A PARAMETRIC SONAR PERFORMANCE CALCULATOR

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Résumé.- La prédiction des niveaux de source de fréquence différence et des ouvertures du faisceau, engendrés par un émetteur sonar paramétrique est un processus compliqué, qui entraîne de nombreux calculs et l'emploi de plusieurs courbes et nomogrammes. Le savant ou l'ingénieur ayant souvent besoin d'une estimation rapide et facile de ces paramètres, on a mis au point un calculateur de type règle à calcul destiné à aider à la conception d'un tel émetteur. Le calculateur donne des valeurs précises pour des réseaux paramétriques fonctionnant en régime non-saturé en utilisant les équations développées pour ce cas par F.H. FENLON. Cette étude décrira les hypothèses utilisées pour la réalisation des échelles ainsi que les limitations à prendre en compte lors de l'emploi du calculateur.

Abstract.- Predicting the difference frequency source levels and beamwidths which will be developed by a given parametric sonar projector is a somewhat complicated procedure involving a number of calculations and the use of several curves or nomograms. Since a scientist or engineer often needs a quick and easy estimate of these parameters, a slide-type calculator was developed as a design aid. The calculator yields accurate values for arrays operating in the unsaturated regime by using equations developed by F.H. Fenlon for this case. In this paper the assumptions used in the preparation of the scales and the limitations to be kept in mind while using the calculator will be described.

1. Introduction and background.- When a scientist is using or considering the use of a parametric sonar he will need to be able to determine the source level and beamwidth at the difference frequency. Unfortunately these quantities must be calculated by the use of equations which are somewhat complex. A number of investigators have developed equations for calculating source level and beamwidth at a parametric sonar's difference frequency or have developed families of curves, nomographs and other aids to help the sonar engineer determine these quantities. In this paper, a slide-type calculator is introduced that should prove to be a useful design tool. The philosophy followed in this design will be presented along with explanations regarding its use.

2. Parametric gain.- The entire front face of the slide calculator has been devoted to the calculation of difference frequency source level in the form of parametric gain. This is simply the difference between the source levels at the difference and mean primary frequencies, that is :

$$G = SL_- - SL_0$$

where :

G   = parametric gain

SL_- = difference frequency source level

SL_0 = mean primary source level

The equation developed by Dr. F.H. Fenlon, currently with ARL-Penn State, for the difference frequency

source level of a parametric source /1/ is :

$$SL_- = 20 \log_{10} (\overline{P}_0 r_0) - 20 \log_{10} (\frac{\Delta}{\Delta'} r) + 20 \log_{10} \left\{ \frac{\sqrt{2}}{\overline{X}_0} \frac{I_1 (X_1) I_1 (X_2)}{I_0 (X_1) I_0 (X_2)} \right\}$$

where :

$\overline{P}_0$ = total rms primary amplitude at the source

r_0 = Rayleigh distance

f_0 = mean primary frequency

f_- = difference frequency

Δ = $E_1 (\alpha_T r_0) \exp \{\alpha_T r_0\}$
     where $\alpha_T = 2\alpha_0 - \alpha_-$ = total attenuation coefficient

$\Delta' = E_1 (\alpha_T r_0 \frac{f_0}{f_-}) \exp \{\alpha_T r_0 \frac{f_0}{f_-}\}$

$$E_1 (y) = \int_y^\infty \frac{1}{x} e^{-x} dx$$

$$\overline{X}_0 = \frac{\beta k_0 \overline{P}_0 r_0}{\rho_0 c_0} \Delta$$

where : $\beta = 1 + \frac{B}{2A}$ = nonlinear parameter for liquids

$k_0 = 2\pi f_0/c_0$ = mean primary wave number

c_0 = small signal speed of sound

ρ_0 = density of the medium

$$X_{1,2} = \frac{\sqrt{2} \beta k_0 P_{0\ 1,2} r_0}{\rho_0 c_0}$$

I_0, I_1 = modified Bessel functions.

Unfortunately this equation has proven to be too complex for evaluation on a slide-type calculator. However, if we restrict ourselves to primary source levels below the levels where shock waves begin to develop, i.e., to the unsaturated regime, the equations simplify immensely. This is not too severe a restriction as most current parametric sonars operate in this regime. The simplification occurs because X_0, X_1, X_2 <<< 1 in this region and the equation for parametric gain becomes :

$$G = SL_0^* - 20 \log_{10} \frac{f_0}{f_-} + 20 \log_{10} \Delta' - 287$$

where $SL_0^* = SL_0 + 20 \log_{10} f_0$

This is the equation solved on the front face of the calculator.

a. Rayleigh Distance, r_0

The Rayleigh distance at the mean primary frequency is the straightforward calculation normally used,

$$r_0 = A/\lambda_0 = A \, c_0/f_0$$

where A = array area

λ_0 = wavelength at the mean primary frequency

c_0 = small signal sound speed

Sound speeds of 1481 and 1520 m/sec were used for fresh and sea water, respectively.

b. Absorption Coefficients

The Shulkin and Marsh /2/ equations were used in the formation of the absorption scales, namely :

For fresh water : $\alpha = 2.68 \times 10^{-2} \dfrac{f^2}{f_T}$

and for sea water :

$$\alpha = \begin{cases} \dfrac{0.1\ f^2}{1 + f^2} + \dfrac{40f^2}{4100 + f^2} & \text{for } f < 5 \text{ kHz} \\[4mm] 1.86 \times 10^{-2} \dfrac{S f_T\ f^2}{f_T^2 + f^2} + 2.68 \times 10^{-2} \dfrac{f^2}{f_T} & \text{for } f > 5 \text{kHz} \end{cases}$$

where $f_T = 21.9 \times 10^{6 - 1520/(T+273)}$

T = temperature

S = salinity in ppt

A temperature of 10°C (50°F) and a salinity of sea water of 35 ppt were the values used in the preparation of these scales.

c. Effective Array Length Δ'

The parameter Δ', sometimes called effective array length, is given by :

$$\Delta' = E_1 \left(\alpha_T\, r_0\, \frac{f_0}{f_-}\right) \exp \left(\alpha_T\, r_0\, \frac{f_0}{f_-}\right)$$

where $E_1 (X) = \displaystyle\int_X^\infty \frac{1}{Z} e^{-Z}\, dZ$

The exponential integral was evaluated using numerical techniques /3/ to generate the scale for calculating 20 $\log_{10}$ Δ'. Equivalent array length can also be used to determine the approximate nonlinear interaction range, i.e., the distance required for the difference frequency souce level to build up to its ultimate value (the value predicted by the calculator). The product of equivalent array length, the Rayleigh distance, and the downshift ratio will yield the approximate interaction range,

$$r_i \approx \Delta' \, r_0 \, \frac{f_0}{f_-}$$

d. Scaled Primary Source Level SL_0^*

The scaled primary source level is simply :

$$SL_0^* = SL_0 + 20 \log_{10} f_0$$

and is evaluated in a straightforward manner on one of the front side scales.

e. Parametric Gain

The last scale on the front face of the calculator evaluates :

$$G = SL_0^* - 20 \log_{10} \frac{f_0}{f_-} + 20 \log_{10} \Delta' - 287$$

which gives the difference frequency source level since $G = SL_- - SL_0$.

3. Directivity and beamwidth.-

On the reverse side of the calculator there are scales for calculating the approximate directivity index and beamwidth at the difference frequency.

a. Directivity Index

The directivity index of the difference frequency beam will be very close to the directivity index of the mean primary frequency beam pattern so the equation used for this is :

$$DI = 10 \log_{10} \frac{4\pi\ A\ f_0^2}{c_0^2}$$

and again, small signal sound speeds of 1481 and 1520 m/sec were used for fresh and sea water, respectively.

b. Beamwidth

The half-power beamwidth is given by one of two equations, depending upon whether the array is spreading loss limited or absorption limited.

For an array which is spreading loss limited the beamwidth is given by :

$$\theta_{3\,dB} = \begin{cases} 2\,\sin^{-1}\dfrac{2.7\,c_0}{2\,f_0\,d} & \text{for a rectangular or square array} \\[2ex] 2\,\sin^{-1}\dfrac{c_0}{2\,f_0\,d} & \text{for a circular array} \end{cases}$$

where d = side length or diameter as appropriate and for an absorption limited array it is :

$$\theta_{3\,dB} = 4\sqrt{\dfrac{\alpha_T\,c_0}{4\pi\,f_-}}$$

for square or circular arrays.

Small signal sounds speeds of 1481 and 1520 m/sec were used in the preparation of the scales.

Whether the array under consideration is spreading loss or absorption limited the beamwidth can be found by obtaining values from both scales and choosing the larger. This will be the appropriate value.

4. Critical source level SL_c.- The critical source level is the mean primary source level at which shock waves begin to form. At source levels above this level the equations being solved on the front face of the calculator no longer apply. Actually, since the accuracy becomes worse the nearer SL_0 comes to SL_c ,the results should be treated with caution when SL_0 is within, say 3 or 4 dB of the critical level (see figure 1). The equation which was used to calculate this quantity /1/ :

$$SL_c = 10\log_{10}L - 20\log_{10}f_0 + \begin{matrix} 191.1 \text{ in fresh water} \\ 192.2 \text{ in sea water} \end{matrix}$$

and the parameter L is the solution to :

$$\exp\left(-\dfrac{\pi}{2L\beta}\right) = 4.5\left(\dfrac{\alpha_0\,r_0}{\beta L}\right)$$

where β = nonlinearity parameter = $\begin{matrix} 3.6 \text{ for fresh water} \\ 3.8 \text{ for sea water} \end{matrix}$

The transendental equation for L was solved using interative techniques and the results for various $\alpha_0\,r_0$ values included as a separate scale on the reverse side. The scale for SL_c is immediately below it and completes the solution.

5. Conclusions.- It is hoped that this calculator will prove to be a handy tool for engineers and scientists involved in finite-amplitude sonar. It is intended to give them an easy way to quickly estimate parametric conversion efficiencies, beam-widths and the like which were previously somewhat inconvenient to obtain.
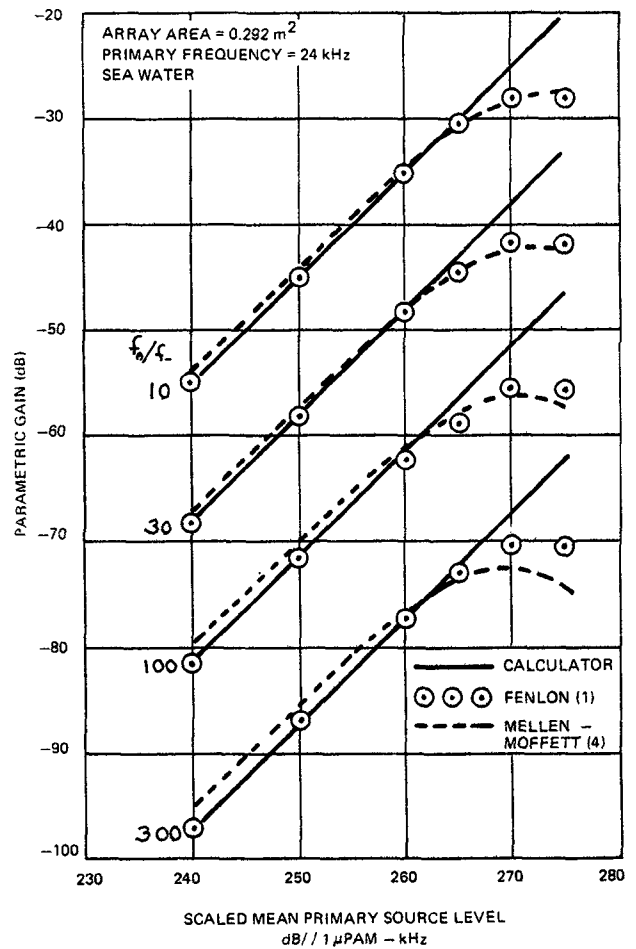

Fig. 1 - Comparison of theoretical results

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