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STRENGTH OF NONLINEAR INTERACTION BETWEEN COLLINEAR SURFACE ACOUSTIC WAVES

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Résumé. - Une méthode optique a été développée pour la détermination du coefficient de couplage non-linéaire dans la propagation colinéaire de deux ondes acoustiques de surface. Cette méthode simple fournit sans exiger la connaissance de la puissance acoustique, la valeur de l'amplitude et de la phase de ce paramètre non-linéaire.

On présente ici les résultats expérimentaux concernant la détermination du coefficient de couplage non-linéaire sur des échantillons de YZ -LiNbO₃ relativement aux cas de génération paramétrique d'ondes acoustiques aux fréquences addition (f_1+f_2 - et différence (f_1-f_2) des fréquences fondamentales f_1 et f_2 .

Dans le premier cas on a trouvé que l'amplitude du coefficient non-linéaire ne dépendait pas du rapport des fréquences f_2/f_1 et était deux fois supérieure à la valeur du coefficient non-linéaire mesuré dans le cas de génération du second harmonique.

Au contraire, dans le cas de génération d'ondes acoustiques de fréquence différence (f_1-f_2) on a trouvé que l'amplitude du coefficient non-linéaire dépendait fortement du rapport f_2/f_1 et avait une valeur toujours inférieure à celle mesurée dans le cas de génération paramétrique à la fréquence addition $f_1 + f_2$.

Abstract. - An optical method has been developed for the determination of the coupling coefficient in the case of nonlinear mixing of two collinear surface acoustic waves. The method is simple enough and permits the evaluation of the amplitude and phase of the coupling coefficient without requiring any knowledge of the acoustic power flows. The value of the coupling coefficient has been measured for YZ LiNbO₃ in the cases of sum- and difference-frequency generation. It has been found that in the former case the magnitude of the coefficient is greater than that of second harmonic generation by a factor of two and is not very sensitive to the variations of the frequency ratio f_2/f_1 of the two interacting waves. However, in the case of difference-frequency generation the magnitude of the coefficient is smaller than the one measured for the sum-frequency generation and also strongly dependent on the ratio f_2/f_1 .

1. INTRODUCTION. - The present paper deals with the mixing of two collinear surface acoustic waves propagating on a nonlinear medium. In the past, Lean and co-workers /1,2/ reported mixing of collinear surface waves on α -quartz and YZ LiNbO₃, whereas others investigated the mixing of non collinear waves /3,4/. The efficiency of this type of nonlinear interaction can be measured in terms of a phenomenological coupling constant, the knowledge of which is important for the design of practical devices /5/.

The novelty of the present work is the introduction of a simple and accurate method for measuring the amplitude and phase of the complex coupling coefficient between two collinear surface waves. First indicative measurement of the value of the coefficient for YZ LiNbO₃ crystals have been also performed. The theory presented here is a combination of the coupled system of equations introduced by Shiren for the explanation of the parametric amplification of bulk waves /6/ and the optical theory of Alippi et al. /7/ which has been successful in the determination of the coupling

coefficient in the case of surface wave second harmonic generation. An exact analytical treatment of the surface wave mixing problem is difficult and cumbersome because of the inhomogeneous nature of the waves involved in the interaction /8/ and compares unfavourably with the present phenomenological approach which is capable of determining the strength of the nonlinear interaction with an estimated error of the order of 5 per cent.

2. ANALYSIS. - Consider two surface waves propagating along the direction and decaying in the -z-direction of an elastically nonlinear medium. Because of the elastic nonlinearity, the two waves will generate, apart from their own second harmonics, surface waves at their sum- (up conversion) and difference-frequencies (down conversion). Following Shiren /6/ or Lean and Tseng /2/, one can write

$$\frac{dA_3(x)}{dx} = \Gamma_{312} A_1(x) A_2(x) \exp \left[i(K_3 - (K_1 + K_2)) \right] - \alpha_3 A_3(x) \quad (1a)$$

$$\frac{dA_4(x)}{dx} = -\Gamma_{412}A_1(x)A_2^*(x)\exp[i(K_4-(K_1-K_2))x] - \alpha_4A_4(x) \quad (1b)$$

where $A_1(x)$ and $A_2(x)$ are the surface values of the transverse elastic displacement of the two interacting waves whose frequencies are f_1 and f_2 ($f_1 > f_2$), respectively; $A_3(x)$ and $A_4(x)$ are the surface values of the transverse elastic displacement of the parametrically generated waves at frequencies $f_3 = f_1 + f_2$ and $f_4 = f_1 - f_2$, respectively.

Γ_{j12} ($j = 3, 4$) is the complex coupling coefficient and α_j and K_j stand for the attenuation coefficient and wavenumber, respectively, of the $A_j(x)$ wave. It can be shown that the general solution of eqns. (1a) and (1b), under the assumption that $A_1(x)$ and $A_2(x)$ are affected little by the presence of the parametric waves, is given by:

$$A_3(x) = \frac{\Gamma_{312}A_1(0)A_2^*(0)}{\alpha_3 - (\alpha_1 + \alpha_2) + i(K_3 - (K_1 + K_2))} \left[\exp - [(\alpha_1 + \alpha_2) + i(K_3 - (K_1 + K_2))]x] \cdot \exp(-\alpha_3 x) \right] \quad (2a)$$

and

$$A_4(x) = \frac{\Gamma_{412}A_1(0)A_2(0)}{\alpha_4 - (\alpha_1 + \alpha_2) + i(K_4 - (K_1 - K_2))} \left[\exp - [(\alpha_1 + \alpha_2) + i(K_4 - (K_1 - K_2))]x] \cdot \exp(-\alpha_4 x) \right] \quad (2b)$$

where it has been assumed that the waves are varying according to $\exp i(\omega_j t - K_j x)$. For optically polished single crystal materials in the MHz region of frequencies the wave dispersion and attenuation are small, and the above equation can be simplified to

$$A_3(x) = \Gamma_{312}A_1(0)A_2^*(0)x \quad (3a)$$

and

$$A_4(x) = -\Gamma_{412}A_1(0)A_2^*(0)x \quad (3b)$$

even for relatively large propagation distances x . Equations (3a) and (3b) show that the amplitudes of the parametrically generated waves are proportional to the product of the amplitudes of the interacting waves and grow linearly with the travelled distance.

One of the most powerful techniques for monitoring the amplitude of surface waves along their propagation path is by probing the crystal surface

with a narrow monochromatic light beam [7,9]. In the subsequent analysis the diffraction spectrum of a light beam reflected by the surface of a crystal on which two surface waves and the sum-frequency wave are propagating will be examined.

The basic reflection geometry is shown in Fig.1. A light beam impinges at a small incidence angle θ on the crystal surface which is corrugated by the surface waves. The reflected light wavefront, E , is phase modulated, so that it replicates the surface corrugation. Thus, one can write [7]

$$E = E_0 \exp \left[i2k \cos \theta \left\{ |A_1(x)| \cos(\omega_1 t - K_1 x + \phi_1) + |A_2(x)| \cos(\omega_2 t - K_2 x + \phi_2) + |A_3(x)| \cos(\omega_3 t - K_3 x + \phi_3) \right\} \right] \quad (4)$$

where E_0 and k are the amplitude and wave-number of the reflected light wavefront, and the factor 2 takes account of the doubling of the phase corrugation resulting from the reflection geometry.

$A_j(x)$, ω_j , K_j and ϕ_j are the amplitude, angular frequency, propagation constant and phase of the A_j ($j = 1, 2, 3$) acoustic wave, with $\omega_3 = \omega_1 + \omega_2$ and $K_3 = K_1 + K_2$.

Equation (3a) shows that the phase angle ϕ_3 can be written as $\phi_3 = \phi_1 + \phi_2 + \psi_{312}$ where ψ_{312} is the phase of the complex coupling coefficient Γ_{312} . If the diameter of the scanning light beam is small enough, so that within the illuminated part of the crystal surface the elastic displacements can be taken to be constant, the Fourier spectrum of the reflected wavefront is composed of a discrete number of terms spatially displaced by amounts proportional to the ratio K_j/k between the sound and light wave-numbers. Using the identity

$$\exp(iz \cos \phi) = \sum_{n=-\infty}^{+\infty} i^n J_n(z) \exp(in\phi)$$

where J_n is the n -th order Bessel function of the first kind, can be shown easily that the intensity of the diffraction orders which contain information about the sum-frequency wave is given by

$$I_{\pm 3} = I_0 \left| -J_1(2k \cos \theta |A_1|) J_1(2k \cos \theta |A_2|) J_0(2k \cos \theta |A_3|) + i J_0(2k \cos \theta |A_1|) J_0(2k \cos \theta |A_2|) J_1(2k \cos \theta |A_3|) \exp i \psi_{312} \right|^2 \quad (5)$$

From the physical point of view, the first term on the right-hand side of eqn. (5) represents the part of $I_{\pm 3}$ which results from the double scattering of the impinging light caused by the two fundamental surface waves, whereas the second term represents the contribution to $I_{\pm 3}$ of the light scattered directly by the parametric wave A_3 . In the series expansion of Bessel functions all terms higher than the first can be ignored because, at ultrasonic frequencies, $2k|A_j| \ll 1$, even for a large acoustic power flows. Hence, substitution of eqn. (3a) into eqn. (5) results in

$$I_{\pm 3} \simeq I_0 (2k \cos \vartheta |A_1 A_2|_{x=0})^2 \left[|\Gamma_{312}|^2 x^2 \pm |\Gamma_{312}| 2k \cos \vartheta \sin \psi_{312} x + (k \cos \vartheta)^2 \right] \quad (6a)$$

where $I_0 = E_0^2$ is the amplitude of the intensity of the reflected light. Following a similar analysis for the difference frequency wave, it can be shown that the intensity of the corresponding diffraction orders, $I_{\pm 4}$, is given by

$$I_{\pm 4} = I_0 (2k \cos \vartheta |A_1 A_2|_{x=0})^2 \left[|\Gamma_{412}|^2 x^2 \pm |\Gamma_{412}| 2k \cos \vartheta \sin \psi_{412} x + (k \cos \vartheta)^2 \right] \quad (6b)$$

The magnitude and phase of the coupling coefficient can be calculated directly from the last two equations, each of which represents a pair of parabolas, using the relations

$$\cos^2 \psi_{j12} = I_{-j}(x_{\min}) / I_j(0) \quad (7a)$$

and

$$|\Gamma_{j12}| = k \cos \vartheta \sin \psi_{j12} / x_{\min} \quad (7b)$$

where $x_{\min}$ is the distance at which I_{-j} attains its minimum value and $I_j(0)$ is the value of I_j at the origin ($I_{+3}(0) = I_{-3}(0) = I_{+4}(0) = I_{-4}(0)$). It must be noted that the method described here has an intrinsic uncertainty of π in the calculation of the phase angle, i.e. the latter can be ψ_{j12} or $\pi - \psi_{j12}$. This uncertainty can be removed by using dispersive crystals or, in the case of transparent media, by performing light transmission measurements. //

Fig. 1 Schematic representation of the diffraction orders caused by two collinear surface acoustic waves and their sum frequency wave.

3. RESULTS AND DISCUSSION. - The interdigital transducers of central frequencies $f_1 = 89.9$ MHz and $f_2 = 18$ MHz have been constructed on a dispersionless Y-cut Z-propagation LiNbO₃ sample. Each transducer was composed of five finger pairs whose width was 150 and 30 wavelengths long, respectively. Two collinear surface wave trains were launched at the above frequencies and the high frequency pulses were properly delayed so that they were overlapping with those of low frequency launched by the rear transducer. The propagation surface has been scanned by a narrow collimated He-Ne laser beam ($\lambda_L = 6328 \text{ \AA}$) which was impinging at a small incidence angle $\vartheta \approx 5^\circ$. A photomultiplier, whose output was measured using a box-car integrator, has been employed for the detection of the various diffraction orders. No absolute measurements of the light intensity have been made since this is not required by the method presented here.

The two propagating waves interacting via the elastic nonlinearity of the medium produce a surface acoustic wave at the sum-frequency $f_3 = f_1 + f_2 = 107.9$ MHz and the second at the difference-frequency $f_4 = f_1 - f_2 = 71.9$ MHz. The measured values of the light intensity diffracted by the two parametric waves at various positions along the propagation direction are shown in Fig. 2. The theoretical

curves obtained by best-fitting eqns. (6a) and (6b) on the measured values are also plotted on the same figure, for comparison. The calculated values of the amplitude and phase of the coupling coefficient for the sum-frequency wave are $|\Gamma_{312}| = 2.28 \times 10^3 \text{ mm}^{-2}$ and $\psi_{312} = 300^\circ$, respectively. Similarly, for the difference-frequency wave it has been found that $|\Gamma_{412}| = 1.76 \times 10^3 \text{ mm}^{-2}$ and $\psi_{412} = 312^\circ$. The estimated error in the value of $|\Gamma_{j12}|$ and ψ_{j12} is of the order of 5 per cent and 4° , respectively, for every case reported here. In the case of dispersionless media the amplitude of Γ_{j12} is proportional to the product $K_1 K_2$ of the wave vectors of the interacting waves [6,7]. A more meaningful measure of the strength of the nonlinear interaction is the quantity $G_{j12} = |\Gamma_{j12}| / K_1 K_2$, which depends only on the material constants and possibly on the profiles of the waves which participate in the interaction. Thus, it has been found that $G_{312} = 0.44$ and $G_{412} = 0.34$ for the up and down conversion, respectively. In the present case, for which the frequency ratio $f_2/f_1 = 0.2$ is small, the value of G_{312} is 23 per cent greater than that of G_{412} . This rather small difference in the strength of the interaction between the sum- and difference-frequency generation cases can be seen directly from Fig. 2 which show that the rate of growth with distance is similar for both parametric waves.

In order to check the effect of the frequency ratio f_2/f_1 on the strength of the nonlinear interaction, the measurements have been repeated for $f_1 = 90.9 \text{ MHz}$ (170 wavelengths wide, five elements transducer) and $f_2 = 54.6 \text{ MHz}$ (100 wavelengths wide five elements transducer), which give a frequency ratio $f_2/f_1 = 0.6$ considerably greater than the previous one. The experimental results, plotted in fig. 3, are again in good agreement with those predicted theoretically. Comparison between Figs. 2 and 3 shows that in the latter case the sum-frequency wave grows with distance much faster than the difference-frequency wave, whereas in the former case the difference between the rate of growth of the two waves is rather small. In fact, for the case shown in Fig. 3 the calculated values of the amplitude and phase of the coupling coefficient are $G_{312} = |\Gamma_{312}| / K_1 K_2 = 0.43$, $\psi_{312} = 333^\circ$ and $G_{412} = |\Gamma_{412}| / K_1 K_2 = 0.19$, $\psi_{412} = 328^\circ$ for the up- and down-conversion, respectively, i.e. the difference between G_{312} and G_{412} is 56 per cent, twice as much as the calculated for $f_2/f_1 = 0.2$.

Fig.2 Theoretical curves and measured values of the intensity of the light diffracted by the sum and difference frequency waves as a function of propagation distance, for a frequency ratio $f_2/f_1 = 0.2$

Second harmonic generation can be considered as mixing of two identical waves. It is therefore interesting to compare the value of the coupling coefficient $G_{211} = |\Gamma_{211}| / K_1^2$ for the second harmonic generation with those derived here for the up-conversion case. Following the method described in ref. [7], it has been found that $G_{211} = 0.23$ and $\psi_{211} = 317^\circ$ for both low ($f_1 = 18 \text{ MHz}$) and high ($f_1 = 89.9 \text{ MHz}$) frequencies. In ref. [7] has been reported $G_{211} = 0.31$ and $\psi_{211} = 319^\circ$, for the same material and orientation. Although the measured phase angle is similar to the one reported here a considerable difference exists between the two values of the magnitude of the coupling coefficient. This difference is too large to be justified by experimental errors or phenomena like attenuation, dispersion or the influence of Fresnel's field, which have not been included in the theory. It is more likely that this difference is due to imperfections in the crystalline structure of LiNbO_3 and, also, to the different past history of the samples used in the reported experiments [10]. Comparison between the present values of the amplitude of the coupling coefficient for the sum-frequency and second har-

Fig. 3 Theoretical curves and measured values of the intensity of the light diffracted by the sum and difference frequency waves as a function of propagation distance, for a frequency ratio $f_2/f_1 = 0.6$

nic generation shows that the former is greater than the latter by a factor of two, or so. The physical meaning of this relationship is that second harmonic generation is a mixing process in which the one half of the acoustic energy of the fundamental wave interacts parametrically with the remaining half.

4. CONCLUSION. - An optical method for the determination of the coupling coefficient for the up- and down- conversions of two collinear surface acoustic waves propagating on an elastically nonlinear medium has been developed. The method is simple and accurate and does not require any knowledge of the acoustic power flows; it uses only the relative values of the intensity of the light diffracted by the surface waves. The validity of the proposed phenomenological theory has been verified experimentally using a YZ LiNbO_3 crystal, for two different values of the ratio f_2/f_1 between the frequencies of the two interacting waves. The results indicate that in the case of sum-frequency

generation the amplitude of the coupling coefficient is practically constant and greater than that of second harmonic generation by a factor of two, whereas in the case of difference frequency generation the amplitude of the coefficient is smaller and appears to decrease as the ratio f_2/f_1 ($f_1 > f_2$) increases. Further measurements are needed for the determination of the functional behaviour with f_2/f_1 of the amplitude and phase of the coupling coefficient for both, up- and down-conversions. An accurate knowledge of these functions is necessary for the construction of several nonlinear devices, such as parametric amplifiers [11] and acoustoelectric correlators [5].

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