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Résumé. — Les propriétés fondamentales des ondes de gravitation, prédites par la relativité générale, sont décrites et comparées avec celles du rayonnement électromagnétique. On indique quelques résultats récents sur les processus radiatifs au voisinage des trous noirs. On souligne que quelques-uns de ces processus conduisent à des méthodes de conversion de la masse au repos en énergie de rayonnement plus efficaces que tout autre processus connu, à l'exception de l'annihilation matière-antimatière. Comme on s'attend à l'existence d'un grand nombre de trous noirs dans l'univers, ces processus de conversion peuvent être importants en astronomie. Les recherches sur les ondes de gravitation, aussi bien théoriques qu'expérimentales, doivent donc être poursuivies avec vigueur.

Abstract. — The basic properties of gravitational radiation, as predicted by general relativity, are described and compared with those of electromagnetic radiation. Some recent results on radiative processes in the vicinity of black holes are outlined. It is stressed that some of these processes lead to more efficient methods of converting rest mass into radiative energy than any other known process except matter-antimatter annihilation. Since one would expect there to be a large number of black holes in the universe, these conversion processes may be important in astronomy. Research into gravitational radiation, both theoretical and observational, should therefore be conducted vigorously.

1. **Introduction.** — Fourteen years after the foundation of the Physical Society in 1874 and nine years after the death of Clerk Maxwell, Heinrich Hertz produced and detected electromagnetic waves in the laboratory and showed that they moved with the speed of light. Twenty-seven years later Einstein predicted from general relativity the existence of gravitational waves also moving with the speed of light. Now, fifty-eight years later still, this prediction has not yet been verified. As we shall see, it would be a hopeless enterprise to attempt to produce a detectable flux of gravitational waves in the laboratory, but naturally occurring astronomical explosions might be expected to produce a flux exceeding the sensitivity of gravitational wave detectors now being built. An acceptable event rate (say, several per year) may require a further increase of sensitivity by a factor $\sim$ 100, but this would not appear to be technologically impossible to achieve. It is at least a reasonable expectation that, say, ten years from now gravitational wave astronomy will be an accepted branch of science.

We owe the realization of this remarkable prospect almost entirely to the pioneering work of one man — Professor Joseph Weber of the University of Maryland. Weber has taught us how to construct a detector of gravitational waves sensitive enough to detect a gravitational pulse containing the rest energy of 10 solar masses released at the centre of the galaxy and radiated isotropically. Such a pulse would induce a displacement $\sim 10^{-14}$ cm in the metre-long cylindrical bar which constitutes Weber's detector, this displacement being detected by piezoelectric transducers attached to the bar. As is well known, Weber has claimed to have detected such pulses, with the enormous event rate of about one per day. Independent attempts to confirm this claim by experimental groups at Glasgow, Moscow, Munich, Frascati, the Bell Telephone Laboratories and the IBM Thomas J. Watson Research Center have not led to positive results and this conflict remains to be cleared up (*).

In the meantime we can ask the question : what conventional sources of gravitational radiation would be expected and what sensitivity would be needed to detect them ? One immediately thinks of supernova explosions as a possible source. In round terms, one such explosion might involve a star of ten solar

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(*) A good account of these experiments is given by J. L. Logan (1973).

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masses and one might expect about one per cent of this mass to be converted into a gravitational wave pulse (such efficiency estimates are discussed later in this article). Thus one might be dealing with a pulse of energy $\sim 0.1\ Mc^2$, released somewhere in the galaxy. Since we are near the edge of the galaxy, a comparison with the sensitivity of Weber's existing detectors shows that an increase of sensitivity by a factor ~ 100 would enable us to detect most supernova explosions in the galaxy. However, the rate of these catastrophic events is low, perhaps only one every 30 years. To obtain an acceptable event rate we would need to be able to detect supernova explosions in the nearest large cluster of galaxies. The obvious candidate is the Virgo cluster, which is a thousand times further away than the galactic centre (10 megaparsecs instead of 10 kiloparsecs). This cluster contains over a thousand galaxies and, although not all of them may be as productive as our own galaxy, we might still expect an event rate of several per year, which would be acceptable. However, the sensitivity required to detect these events would be 10^8 times greater than Weber's sensitivity.

This might seem an alarming requirement, but in fact detectors now being constructed at the Universities of Stanford, Louisiana State, and Rome, which involve using a superconductive suspension for the metal cylinder and cooling it to well below 1 K, are hoped to achieve one per cent of this required sensitivity within perhaps five years. I know of no technological limit which would rule out a further improvement by a factor of 100 in later generations of such supercooled detectors.

In view of these prospects, it seems that physicists might be interested in a non-technical account of the main properties of gravitational waves as predicted by general relativity. The purpose of this article is to provide such an account. More detailed discussions can be found in Sciama [4], Weinberg [5], Foures-Bruhat [1] and Misner, Thorne and Wheeler [3], where extensive references to earlier work are given. It will be convenient to divide our account into three parts : *a*) propagation, *b*) absorption and *c*) emission of gravitational waves.

2. **The propagation of gravitational waves.** — It may be helpful to consider first the ways in which propagating gravitational waves are like electromagnetic waves and then the ways in which they are not. Amongst the similarities are the following :

a) The waves may be regarded as carrying information concerning changes in the source which in particular lead to a change in their Coulomb or Newtonian field (this point is particularly clear if the source begins and ends in different static configurations).

b) The waves may be regarded as carrying energy essentially to infinity, unless they are absorbed on the way. The point here is, of course, that the radiative part of the electromagnetic field decreases with distance r from the source like $1/r$, not like $1/r^2$, so that the energy flux decreases like $1/r^2$ and the total flux across the surface of a sphere is independent of the radius of the sphere. In this sense the energy may be regarded as flowing out to infinity. In the gravitational case there is a slight subtlety because the energy of a varying gravitational field cannot be localized in space. However, if the wavelength of the radiation is small compared with the length-scale of the background gravitational field, one can define a localizable energy in terms of an average over several wavelengths. This averaged energy leads to a flux that is nearly proportional to $1/r^2$, so again one is led to the concept of a flow of energy to infinity.

c) The gravitational waves move with the speed of light.

d) The gravitational waves are transverse.

e) Far from its source a gravitational wave approaches a plane wave in the same sense as for electromagnetic waves and indeed there are *exact* plane wave solutions (both purely gravitational and also gravitational-electromagnetic) of Einstein's field equations.

Amongst the differences are the following :

i) If gravitons exist (for it is still not certain that the gravitational field should be quantized) they would have spin 2, rather than the spin 1 characteristic of photons. The reason is that the gravitational potential is described by a second rank tensor, rather than a vector.

ii) The simplest type of gravitational radiation is quadrupole, rather than dipole. In the electromagnetic case the absence of monopole (spherically symmetric) radiation is a consequence of the conservation of charge. Dipole radiation is permitted because there is no conservation law for the electromagnetic dipole moment. In the gravitational case there is conservation of both mass and momentum (which is related to the mechanical dipole moment of the system) but no conservation of quadrupole moment. Thus quadrupole radiation is the simplest permitted type.

iii) The polarization properties of gravitational waves are different from those of electromagnetic waves. This difference is described in the next section on absorption.

iv) Gravitational waves in vacuo scatter one another, whereas electromagnetic waves do not. This difference arises from the fact that Maxwell's equations are linear while Einstein's equations are non-linear. This means that electromagnetic waves are electromagnetically neutral, whereas gravitational waves are themselves sources of gravitation. The physical reason for this is simply that gravitational waves carry energy and *all* kinds of energy are coupled to the gravitational field (as follows from the principle of equivalence).

3. **The absorption of gravitational waves.** — It also follows from the principle of equivalence that the absorption of gravitational waves is a somewhat different process from the absorption of electromagnetic ones. The existence of particles with different values of e/m means that a time-varying electromagnetic field acting on matter can produce relative motions in a material system which can be used to do work and so to extract energy from the electromagnetic wave. However, in the gravitational case the equivalent of e/m is the same for all particles (« all bodies fall equally fast ») and at first sight it would seem that no relative motions are set up. Fortunately the field of a gravitational wave is *non-uniform*, so when it acts on a material system the different parts of the system can move differentially. In Newtonian language we would say that the *tidal* force of the gravitational wave sets up differential motions ; in Einstein's language it would be the Riemann-Christoffel curvature tensor that is responsible. As in the electromagnetic case, these differential motions can be used to do work and so energy can be extracted from the gravitational wave, which therefore suffers absorption. Of course, gravitational wave detectors depend on the same principle. Weber's bar can be set into longitudinal oscillation by the varying tidal force of an impinging gravitational wave and it is this oscillation that he seeks to detect.

We are now in a position to specify the polarization properties of a gravitational wave. Consider four test particles lying in a plane at right angles to the direction of propagation of a linearly polarized gravitational wave. If they are at rest before the wave reaches them, then after it has passed their differential motions will be as shown in figure 1 for one polarization mode, while for the other the pattern of motions is rotated by 45°. Circular polarization would correspond to taking linear combinations of these modes. It must be emphasized that the field of differential motions of figure 1 is that applying in the neighbourhood of *any* point on the wave front. The symmetry of a plane wave is such that all these points are equivalent ; a plane wave does not single out a preferred observer on the wave front.

FIG. 1.

We now come to the fifth difference from the electromagnetic case.

v) The gravitational interaction is much weaker than the electromagnetic one. For instance, the ratio of the Newtonian to the Coulomb force between an electron and a proton (GMm/e^2) is about 10^{-39}. Thus the gravitational absorption coefficient for matter is extremely small. This may be illustrated by the absorption cross-section of the earth for 1 Hz gravitational radiation, which has been calculated to be about 10^{-3} cm^2.

4. **The emission of gravitational radiation.** — We consider first the weak field limit of Einstein's field equations, which corresponds essentially to neglecting their non-linearities. The gravitational waves may then be regarded as propagating in the Minkowski space-time of special relativity. In fact, the field equations reduce to a form identical to Maxwell's equations, except that the vector potential is replaced by a tensor potential and the charge-current vector by the energy-momentum tensor. A calculation of the gravitational power radiated by a time-dependent source proceeds as in the electromagnetic case, with due allowance being made for the extra indices. As we have seen, the main consequence of the extra indices is that the simplest allowed radiation pattern is a quadrupole one. For the same reason the radiated power depends on the rate of change of the moment of inertia I of the source, being proportional to $<(I)^2>$, where $<\ >$ indicates a time average, and I is so defined that it is zero if the system remains spherical (in effect a trace term is substracted from the usual moment of inertia).

Because the gravitational coupling constant is so small, the gravitational power radiated by typical systems behaving non-catastrophically is also very small. For example, the power radiated by the planets in orbiting the sun is only about 4×10^9 ergs/s (mainly due to Jupiter). By chance, the gravitational power radiated by the atoms of the sun itself due to their mutual collisions is of the same order. By contrast, the optical power of the sun is 10^{24} times greater. It is not surprising, therefore, that any gravitational waves that could be produced in the laboratory would be far too weak to be detectable.

Since in these radiation processes energy is transmitted to infinity, the mass of the source must consequently decrease. This mass loss provides another example of the non-linearity of Einstein's equations. In a linear theory different multipole moments would behave independently of one another. In the gravitational case, however, a changing *quadrupole* moment of the source leads to a radiation of energy, which then shows up in a reduction of the *monopole* moment (the mass). It must be emphasized that this change in mass does *not* drive the radiation (there is no monopole radiation) but is driven *by* it.

Since radiation sources in weak gravitational fields are not very interesting, we now turn to strong gravitational fields. We might expect such fields to be realized in Nature when massive stars collapse after they have reached the end point of their thermonuclear evolution (the rate of such collapses in the galaxy might be as large as one per year). In practice, the collapse is unlikely to be spherically symmetrical because of stellar rotation, magnetic fields and instabilities. As a result, the collapsing star would have a changing quadrupole moment, and so would radiate gravitational waves. We would expect most of the radiation to be emitted close to the critical radius $2\,GM/c^2$, although not so close that the gravitational red shift robs the radiation of most of its energy. (This red shift would be infinite at $2\,GM/c^2$ for a spherical star, at which radius the star would become a black hole. For a star of solar mass this critical radius is three km.) We would therefore expect most of the radiation to be emitted in a pulse whose duration is of the same order as the time taken by the radiation to move through the critical distance, that is about $2\,GM/c^3$ or $10^{-5}\,M/M_\odot$ s. The highest frequencies present would thus be of the order of $10^5\,M_\odot/M$ Hz.

We now reach the critical question : what fraction of the initial rest-mass of the star would be radiated in gravitational waves before the black hole condition is reached ? The maximum efficiency would occur if all the rest-mass were radiated away, that is, if Mc^2 of energy were radiated in a time of $2\,GM/c^3$. The resulting power would be of order c^5/G. This is the maximum power at which a system can radiate and it is important to note that it is independent of the mass of the source. We therefore have

$$\begin{aligned} L_{\max} &\sim c^5/G \\ &\sim 4 \times 10^{59}\ \text{ergs/s} \\ &\sim 2 \times 10^5\,M_\odot\,c^2/\text{s}\,. \end{aligned}$$

By this standard the efficiency of the solar system is only about 10^{-50}.

To obtain more interesting efficiencies we shall consider a number of processes involving black holes. Some of them are probably guides to the efficiencies involved in realistic collapses towards a black hole configuration, which would be very difficult to calculate directly. The simplest case to compute is that of a particle spiralling into a massive spherical (Schwarzschild) black hole, the inward motion being a consequence of gravitational radiation damping (compare pre-Bohr models of the atom). This is an easy problem if the particle is light and a single orbit approximates to a circular one (a geodesic). The reason is that in a static external gravitational field the energy associated with a geodesic is a constant of the motion. Thus all one has to do is to compute the energy associated with the last stable circular orbit before the particle plunges rapidly into the black hole, and compare it with the energy of the particle at infinity. In this way one obtains an efficiency of 5.7 %, that is, 5.7 % of the initial rest mass of the particle is radiated in gravitational waves before the particle is captured by the black hole.

A more realistic case would be that of a gaseous disc rotating around a black hole. Viscosity would lead to a transfer of angular momentum outwards in the disc, with the result that the inner parts of the disc would move towards the black hole. The gravitational potential energy released would be radiated away, in this case in electromagnetic as well as gravitational radiation, and efficiencies of 5.7 % would again be expected. Note that this is a larger fraction of the rest mass of the disc than would be released by thermonuclear reactions converting a hydrogen disc to an iron one (Fe^{56} being the most tightly bound nucleus). This thermonuclear efficiency is less than 1 %.

One can obtain still greater efficiencies if one considers *rotating* black holes. This is also more realistic since the stars of the galaxy are known to rotate. Some of the problems concerning rotating black holes can be readily solved thanks to the remarkable recent discovery that the empty space-time outside a stationary rotating black hole depends only on the mass M and the angular momentum J of the black hole. Any further details of the distribution of mass and velocity in the source do not show up in the field outside after transient gravitational waves have died out. The exact external stationary solution involving M and J is known, and is called the Kerr solution. In gravitational units ($G = c = 1$) the Kerr solution leads to the black hole condition, where radiation inside a critical region is trapped and cannot escape to the outside world, only if $J \leqslant M^2$. Many relativists believe that a realistic collapse to a singularity always leads to a black hole, but this has not yet been proved, and is indeed the most important outstanding problem in black hole physics. If it is correct, then a collapsing star which initially has $J > M^2$ must lose sufficient angular momentum to enable it to reach the black hole condition (e. g. through reaching a state of rotational instability).

Let us consider efficiencies for radiative processes involving a maximal Kerr black hole, that is, one with $J = M^2$. This is realistic because it is known that such a state would be approached if a non-maximal black hole accreted sufficient co-rotating material for most of its final mass to be due to the accreted material. (It also so happens that our own sun has $J \sim M^2$, unless Dicke is correct in his suggestion that the interior of the sun is rotating much more rapidly than the exterior.) If we now consider a particle or a gaseous disc spiralling into the black hole, then it if is co-rotating one obtains a radiative efficiency of 43.2 %, instead of the 5.7 % characteristic of a non-rotating black hole.

In addition to this increase in efficiency, there is the new possibility that the rotational kinetic energy of the black hole itself can be converted into wave energy. The simplest way of seeing this is to imagine constructing a cubical framework of rigid rods of arbitrary size with the Kerr black hole at the centre. The gravitational interaction between the black hole and the framework would set the framework into rotation about the axis of the black hole. The framework would then have a varying quadrupole moment and would radiate gravitational waves. The energy in these waves can only come from the rotational kinetic energy of the black hole, and as a result this rotation would get damped out. The process would continue until all the rotational kinetic energy of the black hole is converted into gravitational wave energy. For a maximal Kerr black hole one can extract in this way $(1 - (1/\sqrt{2}))$ or 29 % of Mc^2 in the form of gravitational waves.

Again one can compare this with thermonuclear efficiencies. Since the sun has $J \sim M^2$, if it were to collapse into a black hole at the end of its thermonuclear evolution without losing appreciable amounts of mass or angular momentum, one could then extract its rotational kinetic energy and so convert 29 % of its present rest-mass energy into the form of gravitational radiation. This efficiency is over 30 times greater than the *maximum* efficiency obtainable by thermonuclear processes.

Thus gravitation, the weakest of all the known interactions in Nature, can manifest itself in a form in which more of the rest-mass energy of matter can be released than through any other known process, except matter-antimatter annihilation itself. In view of the likely prevalence of collapsed objects in the universe, it would be surprising if Nature has not taken advantage of this possibility, perhaps on a grand and dramatic scale. The investigation of gravitational radiation, both theoretical and observational, is therefore full of promise and should be vigorously prosecuted.

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