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ON LANDAU DAMPING IN INTERACTING THE REGULAR WAVE WITH NOISE

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Résumé.- On considère l'interaction non linéaire d'une onde régulière et du bruit. Dans l'approximation des trois ondes on a obtenu par la méthode du champ moyen l'expression du coefficient d'absorption d'une onde, qui se propage dans une zone perturbée par un bruit intense. L'analyse de cette expression montre que l'interaction entre l'onde et le bruit basse fréquence mène à l'atténuation de cette onde tandis que dans le cas du bruit haute fréquence l'onde peut se renforcer du fait d'un transfert d'énergie du champ de bruit à l'onde. Si la fréquence du bruit est beaucoup plus grande que la fréquence de l'onde, le caractère du processus est déterminé par l'interaction entre l'onde et le paquet d'ondes du champ de bruit dont la vitesse de groupe coïncide avec la vitesse de phase de l'onde considérée. Comme dans l'effet Landau l'onde peut s'atténuer ou s'amplifier en fonction des propriétés du spectre de bruit.

Abstract.- The interaction of the regular wave with the noise is considered in the three-wave approximation of the nonlinear wave theory. It is the shown that the interaction of the wave with the low-frequency noise leads to the wave attenuation, while in the case of the high-frequency noise the wave can intensified as the result of the energy transfer from the noise field to the wave. This result comply closely with the well-known Landau expression for the damping of the electromagnetic wave in plasma.

The nonlinear interaction of the regular wave with noise is considered. In three-wave approximation by the method of the average field the expression for the wave absorption coefficient, propagating in the medium, disturbed by the intensive noise is obtained. The analisis of this expression shows that the wave interaction with low-frequency noise tends to its damping, while in the case of high-frequency noise the wave can amplify as the result of the energy transmission from the noise field to the wave.

If the noise frequency is essentialy exceeded the wave frequency, then the process nature is determined by the wave interaction with the wave packet of the noise field, the group velocity projiection of which on the propagation wave direction is coincidenced with the phase velocity of the considered wave.

In this case as in Landau damping effect the wave can damp or amplify depending on the noise spectrum propeties.

When the wave is propagated in the medium, disturbed by the intensive noise due to the nonlinear effects, the interaction of the wave and the noise occurs, tending to the energy exchange between them. Under certain conditions this process tends to the Landau effect damping type /1,2/.

The interaction of the wave with those components of the noise field, which produce the resonance or almost resonance triplets, makes the main contribution to this process. This allows to describe the interaction of the wave

$$a_{\vec k} \exp\,(i\,\vec k\,\vec r - i\,\omega_{\vec k}\,t)$$

and the noise which can be represented in the form of the discrete wave harmonics

$$\sum_{\vec k'} C_{\vec k'} \exp\,(\,i\,\vec k'\vec r - i\,\omega_{\vec k'}\,t)$$

in three-wave approximation of nonlinear wave theory. In this case it is convenient to apply the average field method and to represent the amplitude of the considered wave frequency $\omega_{\vec k}$ in the form of the sum of its mean value and the fluctuation addition

$$\bar a_{\vec k} + b_{\vec k} \tag{1}$$

Thus the amplitude of the wave propagating in the direction $\vec k'$, composes of three components :

$$a_{\vec k}\,\delta_{\vec k\vec k'} + C_{\vec k'} + b_{\vec k'} \tag{2}$$

Substituting this expression in the equation, describing three-wave interaction

$$\dot{a}_{\vec{k}} = \sum_{\vec{k}'} V_{\vec{k}\vec{k}'\vec{k}-\vec{k}'}\, a_{\vec{k}'}\, a_{\vec{k}-\vec{k}'}\, e^{i\Delta\omega_{\vec{k}} t}$$

where $V_{\vec{k}\vec{k}'\vec{k}-\vec{k}'}$ - is the interaction potential and $\Delta\omega_{\vec{k}}$ - is expressed in terms of the interacting wave frequencies, we obtain two equations.

One of them describes the change of the fluctuation addition amplitude

$$\dot{b}_{\vec{k}'} = \sum_{\vec{k}'-\vec{k}} V_{\vec{k}'\vec{k}\vec{k}'-\vec{k}}\, a_{\vec{k}}\, C_{\vec{k}'-\vec{k}} \qquad (3)$$

The summation is performed using all noise components, producing the resonance triplet with considered wave. The other equation refers to the average wave amplitude.

In the linear approximation on the fluctuation addition it is of the form :

$$\dot{a}_{\vec{k}} = \sum_{\vec{k}'} V_{\vec{k}\vec{k}'\vec{k}-\vec{k}'}\, \tilde{C}_{\vec{k}'}\, C_{\vec{k}-\vec{k}'}\, e^{i\Delta\omega_{\vec{k}} t} \qquad (4)$$

Here $\tilde{C}_{\vec{k}'} = C_{\vec{k}'} + b_{\vec{k}}$ - is the modified amplitude of the noise field.

Substitution in the last relation the expression for the fluctuation addition (3) tends to the equation for the average amplitude, the solution of which can be written in the form /3/ :

$$\bar{a}_{\vec{k}} = a_{o\vec{k}}\, e^{-\beta_{\vec{k}} t} \qquad (5)$$

where

$$\beta_{\vec{k}} = -\sum_{\vec{k}'} V_{\vec{k}\vec{k}'\vec{k}-\vec{k}'}\, V_{\vec{k}'\vec{k}\vec{k}'-\vec{k}}\, N_{\vec{k}'-\vec{k}}\, \delta(\Delta\omega_{\vec{k}}) \qquad (6)$$

Here $N_{k'-k} = C_{k'-k}\, C^{*}_{\vec{k}'-\vec{k}}$ - is the number of the interacting noise waves, related to the density of the noise energy by the usual relation /2/ :

$$N_{\vec{k}'}\, \omega_{\vec{k}'} = \varepsilon_{\vec{k}'}$$

in this case

$$\int \varepsilon_{\vec{k}'}\, d\vec{k}' = \int \varepsilon_{k'}\, dk' = E ,$$

where E - is the energy density of the noise field, $\Delta\omega_{\vec{k}} = \omega_{\vec{k}'} + \omega_{\vec{k}-\vec{k}'} - \omega_{\vec{k}}$ - is the magnitude of the interacting wave missmatched.

Expression (6) is the convenient initial point to investigate the properties of the process of the wave interaction with the noise. For this it is convenient to rewrite whis relation in more symmetrical form :

$$\beta_{\vec{k}} = -\frac{1}{2}\sum_{\vec{k}'} V_{\vec{k}\vec{k}'\vec{k}-\vec{k}'}\, \delta(\Delta\omega_{\vec{k}}) \left[V_{\vec{k}'\vec{k}\vec{k}'-\vec{k}}\, N_{\vec{k}'-\vec{k}} - V_{\vec{k}-\vec{k}'\vec{k}-\vec{k}'}\, N_{\vec{k}'}\right] \qquad (7)$$

The received expression allows to determine the process of the wave interaction with the noise occurs alternately for the case of the low frequency and high frequency noise.

That is, if the noise is low frequency, and the relations $\omega_{\vec{k}} > \omega_{\vec{k}'}$, $\omega_{\vec{k}} > \omega_{\vec{k}-\vec{k}'}$ are valid, then due to the relations

$$V_{\vec{k}'\vec{k}\vec{k}'-\vec{k}} = V_{\vec{k}-\vec{k}'\vec{k}_1-\vec{k}'} = -V_{\vec{k}\vec{k}'\vec{k}-\vec{k}'}$$

we obtain

$$\beta_{\vec{k}} = \sum_{\vec{k}} |V_{\vec{k}\vec{k}'\vec{k}-\vec{k}'}|^2\, \delta(\Delta\omega_{\vec{k}})\, N_{\vec{k}'-\vec{k}} \qquad (8)$$

thus $\beta_{\vec{k}} > 0$ and the wave damping occurs.

If the wave interacts with the high frequency noise so as

$$\omega_{\vec{k}'} > \omega_{\vec{k}} \qquad \omega_{\vec{k}-\vec{k}'} > \omega_{\vec{k}}$$

then to obtain the final result if is convenient to consider the next factor.

Within the considered approximation and the conditions of the random phases the regular wave interact with every pair of the noise component, forming the triplets, non interacting with each other.

Each of the triplets in formula (7) takes into account twice when summing on all noise components, forming the resonance triplets. So we can not changing the result, pass to once summing, producing it, for example, using only those values $\vec{k}'$, for which $\omega_{\vec{k}} > \omega_{\vec{k}-\vec{k}'}$ and simultaneously omit the factor 1/2 ahead of formula.

Considering that under the condition $\omega_{\vec{k}'} > \omega_{\vec{k}}$, $\omega_{\vec{k}-k'} > \omega_{\vec{K}}$, $\omega_{\vec{k}'} > \omega_{\vec{k}-\vec{k}'}$, we have

$$V_{\vec{k}'\vec{k}\vec{k}'-\vec{k}} = - V_{\vec{k}-\vec{k}'\vec{k}_1-\vec{k}'} \; ; \; V_{\vec{k}\vec{k}_1'\vec{k}-\vec{k}'} = - V_{\vec{k}'\vec{k}\vec{k}'-\vec{k}}$$

For damping coefficients β_k we obtain :

$$\beta_{\vec{k}} = \sum | V_{\vec{k}\vec{k}'\vec{k}-\vec{k}'} |^2 (N_{\vec{k}'-\vec{k}} - N_{\vec{k}'}) \, \delta \, (\Delta\omega_{\vec{k}})$$

As you can see in this case the term $\beta_{\vec{k}}$ may become negative (under the condition $N_{\vec{k}'-\vec{k}} < N_{\vec{k}'}$), that means the energy in this case would transmit from the noise field to the wave.

Thus, the noise interaction may tend to both wave damping and wave amplification, as it shown particularly in /4-6/.

The effect of the wave amplification by noise is essentially the reflection of the fact of decay of high-frequency noise components relative to the low-frequency disturbance in the form of the wave. In this case the wave, amplifying due to the noise energy, obtains all the statistical features of the noise component and becomes indistinguishable from its other components, in the case when the wave and the noise are the waves of the type.

In this sense the process of the wave amplification by noise can also be considered as "heating" of low-frequency degrees of freedom till "the temperature" of the noise field, so the wave amplitude takes the equilibrum value :

$$a^2_{k\infty} = T\omega^{-1}_{\vec{k}}$$

where T - is the field temperature /2/.

This circumstance imposes the restrictions on the process of the wave amplification as the result of its interaction with the noise. If the initial wave energy is smaller than the equilibrium value then it amplificates till the equilibrium value. If the initial value of the wave amplitude is more than the equilibrium value, then the processes of four-wave interaction as the course of which the swing of low-frequency noise occurs ferstly due to decay of high-frequency noise components will be the main processes, and then the wave absorption in the low-frequency noise takes place.

Further we note, that if the noise frequency is essentially exeeded the frequency of the considered wave, then the next relation results from the synchronism condition (figure 1) :

$$\omega_{\vec{k}'} \geqslant \omega_{\vec{k}-\vec{k}} >> \omega_{\vec{k}}$$

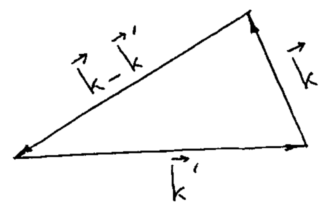

Fig. 1

from which is followed, that the frequencies of the noise components, taking part in the interaction with the considered wave, little differ from one another.

This allows to produce the expansion

$$N_{\vec{k}'-\vec{k}} = N_{\vec{k}'} - \frac{\partial N_{\vec{k}'}}{\partial \vec{k}'} \vec{k} \, , \; \omega_{\vec{k}'-\vec{k}} = \omega_{\vec{k}'} - \frac{\partial \omega}{\partial \vec{k}} \vec{k}$$

that leads to the next expression for $\beta_{\vec{k}}$:

$$\beta_{\vec{k}} = - \frac{1}{2} \sum_{\vec{k}'} |V_{\vec{k}\vec{k}'\vec{k}-\vec{k}}|^2 \left(k \frac{\partial N_{\vec{k}'}}{\partial \vec{k}'}\right)\Big|_{\omega_{\vec{k}'} = \vec{k}\vec{U}} \qquad (10)$$

where $\vec{U}$ - is the group velosity.

The received expression means that the wave damping occurs due to its interaction with the wave packets of the noise field (formed by two noise harmonics), the group velocity projection of which on the direction of the wave vector coincidences with the phase velosity of the considered wave. This result closely corresponds to the known Landau expression for the electromagnetic wave damping in plasma due to its interaction with the particles, the velocity of which coincidences with the phase wave velosity.

Moreover as well as in the considered Landau phenomena, the process character appears to depend on the properties of the noise spectrum (the distribution fenction) $\frac{\partial N_{\vec{k}'}}{\partial \vec{k}'}$.

If $\frac{\partial N_{\vec{k}'}}{\partial \vec{k}'} < 0$ then we have $\beta_{\vec{k}} > 0$ and the wave damping occurs, if $\frac{\partial N_{\vec{k}'}}{\partial \vec{k}} > 0$ $\beta_{\vec{k}} < 0$, and the wave amplifies.

Particularly, for the equilibrium spectrum we obtain $N_{\vec{k}'} = T\omega_k^{-1}$, and $\frac{\partial N_{\vec{k}'}}{\partial k'} < 0$, for all cases, when $\omega_{\vec{k}'}$ increases with increasing $\vec{k}'$, so the equilibrium noise absorbs the wave in spite of its energy spectrum appears to increase in accordance with Rayleigh-Jins law.

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