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FLUX-FLOW VOLTAGE PULSES IN SUPERCONDUCTORS

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Résumé.- On présente une nouvelle méthode de calcul de l'impulsion de tension produite par le mouvement d'un vortex ou tube de flux.

Abstract.- A new theoretical method for the calculation of the measured voltage pulse produced by a moving flux vortex or flux tube is described.

INTRODUCTION.- A fundamental question of continuing interest is that of the origin of the measured voltage V across a magnetic flux-containing superconductor when a sufficiently high current drives it into the flux-flow regime. Although the Josephson relation, $1/V = (\hbar/2e) d\theta/dt$, where θ is the difference in the phase of the order parameter between two contacts, often has been used, ^{/2,4/} caution must be taken in using this expression, because the right-hand side depends upon the gauge chosen ^{/5/}. The purpose of this paper is to derive a new properly gaugeinvariant expression, of the same form as the above equation but with a different interpretation, which should prove useful for calculations of the measured voltage V .

DERIVATION OF THE RESULT.- We consider the usual case of a superconducting specimen and a measuring circuit whose linear dimensions are all large by comparison with the field-dependent penetration depth λ_B . Inside the superconductor, the local flux density contribution $\vec{b}_i$ from vortex i is large only within about λ_B of the vortex axis or the surface. As sketched in figure 1, the associated magnetic flux $\phi_0 = hc/2e$, which flows up through the specimen's interior, returns around the specimen's edges through the surrounding space. When the vortex moves, carrying its flux density contribution $\vec{b}_i$ with it, the corresponding voltage contribution V measured across terminals A and B of a sensitive voltmeter connected by leads C_M to contacts a and b is, to good approximation ^{/2,8/},

$$V = V_A - V_B = -c^{-1} d\phi_{MS}/dt. \quad (1)$$

Here, $\phi_{MS} = \int dS \vec{b}_i \cdot \hat{n}$ is carried out over the surface S_{MS} bounded by the contours C_M and C_S . The contour C_S must be chosen to penetrate perpendicularly

into the specimen and then to remain several λ_B from the surface as it passes between a and b as shown in figure 1.

Fig. 1 : Sketch of measuring circuit and vortex i .

We next consider the self field $\vec{b}_M$ produced by a virtual current I_M through the measuring circuit, taking into account the Meissner effect, as indicated in figure 2. Large values of $\vec{b}_M$ are confined to the region outside the specimen and within λ_B of the surface.

We now consider the integral of the field energy density $b^2/8\pi$, where $\vec{b} = \vec{b}_i + \vec{b}_M$. The interaction energy between vortex i and the virtual current I_M is $U_{int} = \int d^3r \vec{b}_i \cdot \vec{b}_M / 4\pi$. We take the integral only over the space outside both the specimen and the leads and ignore the remaining contributions. We express $\vec{b}_i$ and $\vec{b}_M$ in terms of magnetic potentials via $\vec{b}_i = -\nabla\phi_i$ and $\vec{b}_M = -\nabla\phi_M$ and then evaluate U_{int} in two ways. First, by using the identity $\nabla\phi_i \cdot \nabla\phi_M = \nabla \cdot (\phi_i \nabla\phi_M)$ and applying the divergence theorem, we find that $U_{int} = 0$, because $\vec{b}_M = -\nabla\phi_M$ is parallel to the specimen surface.

Next we use the identity $\nabla\phi_i \cdot \nabla\phi_M = \nabla \cdot (\phi_M \nabla\phi_i)$ and again apply the divergence theorem to express U_{int}

in terms of integrals over the surfaces shown in figure 3 : the lead surfaces, the specimen surface S_S and so-called cut surfaces S_{C1} and S_{C2} , which are necessary to account for the fact that ϕ_M changes by $-4\pi I_M/c$ in executing a circuit in the right-hand sense around the lead.

Fig. 2 : Sketch showing flux density $\vec{b}_M$ produced by virtual current I_M .

The result is

$$U_{int} = \int dS_s \vec{b}_i \cdot \vec{n}_s \phi_M / 4\pi + I_M \phi_c / c = 0, \quad (2)$$

where $\phi_c = \phi_{C1} + \phi_{C2} = -\phi_{MS}$ and $\phi_{Cj} = \int dS_{Cj} \vec{b}_i \cdot \vec{n}_{Cj}$. Note also that $\vec{b}_i \cdot \vec{n}_s = 0$, except very close to the top t_i or the bottom b_i of vortex i , where the surface integral of $\vec{b}_i \cdot \vec{n}_s$ is ϕ_0 or $-\phi_0$, respectively.

From eqs. (1) and (2) we then obtain

$$V = (\hbar/2e) d\theta/dt, \quad (3)$$

provided we define

$$\theta = (c/2I_M) \int_{b_i}^{t_i} \vec{dr} \cdot \vec{b}_M, \quad (4)$$

where the line integral is taken over the path C shown in figure 3.

Fig. 3 : Sketch showing integration surfaces.

Note that $\theta=0$ when the vortex first enters the left side of the specimen and, by Ampere's law, that $\theta=2\pi$ when it exits at the right side, provided it

passes between a and b. Thus θ can be regarded as the gaugeinvariant phase difference of the order parameter of vortex i between the points a and b. Note, however, that θ depends not only upon the relative positions of the top and bottom of vortex i and the contacts a and b, but also upon the shape of the specimen and the spatial configuration of the measuring circuit leads as they rise from a and b up to the voltmeter terminals A and B.

The above treatment is easily applied to the case of a moving flux tube of flux $\phi = N\phi_0$ in a type-I superconductor, the result being the same as eq. (3) except that the right-hand side then includes an extra factor N .

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