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HEAVY ION POTENTIALS: A CRITICAL REVIEW OF VARIOUS APPROACHES

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Résumé.— Des méthodes semi classiques démontrent explicitement que l'amplitude de la diffusion élastique de particules alpha peut avoir des contributions significatives des ondes pénétrant à l'intérieur du noyau cible. Au contraire, la diffusion élastique d'ions lourds est principalement un phénomène de surface. Des méthodes de convolution donnent une bonne description de la forme et même de la grandeur de la partie réelle du potentiel optique. Ces méthodes n'incluent pas les effets d'échanges et de polarisation et des calculs microscopiques indiquent que les corrections dues à de tels effets peuvent être importantes.

Abstract: Semi-classical methods demonstrate explicitly that the elastic amplitude for α -particle scattering can have significant contributions from waves which penetrate the target nucleus. In contrast, heavy-ion elastic scattering is mainly a surface phenomenon. Folding procedures give a good description of the shape and even the magnitude of the real part of the optical potential. These procedures do not include some exchange and polarisation effects and microscopic calculations indicate that corrections from such effects can be large.

INTRODUCTION

Before discussing different approaches for calculating heavy-ion potentials, it is useful to understand the characteristics of a potential which are relevant for an accurate description of experimental data. For this reason in the first part of my talk I will discuss the qualitative relations between optical potentials, partial wave amplitudes and elastic scattering cross-sections. This question has been examined by many authors and Satchler [1] in particular has made an empirical investigation of families of optical potentials which fit experimental data and has noted their common features. From these studies he concludes that the data determine the strength of the real and imaginary potentials at the strong absorption radius. This question will be approached from a different point of view in this lecture, the aim being to use semi-classical ideas developed by Malfliet et al. [2], Knoll and Schaeffer [3] and others to see directly which properties of a

potential influence scattering amplitudes calculated from it. I will also compare and contrast α -scattering with heavy-ion scattering.

The second part of this talk is concerned with different methods for calculating optical potentials for heavy-ion collisions. Folding models and theories based on energy density formalism or on Hartree-Fock have many features in common, but give rather different results. Folding models tend to overestimate the heavy-ion real potential near the strong absorption radius unless there is a free parameter which can be adjusted to give the correct result. Energy density approaches, on the other hand, tend to underestimate the potential at this radius. It is interesting to try to understand the reasons for these differences. Another approach leads to the idea of proximity forces which have been discussed in a number of recent works [4]. An interaction potential in the proximity theory is a product of a simple geometrical factor and a universal function of

separation of the surfaces of the interacting nuclei. The theory is so general that it should include folding approaches and energy density approaches as special cases.

The interaction energy in an energy density formalism is essentially the energy surface of the generator coordinate approach to a microscopic theory of heavy-ion interactions. This formalism leads to a shallow energy-dependent optical potential [5,6]. In the resonating group approach exchange effects lead to a highly non-local energy-dependent potential, but there are indications [7] that this non-local interaction can be replaced by a deep almost local potential. Pauli exchange effects are taken into account by requiring that the wave function of relative motion of the two nuclei should be orthogonal to certain forbidden states [7] of relative motion.

The elastic cross-section for heavy-ion scattering is determined by phase shifts which can in turn be calculated from an interaction potential. An alternative approach to parameterising a potential is to parameterise phase shifts directly. An advantage of a potential description is that a potential determined from elastic scattering can be used in calculations of reaction cross-sections using DWBA or coupled-channel methods. There are, however, approximate formulations of reaction theories in which the elastic scattering potential appears only through the elastic phase shifts [8]. If these formulations are successful one can ask if the potential description of interactions between nuclei is useful or necessary.

POTENTIALS AND SCATTERING AMPLITUDES

This section will be devoted to a discussion of qualitative relations between optical potentials, partial wave amplitudes and elastic scattering cross-sections. The semi-classical methods applied to nuclear scattering problems by Malfliet et al. [2], Knoll and Schaeffer [3] and others [9] give expressions which localise the regions of the optical potential which determine various characteristics of scattering amplitudes. These relations can be understood best by studying some examples. I have chosen examples from α -scattering and heavy-ion scattering [9] which illustrate a range of different effects.

(α , ^{16}O) scattering, $E_{\text{lab}} = 104$ MeV: The angular distribution calculated with Saxon-Woods potential parameters $V_0 = 82$ MeV, $W_0 = 29.7$ MeV, $R_0 = 3.53$ fm and $a = 0.72$ fm is shown in fig.1.

Optical parameters were taken from Harakeh et al. [10] and give a good fit to the experimental data. The angular distribution shows diffraction structure at forward angles and has a pronounced bump for $\theta \sim 50^\circ$ which is characteristic of the nuclear rainbow scattering discussed by Goldberg and Smith [11].

In this example, the grazing or strong absorption angular momentum, defined as the value of ℓ for which the transmission coefficient $T_\ell = \frac{1}{2}$ is $\ell_g = 18$ and fig.2 shows that the real part of the total potential $V(r)$ which is the sum of the nuclear $V_n(r)$, Coulomb $V_c(r)$ and centrifugal potentials $V_\ell(r)$ has no barrier for angular momenta near ℓ_g . This is a case where the simplest version of WKB can be used to calculate the nuclear scattering phase shift

$$\delta = \sqrt{\frac{2\mu}{\hbar^2}} \left[\int_{r_n}^R \{E - V_n - V_c - V_\ell\}^{\frac{1}{2}} dr - \int_{r_c}^R \{E - V_c - V_\ell\}^{\frac{1}{2}} dr \right] \quad (1)$$

In eq.(1) r_n and r_c are the nuclear and Coulomb turning points, i.e. they are zeros of the integrands of the first and second integrals in eq.(1), and R is any point outside the range of the nuclear interaction $V_n(r)$. The nuclear turning point r_n is complex because the optical potential $V_n(r)$ is

Fig.1. Cross-section σ for (α , ^{16}O) scattering ($E_{\text{lab}} = 104$ MeV) from a quantal optical model calculation. σ_a is calculated from the WKB amplitude eq.(1).

complex. The variation of r_n with ℓ is shown in fig.3. Also shown are trajectories of other zeros of $E - V(r)$ which are too far from the real r -axis to make any significant contribution to the scattering amplitude. The crosses in fig.3 indicate poles of the Saxon-Woods potential which lie at the points $r = R_0 \pm i a$. In fig.1 the dashed line is

Fig.2. $\text{Re}V(r)$ for $(\alpha-^{16}\text{O})$ scattering for three angular momentum values. Centre-of-mass energy is $E = 83$ MeV.

the WKB cross-section σ_a . It is very similar to the quantal cross-section σ_q obtained from a numerical solution of the Schrödinger equation using a standard optical model code. Fig. 4 shows the semi-classical deflection function $\theta(\ell) = 2 \frac{d(\text{Re}\delta)}{d\ell}$ and $|\eta_\ell|^2$ where $\eta_\ell = \exp(2i\delta(\ell))$ is the scattering amplitude. The graph of $|\eta_\ell|^2$ indicates that the absorption increases smoothly as ℓ decreases. It is due to the increasing effect of the absorptive potential $\text{Im} V_n(r)$ as the distance of closest approach decreases. The nuclear rainbow angle is $\theta_{nr} \approx -60^\circ$. Partial waves with $7 \leq \ell \leq 17$

Fig.3. Complex turning points for $(\alpha-^{16}\text{O})$ scattering.

scatter into the angular region $20^\circ \rightarrow 60^\circ$ and produce the rainbow peak in the cross-section. The absorption is strong for these angular momenta but is weak enough to allow a peak in the cross-section. The height of the peak is very sensitive to W_0 . As expected, the nuclear rainbow angle corresponds to a classical turning point with $r_a \approx R_0$ (cf. Fig. 3) and is due to the inflection in the nuclear potential at $r = R_0$.

($\alpha, ^{40}\text{Ca}$) scattering, $E_{\text{lab}} = 29$ MeV: Figs. 5-8 illustrate some results calculated with Saxon-Woods parameters $V_0 = 183.7$ MeV, $W_0 = 16.6$ MeV, $R_0 = 4.925$ fm, $r_0 = 0.564$ fm. This is an adaptation of a potential given in ref. [12] which fits the experimental elastic scattering at forward and backward angles, but not for intermediate angles. The potential gives a barrier in $V(r)$ (fig. 5) for angular

Fig.4. The semi-classical deflection function $\theta(\ell)$ and the reflection coefficient $|\eta_\ell|^2$ for $(\alpha-^{16}\text{O})$ scattering.

momenta near the grazing momentum $\ell_g \approx 13$, and it can be shown that the scattering amplitude is a sum of two parts [9]

$$f(\theta) = f_B(\theta) + f_I(\theta) \quad (2)$$

The first part $f_B(\theta)$ corresponds to a wave reflected at the potential barrier and has its main contribution from partial waves with $\ell \geq \ell_g$. The second part $f_I(\theta)$ corresponds to a wave which penetrates into the interior of the target and is reflected from the interior barrier. This part has its main

contribution for low partial waves $\ell < \ell_g$. The semi-classical derivation of eq.(2) in ref. [9] contains some barrier penetration corrections which are omitted in ref.[3]. The pattern of complex turning points for the $(\alpha - {}^{40}\text{Ca})$ scattering is shown in fig. 6. The barrier contribution $f_B(\theta)$ to the scattering amplitude comes from a reflection at the external turning point r_1 while $f_I(\theta)$ corresponds to a reflection from the internal turning point r_3 . There is no reflection from the second turning point r_2 in the upper half of the complex r -plane; but this turning point does affect the barrier penetration corrections. If r_2 is near r_1 then the barrier penetration corrections are important.

Fig.7 gives the contributions $\sigma_B(\theta) = |f_B(\theta)|^2$ and $\sigma_I(\theta) = |f_I(\theta)|^2$ to the total cross-section $\sigma(\theta)$.

Fig.5. Real part of $V(r)$ for $(\alpha, {}^{40}\text{Ca})$ with $\ell = 15$.

The barrier amplitude $f_B(\theta)$ makes the major contribution for small scattering angles while $f_I(\theta)$ gives the dominant contribution at backward angles. The cross-section for intermediate angles is a

Fig.6. Trajectories for the turning points r_1 , r_2 , r_3 for the $(\alpha - {}^{40}\text{Ca})$ potential. The dashed curves are trajectories for the real potential only. Numbers indicated angular momentum values.

result of complicated interference effects between $f_B(\theta)$ and $f_I(\theta)$. The magnitude of the partial wave amplitude for $(\alpha - {}^{40}\text{Ca})$ scattering is shown in fig.8. Both η_B and η_I vary smoothly with ℓ . The structure in $|\eta| = |\eta_B + \eta_I|$ for $\ell \approx 10$ is due to interference between η_B and η_I .

Fig.7. Cross-section for $(\alpha - {}^{40}\text{Ca})$ scattering for the potential parameters in the text. The contributions $\sigma_B(\theta)$ from the barrier wave and $\sigma_I(\theta)$ from the internal wave are also indicated.

Fig.8. Values of the magnitude of the partial wave amplitude $\eta = \eta_B + \eta_I$.

How do these results change for scattering processes involving heavy nuclei? There are several major changes, all of which reflect the diminished importance of the interior region of the optical potential for elastic heavy-ion scattering.

The first change concerns the nuclear rainbow effect which was a prominent feature of the (α , ^{16}O) example discussed earlier in this section and has been seen in many other elastic α -scattering experiments. There are, however, no cases of the nuclear rainbow effect known when heavier projectiles are scattered elastically. A possible reason is that the imaginary part of the optical potential is so large in heavy-ion potentials that only the extreme tail of the optical potential contributes to elastic scattering. Alternatively, Knoll and Schaeffer [3] have pointed out that the nuclear rainbow can occur at smaller angles than the Coulomb one and be hidden by the Rutherford scattering.

Although the nuclear rainbow effect seems not to appear in elastic heavy-ion scattering, there are indications that it may show up in deep inelastic scattering or quasi-fission as a focusing of the reaction products into a certain angular region, for example in the reactions initiated by Kr + Bi at 525 MeV [13].

In cases where the real part of the nuclear potential is strong enough to give a barrier in the total potential, the scattering amplitude can be divided as in eq.(3). The amplitude due to the internal wave $f_I(\theta)$ can make significant contributions in elastic α -scattering, but in heavy-ion systems it is important only for light ions at low energies when resonance effects occur. Normally, the optical model absorption is so strong that the internal wave which penetrates or passes over the barrier is completely absorbed so that $f_I(\theta) \approx 0$ and only the barrier region contributes to the scattering, $f(\theta) = f_B(\theta)$.

The example of (^{16}O , ^{16}O) scattering at 50 MeV studied by Knoll and Schaeffer [3] is a typical example for light-ions at intermediate energies or heavier-ions at low energies. They use a Saxon-Woods potential with parameters $V_0 = 50$ MeV, $W_0 = 20$ MeV, $R_0 = 6.05$ fm and $a = 0.6$ fm. The real part of the nuclear potential is strong enough to give a barrier in the total potential and the turning point pattern resembles fig.6. However, the absorption is strong and the wave $f_I(\theta)$ reflected at the internal turning point r_3 which was important

Fig.9. Cross-section for (^{16}O , ^{16}O) elastic scattering without symmetrization. The barrier penetration correction factor N is included in σ_B and neglected in σ_{BO} .

in the (α , ^{40}Ca) example is totally absorbed in this case. The semi-classical theory gives the partial wave amplitude as [9]

$$\eta = \exp(2i\delta_1)/N(i\epsilon) \quad (3)$$

where δ_1 is to be calculated from eq.(1) with r_n as the most external turning point and $N(i\epsilon)$ is a correction for barrier penetration effects. The quantity ϵ is related to the action integral

$$S(r_2, r_1) = i\pi\epsilon = \int_{r_2}^{r_1} \left\{ \frac{2\mu}{\hbar^2} (E - V(r)) \right\}^{1/2} dr \quad (4)$$

between the turning points r_2 and r_1 on each side of the barrier. The barrier penetration correction factor N is significantly different from unity only if $|\epsilon| \leq 0.5$. A simple estimate gives

$$\min |\epsilon| \approx |\text{Im } V_n(r_B)|/\hbar\omega_B \quad (5)$$

where r_B is the barrier position and $\omega_B \approx \sqrt{V''_B/\mu}$ is the oscillator frequency of a particle with mass μ equal to the reduced mass of the two nuclei moving in a potential obtained by inverting the barrier. In the present example $\min |\epsilon| \approx 0.25$ and the barrier penetration correction makes some difference to the angular distribution only at large angles (fig.9).

Even when the real potential is strong enough to give a barrier in the total potential, the absorption in the barrier region can be so large that most of the incident flux is absorbed by the imaginary part of $V(r)$ outside the barrier. This situation occurs when $\min |\epsilon| \geq 0.5$. Then barrier penetration effects are not important, the elastic scattering is determined by the potential outside the barrier and the barrier height itself is not relevant. A deep real potential with strong absorption in the barrier region is indistinguishable from a shallow real potential with no barrier provided the two potentials are similar in the strong absorption region.

In this surface absorption limit the nuclear phase can be calculated with the simple one-turning point WKB formula (1). Expanding eq.(1) to first order in $V_n(r)$ gives the classical perturbation result and even this is quite accurate [3]

$$2\delta(d) = \sqrt{\left(\frac{2\mu}{k^2}\right)} \int_d^\infty \frac{V_n(r)}{\{E - V_c - V_\ell\}^{1/2}} dr \quad (6)$$

$$= \frac{1}{k v_0} \left(\frac{2d}{1-n/kd} \right)^{1/2} \int_0^\infty V_n(r+u^2) du \quad (7)$$

In eqs.(6) and (7) d is the distance of closest approach for a Rutherford orbit with angular momentum $\lambda = \ell + \frac{1}{2}$, v_0 is the asymptotic relative velocity, $k = \mu v_0/\hbar$ is the wave number and $n = z_1 z_2 e^2 / \hbar v_0$ the Sommerfeld parameter. They are related by $(\lambda/d)^2 = k^2 - 2k n/d$. For an exponential potential

$V_n(r) = - (V_0 + i W_0) \exp - (r-R_0)/a$
eq.(7) gives

$$2\delta(d) = - V_n(d) \frac{1}{k v_0} \left(\frac{2\pi a d}{1-n/kd} \right)^{1/2} \quad (8)$$

The strong absorption radius $d_{1/2}$ [1] is the value of d for which the transmission coefficient $T_{\ell} = \frac{1}{2}$. This corresponds to $\text{Im}(2\delta) = 0.346$. Then eq.(8) can be used to estimate the $d_{1/2}$ if the imaginary part of $V_n(d)$ is known. The elastic scattering of ^{12}C by ^{208}Pb at 96 MeV is an example where

Fig.10. Cross-section for (^{12}C , ^{208}Pb) elastic scattering at 96 MeV [14]

surface absorption is strong. The angular distribution is fitted well [14] by a Saxon-Woods potential with $V_0 = 40$ MeV, $W_0 = 25$ MeV, $R_0 = 10.31$ fm, $a = 0.56$ fm. Solving eq.(8) gives $d_{1/2} = 12.2$ fm which is the same as the value obtained by calculating from a full optical model calculation.

Fig.10 gives a sketch of $\sigma(\theta)/\sigma_R(\theta)$ for this example. The grazing angular momentum is $\ell_g = 52$, $\arg \eta = 2\delta_{\ell_g} \sim 32^\circ$ for $\ell = \ell_g$, and the transmission coefficient falls from 0.9 to 0.1 in a range $\Delta \ell \approx 12$,

ANGULAR DISTRIBUTIONS

There are several different approaches to the problem of calculating angular distributions by semi-classical methods. Their common starting point is the Poisson summation formula which gives [15,16]

$$f(\theta) = \frac{1}{ik} \frac{1}{\sqrt{(2\pi \sin \theta)}} e^{i\pi/4} \sum_{m=-\infty}^{\infty} (-1)^m \{I_m^-(\theta) - iI_m^+(\theta)\} \quad (9)$$

$$\text{with } I_m^\pm(\theta) = \int_0^\infty \lambda \exp [i\lambda(\pm\theta + 2m\pi)] \eta(\lambda - \frac{1}{2}) d\lambda \quad (10)$$

This is a useful result because generally only a few terms in the summation (9) are important and the different terms $I_m^\pm(\theta)$ have a pictorial association with classical trajectories. Some common combinations are illustrated in fig.11 taken from ref. [15]. The cross-sections for (^{16}O , ^{16}O) scattering at 50 MeV (fig.9) for $\theta < 20^\circ$ and (^{12}C , ^{208}Pb) scattering at 96 MeV (fig.10) for $\theta < 45^\circ$ have contributions mainly from $I_0^-(\theta)$ (fig.11). This term has a dominant Coulomb contribution $I_0^-(C)$ and a nuclear contribution $I_0^-(N)$ from the

Fig.11. Classical trajectories corresponding to (a) forward angle oscillations, (b) intermediate angle oscillations and (c) backward angle oscillations.

diffractive and refractive effects of the nuclear surface. These contributions interfere to give a Fresnel type diffraction pattern. For $\theta > 30^\circ$, I_0^- and I_0^+ contribute. Both terms vary smoothly with angle and interfere to give the diffraction structure shown in fig.9. Backward angle oscillations are produced by interference of I_0^+ and I_{+1}^- .

Different semi-classical approaches vary in the methods used to calculate the integrals $I_m^+(\theta)$ and in the interpretation of results. Methods are discussed in many references. In this lecture I will make a few remarks about interpretation. The gross features of elastic angular distributions are determined by two parameters, the Coulomb parameter n and the grazing angular momentum l_g . The ratio of the elastic cross-section to the Rutherford cross-section resembles a Fresnel diffraction pattern when $n \gg 1$ with $\sigma/\sigma_R = \frac{1}{4}$ for the classical grazing angle. When $n \lesssim 1$ the angular distribution is like a Fraunhofer diffraction pattern. The refractive effects of the nuclear potential make subtle but important modifications to these simple patterns.

There are two interpretations of angular distributions in the limit $n \gg 1$ (eg. ^{16}O , ^{208}Pb) in fig. 10). In one view [16] elastic and inelastic heavy-ion collisions are dominated by diffractive scattering of the Fresnel type. There are modifications of the pattern because the cut-off in the scattering amplitude near the grazing angular momentum is smooth rather than sharp, and because there is a real nuclear phase. These modifications produce a perturbation of the basic Fresnel pattern. In this view the scattering is mainly influenced by

the imaginary part of the optical potential. This approach is associated with a method for calculating the scattering amplitude in which the leading Fresnel amplitude is modified by corrections depending on the detailed angular momentum dependence of the partial wave amplitudes. An alternative interpretation is that the forward angle oscillations in a case like fig.10 are produced by interference between two branches of a classical deflection function near the Coulomb rainbow angle [17,18]. The integral (10) is calculated by the stationary phase method [18] or the saddle point method [3] and the interfering classical paths correspond to points of stationary phase or to saddles in one of these methods. Advocates of this method argue [19] that the angular distribution for elastic scattering is governed by the Coulomb rainbow angle in the classical deflection function and that the angular distribution is mainly expected to depend on the real part of the nuclear potential with perturbations due to the imaginary part.

Empirical investigations by Satchler [1] and collaborators [14] seem to indicate that the long range parts of both the real and the imaginary parts of the optical potential are important for fitting elastic scattering data. In examples like (C, ^{208}Pb) the strong absorption radius is most sensitive to the imaginary part of the potential, but the real part of the optical potential at the strong absorption radius is also rather well determined. This second condition on the real potential could be almost equivalent to a requirement that the real potential should give a correct value to the Coulomb rainbow angle [19].

CALCULATING HEAVY-ION POTENTIALS

The discussion in the preceding sections shows that the elastic scattering of heavy-ions is sensitive only to the extreme tail of an internuclear interaction potential, except possibly in some cases involving lighter ions at low energies. A number of approaches have been used for calculating the real part of the potential. Usually the imaginary part is treated phenomenologically, although it seems to be at least as important as the real part of the potential [20].

Folding models have been discussed in a recent review by Satchler [1]. Here I shall make some brief remarks about them and comment on their relation with other microscopic calculations which

include exchange and polarization corrections. In the single folding model one takes an optical potential $U_N(r)$ for scattering of a nucleon from one nucleus and folds this with the density distribution of the other nucleus. The parameters of $U_N(r)$ are chosen empirically from fits to nucleon-nucleus scattering data. In most calculations this model overestimates the heavy-ion potential near the strong absorption radius by a factor of two or more. However, a recent investigation by Rickertsen and Satchler [21] shows that the simple folding model can give better results if a shape, which differs from Saxon-Woods, is used for $U_N(r)$.

The double folding model represents the heavy-ion potential in the form

$$U(r) = \iint \rho_1(r_1) \rho_2(r_2) v(r_{12}) d^3 r_1 d^3 r_2 \quad (11)$$

where $r_{12} = r + r_2 - r_1$ and v is an effective nucleon-nucleon interaction. This model gives a good account of elastic scattering data provided the effective interaction v has a reasonably short range and its strength is taken as a free parameter. Effective interactions v with a range which is too big are excluded because they give heavy-ion potentials which disagree with experimental data. For example, an effective interaction derived from the long range part of the Hamada-Johnston interaction does not give a satisfactory potential in the double folding model [14]. There, the fault lies with the long range one-pion exchange component of the interaction. If the effective interaction has a reasonably short range, the $U(r)$ can be expanded as

$$U = A \left[\int \rho_1 \rho_2 d^3 r - \frac{1}{6} \langle r^2 \rangle \int \nabla \rho_1 \cdot \nabla \rho_2 d^3 r \right] + \dots \quad (12)$$

where A is the volume integral of the effective interaction and $\langle r^2 \rangle$ is its mean square radius. This formula shows that $\langle r^2 \rangle$ is a convenient measure of the range of an effective interaction for the purposes of the double folding model. A value $\langle r^2 \rangle = 11.8 \text{ fm}^2$ appropriate for a one-pion exchange potential seems to be too large to fit the heavy-ion data. An effective interaction derived from realistic g -matrix elements by Love and Satchler [22] has $\langle r^2 \rangle = 3.11 \text{ fm}^2$ and even fits the magnitude of $U(r)$ to within 10%.

The single and double folding models neglect certain exchange effects. They also neglect contributions to the internuclear potential caused by polarization of the nuclei by the nuclear and Coulomb fields. Some idea of the magnitudes of these effects can be formed by studying models

Fig.12. $(^{16}\text{O}, ^{16}\text{O})$ potential from ref.[23]. The various curves are explained in the text.

with simple effective interactions but with effects of exchange and polarization included. An example of such a calculation is the study of the $(^{16}\text{O}, ^{16}\text{O})$ interaction using Skyrme's effective interaction and its self-consistent extension [23]. In this model the total energy of the $(^{16}\text{O}, ^{16}\text{O})$ system at a fixed separation R is identified with the heavy-ion potential. The total energy is computed using completely antisymmetrized wave-functions so that effects of exchange and the Pauli exclusion principle are included exactly. The parameters of the Skyrme interaction were fitted to nuclear binding energies so the model should give a reasonable estimate of the shape and strength of the internuclear interaction. The results illustrated in fig.12 are taken from a review talk by Mosel [23]. The curve marked PES (potential energy surface) gives the total interaction energy of the two nuclei including all anti-symmetry effects. Comparison with the empirical potential WS shows that it is too weak in the surface region ($R \sim 8 \text{ fm}$) by about a factor of two. The curve marked F is obtained by folding the Skyrme interaction with

the unperturbed wave functions of the isolated $^{16}_0$ nuclei neglecting anti-symmetry, while the curve FAS includes some part of the exchange corrections. The difference between the curves FAS and PES is mainly a kinetic energy effect. When the two nuclei overlap the Pauli principle forces some nucleons into higher orbits increasing the mutual kinetic energy and making the mutual attraction less strong. The curve SCPES is the result of a self-consistent calculation. The interacting nuclei were allowed to change their wave-functions to minimize the total energy. Such a self-consistent procedure is a way of calculating static polarization effects. This final curve is in reasonable agreement with the empirical curve. This calculation shows that both exchange and polarization effects can be large; each can change the interaction strength by a factor of two. In the $(^{16}_0, ^{16}_0)$ example they almost cancel, but there is no guarantee that this will happen for other pairs of nuclei. Should polarization effects be included in the heavy-ion potential? The answer to this question depends on the time required for polarization compared with the collision time.

A rather different approach to heavy-ion potentials is based on the idea of proximity forces developed by the Berkeley group [4] which generalizes some earlier ideas of Wilczynski [26]. In this theory the interaction energy of two nuclei is given by a formula

$$U = \int_{\mathcal{S}} e(D) d\sigma$$

The integral is taken over a surface $\mathcal{S}$ between the interacting nuclei, D is the separation distance between the surfaces of the two nuclei measured perpendicular to the surface $\mathcal{S}$, and $e(D)$ is an interaction energy per unit area of two parallel nuclear surfaces separated by a distance D . Approximate calculation of this integral for interacting spherical nuclei with radii R_1 and R_2 gives

$$U(s) = 2\pi \bar{R} \int_s^\infty e(D) d(D) \quad (13)$$

where $s = r - (R_1 + R_2)$ and the average radius $\bar{R} = R_1 R_2 / (R_1 + R_2)$. This corresponds to a force

$$F(s) = -2\pi \bar{R} e(s). \quad (14)$$

A very nice feature of eq.(13) is that the interaction energy $U(s)$ of two nuclei with surfaces separated by a distance s is given in terms of a universal function $e(s)$. If $e(s)$ is known or can be calculated for one pair of nuclei then one immediately has information about other pairs of nuclei. The idea of the proximity formula is based

on the liquid drop model, but the formula is very general and applies also to other theories. For example, suppose the interaction energy is represented by the folding formula

$$U = A \int \rho_1(r_1) \rho_2(r-r_1) d^3 r_1 \quad (15)$$

which corresponds to a double folding model with a δ -function effective interaction v . If the densities ρ_1 and ρ_2 have Saxon-Woods shapes

$$\rho_i(r) = \rho_0 / [1 + \exp(r-R_i)/a]$$

then the integral (15) can be evaluated approximately using the methods in the appendix of ref. [27] to give

$$U(s_0) \approx 2\pi A \rho_0^2 \bar{R} \int_{s_0}^\infty \frac{s ds}{\exp(s/a-1)} \quad (16)$$

where $s_0 = (r - R_1 - R_2)$ and r is the separation between the centres of the two nuclei. This has exactly the proximity form with a particular expression for $e(s)$. Eq.(16) has been derived assuming that s_0 and a are both much smaller than R_1 and R_2 .

Calculations based on the energy density approach [26] or on Skyrme's interaction with the Thomas-Fermi approximation [27] are generally consistent with the predictions of the proximity model, though there are some small deviations. For example, the outer part of the potentials in ref. [27] were fitted by Saxon-Woods potentials. According to the proximity model the surface diffuseness should be constant. The results in ref. [27] give values between $a = 0.56$ for $(^{16}_0, ^{16}_0)$ to $a = 0.66$ for (Pb, Pb). Also, the maximum force between two nuclei should be given by the formula [4].

$$F_{\max} = -2\pi \bar{R} e(0) = 4\pi \bar{R} \gamma$$

where γ is the surface energy coefficient for nuclear matter ($\gamma \sim 1 \text{ MeV/fm}^2$). Values of γ extracted from heavy-ion potentials in ref.[27] vary between 0.70 MeV/fm^2 for $(^{16}_0, ^{16}_0)$ to 0.90 MeV/fm^2 for (Pb,Pb) with most values concentrated near 0.85 MeV/fm^2 .

The proximity formula will certainly be a very useful guide for the study of heavy-ion potentials because of its simplicity and generality. There are, however, a number of reasons why it should be used with some caution. It should be accurate in cases where $s \ll R_1$ and R_2 . At the strong absorption radius for $(^{16}_0, ^{208}\text{Pb})$ $R_1 = 2.5 \text{ fm}$, $R_2 = 6.8 \text{ fm}$ while $s = 3.2 \text{ fm}$ and the above condition is not satisfied. Also there is no reason why

polarization effects should follow the proximity formula.

Bibliography

- [1] G.R. Satchler, Procs. Conf. on Reactions between Complex Nuclei, Nashville (North-Holland 1974) Vol. 2, 174; Symposium on Macroscopic Features of Heavy-Ion Collisions, Argonne, Illinois, April 1976.
- [2] R.A. Malfliet et al. Phys. Lett. 44B (1973)238. T. Koeling and R.A. Malfliet, Phys. Reports 22C (1975) 181.
- [3] J. Knoll and R. Schaeffer, Annals of Phys. 97 (1976) 307.
- [4] J. Blocki et al. Laurence Berkeley Report LBL-5014 (1976).
- [5] D.M. Brink and Fl. Stancu, Nucl. Phys. A243 (1975) 175.
- [6] P.G. Zint and U. Mosel, Phys. Lett. 56B (1975) 424.
- [7] S. Saito, Prog. Theor. Phys. 41 (1969) 705 B. Buck et al., preprint, Oxford 11/76 (1976).
- [8] S. Landowne et al., Nucl. A259 (1976) 99.
- [9] D.M. Brink and N. Takigawa, preprint, Oxford (1976).
- [10] M.N. Harakeh et al, preprint Groningen KV-63 (1976).
- [11] D.A. Goldberg et al., Phys. Rev. C10 (1974) 1362.
- [12] G. Gaul et al., Nucl. Phys. A137 (1969) 177.
- [13] H.H. Deubler and K. Dietrich, Phys. Lett. 56B (1975) 241.
- [14] J.B. Ball et al., Nucl. Phys. A252 (1975) 208.
- [15] N. Rowley and C. Marty, preprint Orsay PNO/TH 76-3.
- [16] W.F. Frahn, Heavy-Ion High-spin states and Nuclear Structure, Vol.1 (IAEA, Vienna,1975) p.157. W.E. Frahn and D.H.E. Gross, to be published. W.E. Frahn and K.E. Rehm, Conference on Nucl.Phys.with Heavy-Ions, Caen(1976)
- [17] R.A. Malfliet et al., Phys. Lett. 44B (1973)238.
- [18] R. da Silvera, Phys. Lett. 45B (1973) 211.
- [19] P.R. Christensen and A. Winther, preprint.
- [20] P.J. Moffa et al., Phys. Lett. 35 (1975) 992.
- [21] L.D. Rickertsen and G.R. Satchler, Conference on Nucl. Phys. with Heavy-Ions, Caen (1976) 5.
- [22] W.G. Love and G.R. Satchler, Conference on Nucl. Phys. with Heavy-Ions, Caen (1976) 5. H. Flocard, Phys. Lett. 49B (1974) 129.
- [23] U. Mosel, Symposium on Macroscopic Properties of Heavy-Ion Collisions, Argonne (1976).
- [24] J. Wilczynski, Phys. Lett. 47B (1973) 484.
- [25] D.M. Brink and N. Rowley, Nucl. Phys. A219 (1974) 79.
- [26] C. Ngo et al., Nucl. Phys. A252 (1975) 237.
- [27] Fl. Stancu and D.M. Brink, Nucl. Phys. to be published.