt \{L^{(2)} + \oint_S dS\, \xi_\alpha x_\beta \Phi_{\alpha\beta}\} = 0 \tag{24}$$

being $\bar{\xi}(\Omega)$ a Lagrange multiplier.

Arbitrary variation with respect to χ yields the continuity equation.

$$\dot{\rho}_f^{(1)} + \partial_\alpha \{\rho_0 \dot{s}_\alpha + \frac{\rho_0}{m} [\partial_\alpha \chi + \frac{p_F^2}{5} (\frac{1}{2} \partial_\alpha \Phi_{\beta\beta} + \partial_\beta \Phi_{\alpha\beta})]\} = 0 \tag{25}$$

Since ρ_0 is a step function , equation (25) implies at the surface the boundary condition

$$x_\alpha \{\rho_0 \dot{s}_\alpha + \frac{\rho_0}{m} [\partial_\alpha \chi + \frac{p_F^2}{5} (\frac{1}{2} \partial_\alpha \Phi_{\beta\beta} + \partial_\beta \Phi_{\alpha\beta}]\}_{r=R} = 0 \quad . \tag{26}$$

In an analogous way an arbitrary variation with respect to $\bar{s}$ leads to the Euler type equation

$$\partial_t j_\alpha = -c_f^2 \partial_\alpha \rho_f^{(1)} + (\frac{7}{15} c_t^2 - c_z^2) \rho_0 \partial_\alpha \bar{\nabla}.\bar{s} - \frac{7}{15} c_t^2 \rho_0 \Delta s_\alpha \tag{27}$$

and to the boundary condition

$$x_\alpha P_{\alpha\beta} \Big|_{r=R} = 0 \tag{28}$$

where

$$P_{\alpha\beta} = -\{\delta_{\alpha\beta} c_f^2 \rho_f^{(1)} + \delta_{\alpha\beta} (c_z^2 - \frac{14}{15} c_t^2) \rho_0 \bar{\nabla}.\bar{s} + \frac{7}{15} \rho_0 c_t^2 (\partial_\alpha s_\beta + \partial_\beta s_\alpha)\} \tag{29}$$

The constants c_f, c_z and c_t are respectively the first sound velocity and the zero sound longitudinal and transverse sound velocities.

$$c_f^2 = \frac{p_F^2}{m^2} (\frac{1+F_0}{3}) \quad , \tag{30}$$

$$c_z^2 = \frac{p_F^2}{m^2} (\frac{3}{5} + \frac{F_0}{3}) \quad , \tag{31}$$

$$c_t^2 = \frac{3}{7} \frac{p_F^2}{m^2} \quad , \tag{32}$$

$$F_0 = \frac{3m}{p_F^2} \sum_\sigma a_\sigma \sigma(\sigma-1) \rho_0^{\sigma-1} \quad . \tag{33}$$

It is interesting to note that the tensor $P_{\alpha\beta}$ may be clearly interpreted, in analogy with continuum classical mechanics, as the stress tensor since the current is equal to the derivative of this tensor and also because the boundary condition (28) means that at the surface the stress is zero as we would expect on account of the nuclear surface being free. Another interesting feature is that the stress tensor can be decomposed in the sum of the stress tensor corresponding to the first sound, which is diagonal, with the stress tensor corresponding to the generalized scaling model. Finally, arbitrary variations with respect to $\rho_f^{(1)}$ and $\Phi_{\alpha\beta}$ lead to the following equations

$$-\dot{\chi} - \frac{p_F^2}{6} \dot{\Phi}_{\alpha\alpha} = \frac{m}{\rho_0} c_f^2 (\rho_f^{(1)} + \rho_0 \bar{\nabla}.\bar{s}) \tag{34}$$

$$\delta_{\alpha\beta} \{\frac{5}{3} \dot{\rho}_f^{(1)} + \rho_0 \bar{\nabla}.\dot{\bar{s}} + \frac{\rho_0}{m} [\Delta\chi + \frac{p_F}{7} (\frac{1}{2} \Delta\Phi_{\alpha\alpha} + \partial_\alpha \partial_\beta \Phi_{\alpha\beta})]\} + \rho_0 (\partial_\alpha \dot{s}_\beta + \partial_\beta \dot{s}_\alpha) +$$

$$+ \frac{2\rho_0}{m} \partial_\alpha \partial_\beta \chi + \frac{\rho_0 p_F^2}{7m} (\Delta\Phi_{\alpha\beta} + \partial_\alpha \partial_\beta \Phi_{\gamma\gamma} + 2\partial_\alpha \partial_\gamma \Phi_{\gamma\beta} + 2\partial_\beta \partial_\gamma \Phi_{\gamma\alpha}) = 0 \tag{35}$$

$$\delta_{\alpha\beta} x_\gamma \{\rho_0 \dot{s}_\gamma + \frac{\rho_0}{m} [\partial_\gamma \chi + \frac{p_F^2}{7} (\frac{1}{2} \partial_\gamma \Phi_{\delta\delta} + \partial_\delta \Phi_{\gamma\delta})]\} +$$

$$+ x_\alpha \{\rho_0 \dot{s}_\beta + \frac{\rho_0}{m} [\partial_\beta \chi + \frac{p_F^2}{7} (\frac{1}{2} \partial_\beta \Phi_{\gamma\gamma} + \partial_\gamma \Phi_{\beta\gamma})] + \xi_\beta\} + \{\alpha \rightleftarrows \beta\} +$$

$$+ \frac{\rho_0 p_F^2}{7m} x_\gamma (\partial_\gamma \Phi_{\alpha\beta} + \partial_\alpha \Phi_{\beta\gamma} + \partial_\beta \Phi_{\gamma\alpha}) \Big|_{r=R} = 0 \tag{36}$$

The set of equations of motion (25), (27), (34) and (35) and the boundary condition (20), (26), (28) and (36), obtained by minimizing the action integral, determine completely the normalmodes of the system.

In this scheme the transverse sound velocity is still the same as in the generalized scaling model since we are just introducing an additional scalar field having no influence on the magnetic modes which are purely transverse. However we have a change with respect to the longitudinal sound velocities which are now c_1 and c_2

$$c^2_{\substack{1\\2}} = \frac{1}{2}\left\{\frac{9}{35}\frac{p_F^2}{m^2} + c_z^2 \pm \sqrt{\left(\frac{9}{35}\frac{p_F^2}{m^2} + c_z^2\right)^2 - \frac{36}{35}\frac{p_F^2}{m^2}c_f^2}\right\} \quad . \tag{37}$$

In fact the introduction of the local equilibrium distortions implies a considerable improvement in the description of electric modes as we will see in the next section.

5 - NUMERICAL RESULTS

We present results in terms of the unambiguous radial functions $j_\pm(r)$, $j_{div}(r)$, $j_{curl}(r)$ defined by

$$\begin{aligned} \bar{j}(\bar{r}) &= j_+(r)\,\bar{Y}_{\ell,\ell+1,0}(\Omega) + j_-(r)\,\bar{Y}_{\ell,\ell-1,0}(\Omega) \quad , \\ \bar{\nabla}\cdot\bar{j} &= j_{div}(r)\,Y_{\ell 0} \quad , \\ \bar{\nabla}\times\bar{j} &= i\,j_{curl}(r)\,\bar{Y}_{\ell\ell 0} \quad . \end{aligned} \tag{38}$$

The radial functions j_{div} and j_{curl} are a way of investigating the relative magnitude of the longitudinal and transverse components of $\bar{j}$.

The numerical results presented in this section are obtained using zero range effective interactions previously used in other macroscopic descriptions /2,7,10/ in order to allow for comparison with other results. We will be using a potential energy functional of the form

$$\sum_\sigma a_\sigma \rho^\sigma = a_2\rho^2 + a_{2+\alpha}\,\rho^{2+\alpha} \quad . \tag{39}$$

With the power α chosen to be 1/6 or 1. For a given α the two constants a_2 and $a_{2+\alpha}$ are adjusted to give saturation properties of nuclear matter as

$$k_F = 1.26\ \mathrm{fm}^{-1}\ (\rho_0 = 0.135\ \mathrm{fm}^{-3})\ ,\quad E/A = -13.8\ \mathrm{MeV} \quad . \tag{40}$$

Then we have

$$\begin{aligned} a_2 &= -\frac{3}{8}\times 3075.8\ \mathrm{MeV\ fm}^3 \ , & a_{2+1/6} &= \frac{1}{16}\times 20216.4\ \mathrm{MeV\,fm}^{3+1/2}\ , \quad (\alpha = 1/6) \\ a_2 &= -\frac{3}{8}\times 1064.4\ \mathrm{MeV\ fm}^3 \ , & a_3 &= \frac{1}{16}\times 17862.9\ \mathrm{MeV\ fm}^6\ , \quad (\alpha = 1)\ . \end{aligned} \tag{41}$$

We will only refer to the results for the electric modes since this scheme introduces no changes with respect to the magnetic modes which were already considered in the references /2,7/. We will be considering A = 208.

$\underline{\ell^\pi = 0^+}$: Monopole modes are purely longitudinal. When the generator Q is defined by (15) the generalized scaling model gives identical results to the more simplified approach /10/ where a local approximation is made for Q. Allowing for the interplay with first sound we obtain also a very collective state which exhausts about 96% ($\alpha = 1/6$) of the energy wheighted sum rule (EWSR) and having an energy of 13.68 MeV which is slightly lower than the one provided by the GSM (13.70 MeV).

$\underline{\ell^\pi = 1^-}$: The lowest 1^- mode is the uniform translation which occurs at $\omega = 0$ in both formulations. The strength is mainly fragmented in two states at energies 13.6 and 25.56 MeV ($\alpha = 1/6$) this last state having about 48% of the EWSR.

Table 1

List of normalmodes ($\ell \leqslant 4$) in ^{208}Pb with the force $\alpha = 1/6$

ℓ_i^π	$\hbar\omega$	EWSR	$\hbar\omega$	EWSR
0_1^+	13.68	0.957	13.70	0.975
0_2^+	16.58	0.021	33.02	0.020
0_3^+	24.63	0.003	50.66	0.003
1_1^-	8.35	0.012	11.06	0.072
1_2^-	13.60	0.383	22.36	0.897
1_3^-	20.26	0.044	35.02	0.021
1_4^-	25.56	0.475	41.15	0.006
2_1^+	8.51	0.819	8.60	0.885
2_2^+	11.87	0.137	15.82	0.109
2_3^+	17.86	0.038	29.59	0.003
3_1^-	2.98	0.352	13.10	0.792
3_2^-	16.65	0.288	22.73	0.191
3_3^-	17.41	0.177	36.62	0.007
3_4^-	23.18	0.158	47.55	0.004
4_1^+	4.51	0.389	17.28	0.747
4_2^+	20.49	0.139	29.41	0.224
4_3^+	23.85	0.179	43.34	0.011
4_4^+	28.90	0.152	53.75	0.008

Table 2

List of normalmodes ($\ell \leqslant 4$) in ^{208}Pb with the force $\alpha = 1$

ℓ_i^π	$\hbar\omega$	EWSR	$\hbar\omega$	EWSR
0_1^+	18.12	0.912	18.19	0.960
0_2^+	20.04	0.051	40.41	0.031
0_3^+	29.74	0.003	61.53	0.006
1_1^-	9.24	0.004	11.89	0.022
1_2^-	16.21	0.190	27.65	0.934
1_3^-	24.61	0.033	35.27	0.036
1_4^-	29.27	0.688	49.58	0.006
2_1^+	8.59	0.864	8.65	0.902
2_2^+	13.18	0.108	17.01	0.095
2_3^+	21.04	0.024	35.89	0.0003
3_1^-	3.03	0.353	13.21	0.821
3_2^-	17.40	0.516	24.51	0.170
3_3^-	19.43	0.020	43.45	0.0001
3_4^-	26.36	0.095	48.72	0.006
4_1^+	4.60	0.390	17.45	0.780
4_2^+	23.11	0.377	31.81	0.202
4_3^+	25.52	0.029	50.40	0.0002
4_4^+	32.21	0.160	55.79	0.010

For the states listed in the first column energies (in MeV) and contributions to the total EWSR are given. The second and third columns show the results obtained from the present nuclear fluid dynamical approach (being the variational function given by the expression (17) where the generators P and Q are defined by (13) and (15)) and the fourth and fifth columns show the results given by the GSM (being the variational function defined by (12) where the generators P and Q are again given by (13) and (15)). We have considered the following excitation operators, r^2Y_{00}, $j_1(qr)\,Y_{10}$ with $qr = 4.49$ and $r^\ell Y_{\ell m}$ for $\ell > 1$.

$\underline{\ell^\pi = 2^+}$: The state with the lowest energy appears at an energy of 8.5 MeV and exhausts about 82% of the EWSR. This state is most similar to the one provided by previous rotational fluid dynamical models /2,3,6,7/.

$\underline{\ell^\pi = 3^-, 4^+}$: For $\ell \geqslant 3$ this scheme succeeds in producing low lying states at the correct energies and with an appropriate strength (for comparison see for instance reference /11/).

A common feature of all the states produced in this "square well" formulation is the dependence of the energy on $A^{-1/3}$ which is due to the fact that the boundary conditions determine a variable $z = kR$ for each mode. For each normalmode we will have the energy and the different wave numbers related by the equation $\omega = k_t c_t = k_1 c_1 = k_2 c_2$.

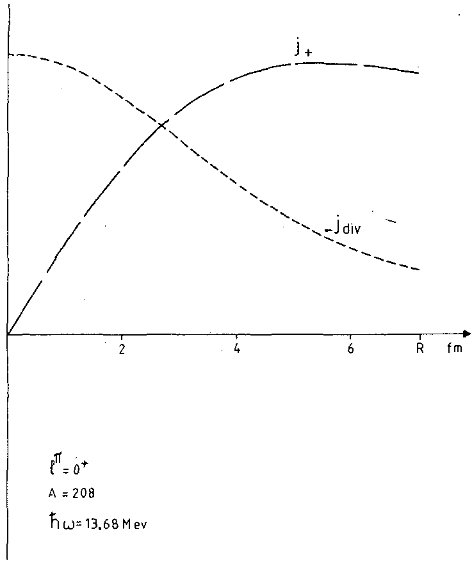

Fig. 1 - The radial functions of $\bar{j}$ and $\bar{\nabla}.\bar{j}$ according to eqs. (38) for the first excited 0^+ state ($\alpha = 1/6$).

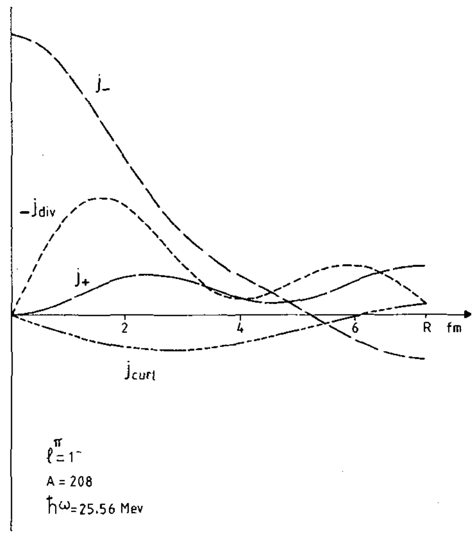

Fig. 2 - The radial functions of $\bar{j}$, $\bar{\nabla}.\bar{j}$ and $\bar{\nabla}\times\bar{j}$ according to eqs. (38)for the fourth excited 1^- state ($\alpha = 1/6$)

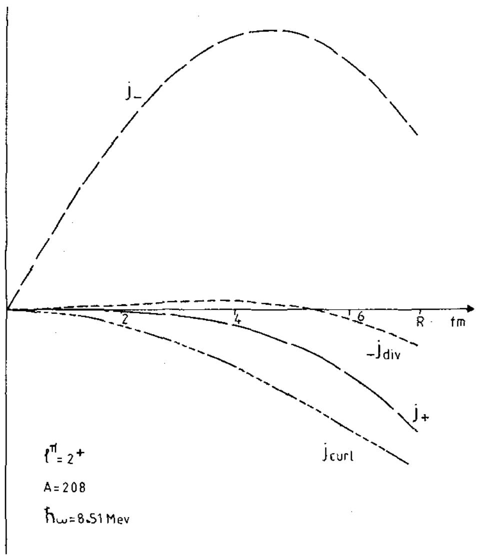

Fig. 3 - Same as fig. 2 for the first excited 2^+ state.

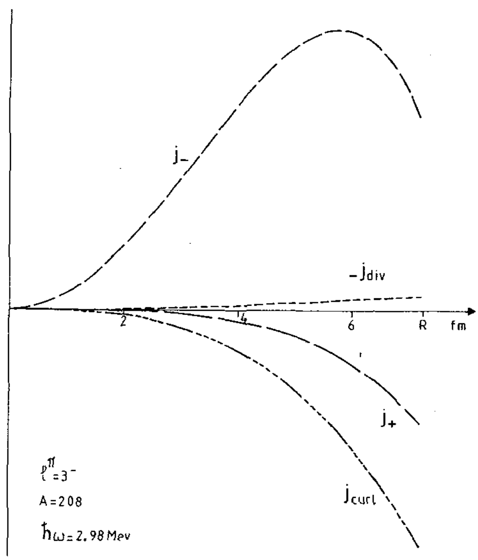

Fig. 4 - Same as fig. 2 for the first excited 3^- state.

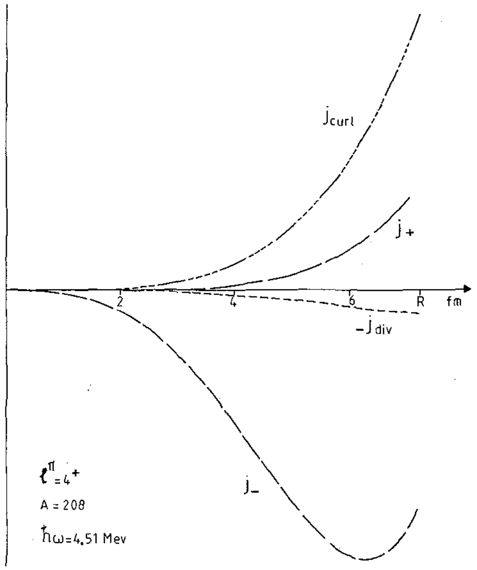

Fig. 5 - Same as fig. 2 for the first excited 4^+ state.

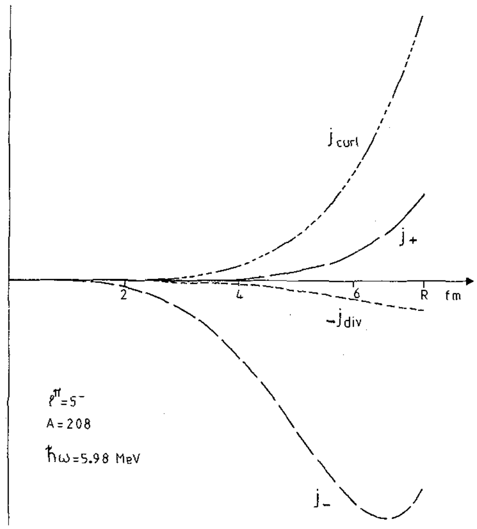

Fig. 6 - Same as fig. 2 for the first excited 5^- state.

Another important aspect is that although the Landau parameter F_0 and the incompressibility K have rather different values for $\alpha = 1/6$ ($F_0 = -0.12$, K = 175 MeV) and for $\alpha = 1$ ($F_0 = 0.66$, K = 327 MeV) the energy and the strength remain approximately constant for the mode with lowest energy for each multipolarity equal or greater than 2.

In figs. 4-6 the low lying states are represented for $\ell = 3,4$ and 5. From the behaviour of the curve j_{curl} it is clear that for these states the transverse components are very significative.

6 - CONCLUSION

In the present contribution we have derived a variational principle taking Slater determinants as variational functions which allows for the interplay between first sound and the generalized scaling model. Due to the special ansatz (15) for the generator Q this model also allows for vorticity which is shown to be specially important for the description of the so called low lying modes ($\ell \geqslant 3$).

In analogy with previous variational approaches /2, 7 ,10/ this formulation is a genuine variational principle in the sense that first we consider a truncated variational space and only afterwards we allow for arbitrary variations within this space.

When comparison is made with previous results /10,11/ of microscopic calculations we find good agreement for the energy levels and the strength distributions. Another success of the present formulation is that all the sum rules correctly reproduced by the GSM are also reproduced in this scheme. Additionally also the inverse energy wheighted sum rule is preserved.

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