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CURRENT-DRIVEN TEXTURAL TRANSITION OF $^3\text{He-A}^\dagger$

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Résumé.- En utilisant une méthode variationnelle, nous étudions la transition hydrodynamique de la texture de $^3\text{He-A}$ dans une géométrie planaire pour des températures proches de T_c et une épaisseur de l'échantillon $\gg 10^{-3}\text{cm}$, sans nous limiter à des textures bidimensionnelles. On trouve que la transition est du deuxième ordre mais très proche d'être du premier lorsqu'on varie le courant.

Abstract.- Using a variational approach, we study the current-driven textural transition of $^3\text{He-A}$ in a slab geometry, for temperatures near the critical, and slab width $\gg 10^{-3}\text{cm}$, without restricting ourselves to only planar textures. The transition is found to be 2nd order, but very close to being first order, as current is varied.

Recently, Fetter has shown that the uniform texture of $^3\text{He-A}$ in a slab has a hydrodynamic instability at a critical wave number Q_c /1/. His assumption of a planar texture for $Q > Q_c$, however, has since been realized to be invalid because the exact equilibrium conditions indicate that a flow can exert a torque to rotate the orbital axis $\hat{l}$ out of a planar configuration /2/. In this paper, therefore, we reexamine this instability without the restriction to planar textures, and also attempt to establish the order of the transition that occurs at Q_c .

The orbital part of the order parameter for $^3\text{He-A}$ is $\Delta_0(\hat{m} + i\hat{n})$ where $\hat{m}, \hat{n}(\hat{l})$ and $\hat{l} = \hat{m} \times \hat{n}$ are a triad of three unit vectors /2/. Here it is convenient to let /3/ :

$$\begin{aligned}\hat{m} + i\hat{n} &= e^{iQy} [A_0 \hat{x} + A_+ (\hat{y} + i\hat{z})/\sqrt{2} + A_- (\hat{y} - i\hat{z})/\sqrt{2}] \\ A_0 &= \sin\chi \exp(iS_1), \\ A_\pm &= (1/\sqrt{2})(\cos\chi \pm 1) \exp[i(S_1 \pm S_2)]\end{aligned}\quad (1)$$

where the slab boundaries are assumed to be located at $x = 0$ and w , and the flow is taken to be along y . This representation gives : $l_x = \cos\chi$, $l_y = -\sin\chi \cos S_2$, $l_z = \sin\chi \sin S_2$. The gradient energy density near T_c , after assuming that $S_{1,2}$ and χ depend only on x , reduces to :

$$\begin{aligned}f_G &= \Delta_0^{-2} \{ (\Gamma_{13} \sin^2\chi + 2\Gamma_2) S_{1x}^2 \\ &\quad + [\Gamma_{13} (\sin^2 S_2 + \cos^2\chi \cos^2 S_2) + 2\Gamma_2] Q^2 \\ &\quad + 2\Gamma_{13} Q \sin\chi \cos S_2 (\cos\chi S_{1x} + S_{2x}) \\ &\quad - 2\Gamma_1 Q (\sin\chi \sin S_2)_x + 4\Gamma_2 \cos\chi S_{1x} S_{2x} \\ &\quad + (\Gamma_{13} + 3\Gamma_2) \chi_x^2 + [\Gamma_{13} \sin^4\chi + \Gamma_2 (2 + \sin^2\chi)] S_{2x}^2 \}\end{aligned}\quad (2)$$

where the parameters $\Gamma_{13} \equiv \Gamma_1 + \Gamma_3$ and Γ_2 have been defined in reference /4/, and we have assumed dipole-locking of the spin axis $\hat{d}$ with $\hat{l}$, thereby restricting ourselves to $w \gg 10^{-3}\text{cm}$. An expression for the current density along $\hat{y}$ may be obtained from /1/ : $j_y = (2m/W) (\partial f_G / \partial Q)$. Minimizing $F_G \equiv \int_G d^3x$ with respect to S_1 gives a current conservation equation and boundary condition (along $\hat{x}$) which may be integrated once to allow S_{1x} to be eliminated in terms of Q , S_2 , and χ . If the resulting $F_G[\chi, S_2]$ is further minimized, one obtains two coupled non-linear differential equations which must be solved numerically. [The boundary condition being $\chi = S_{2x} = 0$ at $x = 0$ and w , where $S_{2x} = 0$ is required to prevent unphysical divergences.] While this numerical solution is currently being attempted, and results will be presented, below we wish to discuss the results of a semi-quantitative study, based on a variational approach : Approximating the trigonometric functions of χ and S_2 in F_G by their leading one-or two-term expansions, depending on an order analysis which ensures that all terms of 4th order or lower, and some of the terms of 6th order and higher, are included, we then minimize $F_G[\chi, S_2]$ using the trial functions $\chi = \alpha \sin(\pi x/w)$, $S_2 = \beta \cos(\pi x/w)$, treating α and β as variational parameters. It is found that the instability is at the same Q_c as found by Fetter /1/ (i.e., $Q_c = \sqrt{5}\pi/2w$ after assuming $\Gamma_1 = \Gamma_2 = \Gamma_3$) but the inhomogeneous texture at $Q > Q_c$ is distinctly non-planar. For $(Q - Q_c) \ll Q_c$ when α and β are small, the analysis may be done analytically by a systematic perturbation approach. Introducing the normalized total gradient energy and current :

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$$\bar{F}_G \equiv (w\Delta_0^2/2\pi^2\Gamma_2)_0 \int_0^w f_G dx, \quad \bar{I} \equiv (w\Delta_0^2/8\pi m\Gamma_2)_0 \int_0^w j_y dx$$

we find : $\beta = (3/5)q\alpha$, $\alpha^2 = (8/11)(q^2 - 5/4)$ where $q \equiv Qw/\pi$, and $\bar{F}_G = 2q^2 - (4/11)(q^2 - 5/4)^2$, $\bar{I} = 2q[1 - (4/11)(q^2 - 5/4)]$. This should be compared with the values for $Q < Q_c$: $\alpha = \beta = 0$, $\bar{F}_G = 2q^2$ and $\bar{I} = 2q$, corresponding to a uniform texture with $\mathcal{E} = \hat{x}$. The slope $\partial\bar{I}/\partial q$ is seen to change from 2 for $Q < Q_c$, to $2/11$ for $Q > Q_c$. Since $\partial\bar{I}/\partial q > 0$ above Q_c , the textural phase transition as predicted by the present approximate study is a second-order transition with no hysteresis as current is varied, but the smallness of $2/11$ suggests that a more precise calculation using numerical methods might predict a first order transition at Q_c (i.e., with $\partial\bar{I}/\partial q < 0$ just above Q_c). This interesting possibility is currently being explored. For larger values of $Q - Q_c$ we have minimized our $\bar{F}_G(\alpha, \beta)$ by numerical method. In figure 1 we have plotted the normalized gradient energy $\bar{F}_G/2q^2$, and the normalized current $\bar{I}$, as function of q . The dotted lines indicate what would have been obtained if planar approximation were used (i.e. if $\beta = 0$ is assumed).

Fig. 1 : The normalized gradient energy $\bar{F}_G/2q^2$, and total current $\bar{I}$, are plotted as functions of $q \equiv Qw/\pi$ (solid lines). The dotted lines indicate the corresponding results if planar approximation were used.

In figure 2 we have plotted the values of α^2 and β^2 which minimize $\bar{F}_G$, as function of q (solid lines), as well as the value of α^2 which minimizes $\bar{F}_G|_{\beta=0}$ (the dotted line). We note also that for $q \gg 1$ (when our approximation breaks down) we expect $\bar{F}_G \rightarrow q^2$ and $\bar{I} \rightarrow q$ corresponding to a uniform texture with $\mathcal{E} = \hat{y}$, except within surface regions of thickness $\sim Q^{-1} \ll w/5$. In conclusion, we point out that the stability of the new inhomogeneous current-carrying state for $q^2 > 5/4$ has yet to be established, as is done in reference /5/ for $q \gg 1$, in order to confirm the textural transition

at Q_c .

Fig. 2 : The squares of the variational parameters α and β which minimize $\bar{F}_G$, are plotted as functions of q (solid lines). The dotted line is the square of the value of α which minimizes $\bar{F}_G|_{\beta=0}$

It is hoped that our current attempt for the exact solution of this problem by numerical method can be extended to include such a stability analysis.

References

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