

THERMAL STRESSES IN GLASS PANELS WITH MULTIPLE HOT ZONES

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Résumé. L'effet de deux zones chaudes de diamètres différents sur les contraintes thermiques existant dans les régions restées froides de plaques de verre rectangulaires est investigué par la méthode des éléments finis. Comparés à ceux obtenus par une expression analytique approchée, les résultats sont trouvés consistants à 15% près. La simplicité de la solution analytique en fait un outil intéressant lors de la conception de panneaux de verre destinés à être chauffés de façon non uniforme. L'expression est aussi utile pour évaluer la probabilité de fracture thermique en comparant les niveaux de contraintes des régions situées entre deux zones chaudes à ceux des bords de la plaque.

Abstract. The interaction of two unequal circular hot zones on thermal stresses in the remaining cold region of rectangular glass panels is examined via finite element analysis. The results are then compared with those given by an approximate analytical solution. The FEM and analytical solutions agree to within 15%. The simplicity of the analytical solution, however, makes it a useful screening tool during the design phase of nonuniformly heated glass panels. The solution is also useful for assessing the probability of thermal fracture by comparing stresses in the region between the hot zones with those at the edge of the panel.

Introduction

The present paper is concerned with the distribution of thermal stresses in a rectangular glass panel with two unequal circular hot zones. In a previous paper, Gulati et al¹ considered the stress distribution due to a single circular hot zone. They demonstrated that the maximum thermal stress at the cold edge was more than twice that given by the simple axisymmetric solution², the added contribution being due to localized bending produced by the expanding hot zone. They also developed an approximate analytical solution which, together with the FEM solution, agreed well with experimental results. More recently, Matsumoto and Sekiya³ have treated the two-dimensional thermoelastic problem involving an oval region with uniform internal heat generation at the center of a square region. Their solution also shows good agreement with strain gage data and with the solution proposed by Gulati et al¹. In the present work, the analytical solution¹ is extended to multiple hot zones, and used to examine the probability of thermal fracture from flaws in the region between the hot zones vs those at the edge of the panel.

Finite Element Solution.

The finite element mesh of the half panel containing two circular zones of radii a_1 and a_2 is shown in Fig. 1. The spacing between the

boundaries of these zones as well as between each boundary and the edge of the panel is denoted by d . We will examine the variation of the tangential stress, σ_y , with d at various points along the axis of symmetry, and in the interregion bounded by points A_{1R} and A_{2L} .

The physical properties, and the zonal temperatures and dimensions, for the specific cases examined are as follows:

- E = Young's Modulus = 83 GPa
- α = Coeff. Linear Expansion = $1 \times 10^{-6} \text{ } ^\circ\text{F}^{-1}$
- $T_{\text{hot zone}}$ = 1070°F
- $T_{\text{cold zone}}$ = 70°F
- $a_1 = 5\text{cm}$, $a_2 = 7.6\text{cm}$, $2.5\text{cm} < d < 7.6\text{cm}$

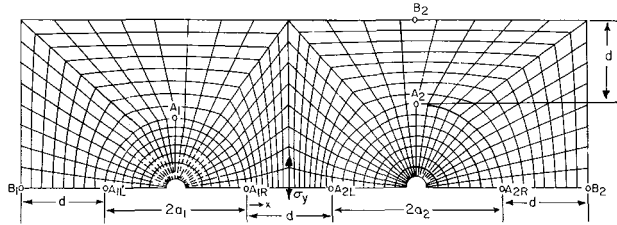


Fig. 1 Finite Element Mesh for 1/2 Glass Panel with Two Unequal Circular Hot Zones

The maximum tangential stress at the edge of the panel occurs at point B_2 located closest to the larger zone. An approximate solution for this stress, which neglects the influence of the adjacent hot zone of radius a_1 , is given by¹

$$\sigma_y(B_2) = 2.25E\alpha \Delta T \left(\frac{a_2}{a_2+d} \right)^2 \tag{1}$$

A comparison of $\sigma_y(B_2)$, as given by FEM and eqn. 1, is shown in Table 1 for different values of interregional spacing d . The agreement between the two solutions is excellent. Thus, the influence of the adjacent hot zone is negligible and eqn. 1 may be used for estimating the maximum stress at the panel edge.

The tangential stress at the boundary points A_{1L} and A_{2R} is also given by a solution similar to that for the case of the single hot zone¹, viz.

$$\sigma_y = K_a \left(\frac{E\alpha\Delta T}{2} \right) \left[1 + \left(\frac{a_1}{a_1+d} \right)^2 \right] \text{ at } A_{1L}$$

$$= K_a \left(\frac{E\alpha\Delta T}{2} \right) \left[1 + \left(\frac{a_2}{a_2+d} \right)^2 \right] \text{ at } A_{2R} \tag{2}$$

where the stress factor K_a depends on the ratio $(a+d/a)$; see Table 2. The values of σ_y given by equation 2 are compared with those obtained from finite element analysis in Table 3. Again, the agreement is excellent.

Table 1. Max. Stress at Panel Edge (point B_2) for Different Values of d

d, cm.	$\sigma_y(B_2)$, MPa.	
	FEM	Eqn. 1
2.5	106.2	104.8
3.8	83.5	82.8
5.0	66.9	67.0
6.3	55.2	55.4
7.6	46.2	46.5

Table 2. Stress Factors K_a and K_{zi}

(a+d/a)	K_a	K_{zi}
1.33	1.66	1.18
1.50	1.67	1.19
1.67	1.69	1.18
1.83	1.73	1.18
2.0	1.77	1.21
2.25	1.91	1.19
2.50	1.96	1.16

Table 3. Tangential Stress at Boundary Points A_{1L} and A_{2R} for Different Values of d

d, cm	$\sigma_y(A_{1L})$, MPa.		$\sigma_y(A_{2R})$, MPa.	
	FEM	Eqn. 2	FEM	Eqn. 2
2.5	109.0	99.8	106.9	107.3
3.8	98.6	93.9	99.3	99.8
5.0	93.8	91.6	95.2	95.1
6.3	91.7	94.6	93.1	92.9
7.6	90.3	94.1	91.7	91.5

The FEM results for tangential stress in the interzonal region are shown in Fig. 2. The maximum stress occurs at points A_{1R} and A_{2L}

which experience an appreciable influence from the adjacent hot zone. These stresses may also be estimated from

$$\left. \begin{aligned} \sigma_y &= k_{zi} k_a \left(\frac{E\alpha\Delta T}{2} \right) \left[1 + \left(\frac{a_1}{a_1+d} \right)^2 \right] \\ &\text{at } A_{1R} \\ &= k_{zi} k_a \left(\frac{E\alpha\Delta T}{2} \right) \left[1 + \left(\frac{a_2}{a_2+d} \right)^2 \right] \\ &\text{at } A_{2L} \end{aligned} \right\} (3)$$

where K_{zi} is the zonal interaction factor shown in Table 2. The various stress factors in eqns. 1-3, which resemble the axisymmetric solution, reflect the added bending effect and/or the influence of the adjacent hot zone.

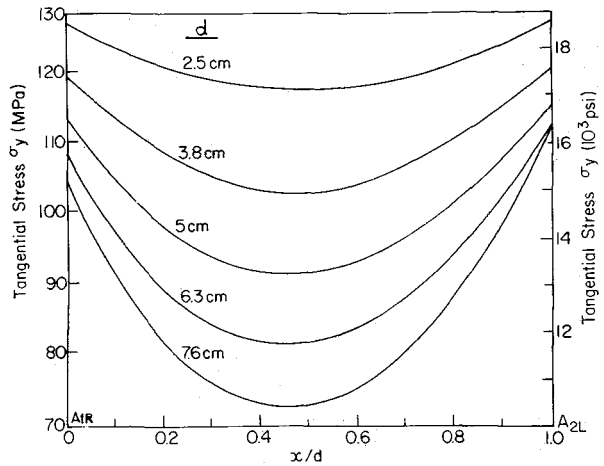


Fig. 2 Distribution of Tangential Stress in Interzonal Region (FEM solution)

Approximate Analytical Solution

Referring to the dual hot zones in Fig. 3, the tangential stress in the interzonal region and the flexural deflection at the common boundary may be approximated by¹

$$\sigma_y = 1/2 E\alpha\Delta T [(a/r)^2 + (a/b)^2] \tag{4}$$

$$\delta = 25/16 [(a^4\alpha\Delta T)/b^2 (a+b)] \tag{5}$$

where r is the radial coordinate and $b = a + d$. Applying eqns. 4 and 5 to each of the hot zones, invoking superposition, and equating flexural deflections at the common boundary, which must remain straight, the combined effect of dual hot zones is given by

$$\sigma_y = 1/2 E\alpha\Delta T [(a_1/b_1)^2 + (a_2/b_2)^2 + (a_1/r_1)^2 + (a_2/r_2)^2] \tag{6}$$

where $r_1 = a_1 + x$, $r_2 = a_2 + d - x$, $b_1 + b_2 = a_1 + a_2 + d$, and $(b_1/b_2)^3 = (a_1/a_2)^4$.

The FEM and analytical solutions (eqn.6) are compared in Fig. 4 for the specific case of $d = 7.6$ cm. They agree to within 15%. A third solution, obtained by adding the FEM results for one hot zone at a time, is less accurate due to the neglect of the adjacent hot zone.

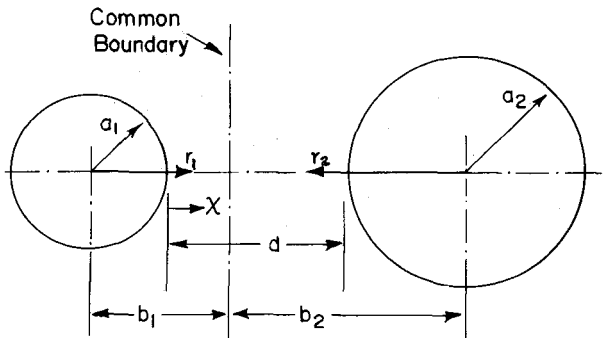


Fig. 3 Coordinate System and Geometry of Dual Hot Zones

Assessment of Thermal-Fracture

Thermal fracture will occur if the maximum thermal stress exceeds the strength of the panel. The latter is a function of flaw severity, fatigue characteristics, and the temporal stability of the panel material. Since the maximum thermal stress at the edge and at the boundaries of the hot zones is of similar magnitude, the failure probability will depend on the severity of damage inflicted in service. The failure may be minimized, however, by selecting a low expansion glass, controlling the peak use temperature, and optimizing the panel dimensions relative to those of hot zones.

Conclusions

The interaction of the adjacent hot zone has been examined; the results indicate that

1) the maximum edge stress occurs at the point closest to the larger hot zone and is unaffected by the presence of the adjacent hot zone;

2) The approximate analytical solution for tangential stresses at the edge and at the boundaries of the hot zones provides satisfactory values for design purposes;

3) The FEM and analytical solutions for tangential stress in the interzonal region agree to within 15%.

The analytical solution is also useful for assessing the failure probability by comparing stresses and flaw severities in the interzonal and edge regions.

References

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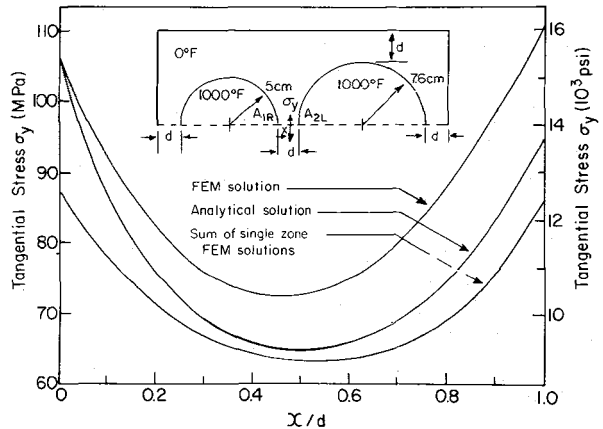


Fig. 4 Comparison of Various Solutions for Tangential Stress in Interzonal Region ($d=7.6\text{cm}$)