CAN THE ANDC METHOD REALLY OVERCOME THE CASCADING PROBLEM IN B.F.S. LIFETIME MEASUREMENTS?

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Résumé - On indique quelques méthodes de détermination des limites d'erreur sur les durées de vie obtenues par la technique ANDC à partir des résultats de BFS. On conclut que si on tient compte de manière correcte de toutes les transitions concernées, on peut avoir confiance dans cette technique, même lorsque les cascades sont très sévères. Comme exemple, on a déterminé les forces d'oscillateur de la transition 5s-5p dans I VII et Xe VIII et on obtient un accord satisfaisant avec des calculs récents (RHF)

Abstract - We indicate how meaningful error limits may be derived for lifetimes calculated from beam-foil data using the ANDC technique. We conclude that, provided all relevant transitions are correctly included, the ANDC technique can give reliable results, even in heavily cascaded situations. As an example, we apply the technique to obtain f-values for the 5s-5p transition in I VII and Xe VIII, and obtain results consistent with recent RHF calculations.

1. INTRODUCTION - Particular attention has been focused upon the problem of cascade repopulation in beam-foil lifetime measurements by recent discussions of the significant discrepancies between beam-foil and theoretical f-values for members of the Cu I, Zn I, Ag I and Cd I isoelectronic sequences. In these cases, the major, if not the dominant, cascade contribution usually comes from the yrast chain which can contain many significant cascade components, and cover a wide range of lifetimes. Under these circumstances, standard multi-exponential curve-fitting techniques may experience difficulty in extracting the primary lifetime from the primary decay curves. It has been shown analytically [1] that these problems can be overcome, in principle at least, by the ANDC technique, provided that ideal decay curves can be obtained for all the direct cascades into the primary level. However, it is not so obvious how reliable the ANDC technique will be in a practical situation, where the data consists of a finite number of points, each subject to Poisson statistics. In this report we will describe ways to establish meaningful error limits for lifetimes derived using the ANDC technique. We will also discuss the application of these methods to synthetic and real beam-foil decay curve data.

2. METHODS FOR APPLICATION OF THE ANDC PRINCIPLE -

As was shown by Curtis et al. [1], the differential equations relating the populations of the various levels involved in a cascade scheme leads, in the case of a single direct cascade, to the result

$$\tau = \frac{P}{\Delta} - \xi \cdot \frac{C}{\Delta} \quad (1)$$

where τ is the lifetime of the primary level, and the quantities P , C and Δ are defined as follows:

$$P \equiv \int_{t_i}^{t_f} I_p(t) dt = \text{area under primary decay curve,}$$

$$C \equiv \int_{t_i}^{t_f} I_c(t) dt = \text{area under cascade decay curve,}$$

$$\Delta \equiv I_p(t_i) - I_p(t_f) = \text{change in primary intensity,}$$

where t_i and t_f are the initial and final times following excitation defining a given panel. ξ is given by the expression:

$$\xi = (A_{ji}\tau) \cdot \frac{E_p}{E_c} \quad (2)$$

where A_{ji} is the transition probability of the primary transition $j \rightarrow i$, and E_p , E_c are the relative detector sensitivities at the primary and cascade wavelengths. If more than one direct cascade into level j is possible, eq. (1) assumes the general form

$$\tau = \frac{P}{\Delta} - \sum_{n=1}^m \xi_n \cdot \frac{C_n}{\Delta} \quad (3)$$

Sets of ANDC data points (P_i , C_{1i} , C_{2i} , ..., C_{mi} , Δ_i) may then be calculated from the decay curves and fitted to eq. (1) or (3), depending on whether the number of direct cascades, m , is equal to 1 or greater than 1, respectively.

There is one fundamental difficulty associated with eqs. (1) and (3). The uncertainties in the values of Δ are generally much greater than those in the values of P or C_n , since Δ is the difference

of two quantities of similar magnitude while P and C_n are obtained by adding two (or more) quantities of similar magnitude. This has the consequence that the uncertainties in the two variables (P/Δ) and (C/Δ) are highly correlated. This correlation must be included in the linear regression analysis if valid values for τ and its uncertainty are to be obtained. We have conducted ANDC analyses based on eq. (1), both including and omitting the correlation between the variables, and have found that the results can be significantly in error if the correlation is ignored. Our program to include correlation utilizes a routine written at Oxford [2]. Unfortunately this routine can not be used for more than two correlated variables. However, we can write eq. (3) in the form:

$$\Delta = K + \frac{P}{\tau} - \sum_{n=1}^m \xi_n \frac{C_n}{\tau} \quad (4)$$

K is a constant which should be zero, but which is included to give one of the validity criteria discussed later in this report. The analysis involves one more variable than is required using eq. (3), but has the important advantage that the variables are almost completely uncorrelated. (There is, of course, a weak correlation between P and Δ , but this will normally have a minimal effect on the analysis.) Since the variables are not significantly correlated it is possible to use standard least-squares techniques [3] to derive meaningful error limits for the fitted constants in eq. (4), provided that the uncertainties in Δ are much larger than those in P and C_n , which is generally the case.

A third variation may be obtained by writing eq. (1) in the form:

$$\frac{\Delta}{P} = \frac{1}{\tau} - \xi \cdot \frac{C}{P\tau} \quad (5)$$

This again gives a simple linear fit. The difference between eq. (1) and eq. (5) is that the correlation between the uncertainties in the variables (Δ/P) and (C/P) is quite low, being typically 0.10, whereas the corresponding correlation coefficient for the variables (P/Δ) and (C/Δ) is typically 0.95. Thus the ANDC analysis can be performed in the case of a single significant direct cascade according to both eq. (1) and eq. (5) to test the self-consistency of the method and the effectiveness of the error correlation routine.

In addition to the various ways just discussed of

conducting an ANDC analysis for a given set of ANDC data points ($P_i, C_{1i}, C_{2i}, \dots, C_{mi}, \Delta_i$), there is also the question of how one may best obtain these input data from the beam-foil decay curves, i.e. how one may best derive the ANDC panels from the beam-foil decay data. Since all valid ANDC analyses must give equivalent results, variations of the method and/or panel arrangement can be tried to test the self-consistency of the analysis. (The error analysis must be modified, of course, in the case of overlapping panels, if rigorous error limits are to be retained.)

3. THE EFFECT OF TIME AVERAGING - In any beam-foil measurement, one does not actually record the intensity at a point d_i from foil. Rather, one measures the intensity from a length of beam, δ , centered on the point d_i from the foil. Obviously the averaging length, δ , should be the same for the primary and cascade decay curves used in a given ANDC analysis. The question remains, however, concerning how large δ can become before the ANDC analysis becomes invalid. Somewhat surprisingly, we have found [4] that the validity of the ANDC analysis is unaffected by the size of δ . However, as δ becomes large compared with the quantity, $v\tau$, (v being the beam velocity) the ANDC points bunch closer together and the uncertainty in the slope of the fitted line or plane through those points increases, with a corresponding increase in the uncertainty in the resulting primary lifetime. (This corresponds to the situation for multi-exponential fitting; as δ increases the analysis remains valid but the uncertainty in τ increases.)

4. VALIDITY CRITERIA - In all the preceding discussion, we have assumed that the ANDC analysis is valid. This will be true provided that all significant direct cascades have been included, and that none of the transitions involved in the analysis is blended or incorrectly assigned. (We are assuming here that the physical parameters in the experiment, such as the beam energy, E , and the averaging length, δ , are the same for all decay curve measurements used in a given analysis.) Obviously it is most important to study the spectrum carefully before beginning the measurement, but this may not always be sufficient to detect blending or an incorrect assignment, particularly in some of the poorly known spectra of the multiply-ionized heavy atoms. Hence there is a need for validity checks within the ANDC analysis itself.

It is not possible to include all details of our studies here, but a summary of the validity criteria we use is shown in Table 1. (For details, see [4].)

TABLE 1. *ANDC VALIDITY CRITERIA*

Method	Quantity	Test
eq. (1)	curvature slope	reject if significant curvature in plot
eq. (5)		reject if ξ is negative
eq. (4)	uncertainties in P_i , C_{ni} and Δ_i	reject if uncertainties in P_i or C_{ni} greater or comparable to that in Δ_i
	K	reject if significantly larger than zero
any	reduced CHI-squared	reject if significantly greater than 1

5. SYNTHETIC DATA ANALYSIS — As a test of the ANDC methods described above, we have used synthetic data corresponding to some moderately or heavily cascade situations that we have encountered in the laboratory. The relationships between the coefficients of the various cascade and primary components, the dependence of the relative initial populations on $(2l+1)/n^3$, and the presence of random scatter according to Poisson statistics have all been included in our data simulations. We have formed several conclusions from these analyses of synthetic data. Most important of all, the uncertainty derived for the primary lifetime appears to be a reliable single standard deviation estimate. In many cases this was not so when the same synthetic data were analyzed using multi-exponential curve fitting techniques, where it was often found that either the fitted lifetime did not agree with the true value within the error estimate, or it did agree but only by virtue of an unduly large error estimate. Not unexpectedly, the uncertainties found by the ANDC analyses were larger for more complex decay schemes, for lower peak intensities and for curves covering shorter beam lengths. The validity criteria have proved to be useful indicators of analyses that may lead to inaccurate results. Finally, the lifetime value obtained in a given situation depends more critically on the particular

primary decay curve used than it does on the cascade decay curve(s). Examples of synthetic analyses on particular cascade schemes may be found in [4].

6. ANDC ANALYSIS FOR THE 5s-5p TRANSITION IN I VII AND Xe VIII — We have conducted ANDC analyses to obtain values for the lifetimes of the 5p levels in I VII and Xe VIII, using data recently measured at Alberta [5,6]. The f-value trend for this Ag I iso-electronic sequence is discussed elsewhere in these Proceedings [5]. The I VII data lead to a 5s-5p multiplet f-value of 0.90 ± 0.06 from an ANDC analysis and of 0.73 ± 0.04 using multi-exponential fitting, while the corresponding Xe VIII f-values are 0.89 ± 0.08 (ANDC) and 0.64 ± 0.04 (m-e fit). Cheng and Kim [7] have recently made a relativistic Hartree-Fock calculation for this sequence, and obtain a 5s-5p f-value equal to approximately 1.08 for both I VII and Xe VIII. They estimate that polarization effects might lower this value by 10-20%. The agreement between the f-values we derive from our ANDC analysis and those calculated by Cheng and Kim is thus satisfactory for these ions, although it would certainly be interesting if core polarization could be included in a future calculation.

7. SUMMARY — We have outlined various ways in which the ANDC principle can be employed in practice, and have discussed their use with synthetic and real beam-foil data. We have suggested some criteria for testing the validity of the ensuing analysis. Provided that the conditions requisite for a valid ANDC analysis are obeyed, the primary lifetime thus determined should always be reliable within its estimated uncertainty. Unfortunately, there is no absolute guarantee that the validity criteria we have discussed will always reveal when a violation of the conditions required for a valid ANDC analysis has occurred. If a significant cascade is omitted, blended or incorrectly assigned, and if the validity criteria fail to reveal this, then the ANDC analysis may be in error. Further work is therefore planned to minimize this possibility of error by devising a more complete set of validity tests.

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