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THE VALUE OF ELECTRONIC HYPERFINE ANOMALIES TO NUCLEAR AND SOLID STATE PHYSICS

N. J. STONE

Mullard Cryomagnetic Laboratory, Clarendon Laboratory, Oxford, UK

Résumé. — L'anomalie hyperfine a été principalement considérée comme un paramètre intéressant en connexion avec la théorie du moment dipolaire nucléaire. Il est montré que cette application est limitée par l'état actuel de la théorie de cet effet. En contrepartie, la comparaison de mesures d'anomalies dans différents environnements électroniques conduit à des informations concernant l'origine des interactions magnétiques hyperfines et ceci sans qu'il soit nécessaire, *a priori*, de requérir des calculs quantitatifs précis de l'anomalie. Des résultats récents d'anomalies, pour les isotopes de Ir et Au dilués dans des alliages ferromagnétiques, indiquant la formation d'un moment local, près de Ir et Au, sont discutés.

Abstract. — The hyperfine anomaly has been primarily considered as a parameter of interest in connection with nuclear dipole moment theory. It is shown that the present state of the theory of the effect limits this application. In contrast the comparison of anomaly measurements in different electronic environments yields information concerning the origin of magnetic hyperfine interactions without requiring a precise quantitative calculation of the anomaly *a priori*. Recent anomaly results on dilute Ir and Au isotopes in ferromagnetic alloys, which indicate local moment formation at the Ir and Au site, are discussed.

1. **Introduction.** — This paper reviews, in outline, the basic theory of the hyperfine anomaly in electronic hyperfine interactions. The complexity of the theory arises from the need to consider the effects of the distribution over the nuclear volume of nuclear magnetism (DNM) and of nuclear charge, the latter determining the variation of the electron wavefunctions within the nucleus. The limited value of the anomaly as a parameter to assist in the choice of appropriate nuclear models is illustrated briefly. It is shown that in favourable cases the anomaly can give more information concerning atomic and solid state problems. Its measurement in different environments can lead to separate knowledge of contact terms and non-contact or orbital terms in the hyperfine interaction. Since the separation is only accurate to the precision with which the ratio of hyperfine anomalies in two environments can be measured, the conditions for existence of large anomalies are reviewed. Details are given of the use of anomaly measurements in this way for Ir and Au isotopes in dilute ferromagnetic alloys, and the interpretation of the results is outlined. With the increasing frequency of precise hyperfine interaction measurements on excited nuclear states, often giving wider variation in DNM than found in stable ground states, it is to be expected that the use of the hyperfine anomaly in this way will become more important.

2. **The hyperfine anomaly.** — Magnetic hyperfine interactions can be produced either by spin or orbital dipolar interactions for electrons with wavefunctions of non-spherical symmetry, or by the «contact»

interaction of electrons having spherically symmetric wavefunctions which are finite at the origin. Considering the nucleus as a point dipole, Fermi and Segre [1] derived the expression

$$\text{Contact hyperfine field} = -\frac{8\pi}{3}\mu_B |\psi(0)|^2$$

where μ_B is the Bohr magneton and $\psi(0)$ is the electron wavefunction amplitude at the origin. A classical derivation is given by Ferrell [2]. The contact interaction arises for s electrons, and for heavy atoms where relativistic components in the wavefunctions are important, for $p_{1/2}$ electrons also.

This expression predicts that the contact hyperfine interactions W_i of two isotopes of a single element would be in the ratio of their nuclear dipole moments i.e.

$$\frac{W_1}{W_2} = \frac{\mu_1 H_c}{\mu_2 H_c} (1) = \frac{\mu_1}{\mu_2} \quad (1)$$

This relation does not hold in practice, the deviation being characterised by a « hyperfine anomaly » $^1\Delta^2$, such that

$$\frac{W_1}{W_2} = \frac{\mu_1}{\mu_2} (1 + ^1\Delta^2) \quad (2)$$

The origins of the anomaly lie in the breakdown of the

(1) Those readers who feel strongly on the matter can read B for H if they so wish. The use of H for the hyperfine field throughout this paper avoids confusion with current usage and its accuracy will not be discussed here.

point nucleus assumption which is implicit in eq. (1). The hyperfine field H varies over the finite nuclear volume since the electron wave function $\psi(r)$ is not constant, and the DNM is non-uniform. Thus the general expression for the contact interaction energy should be

$$W_i = - \int_{\text{nucleus}} H(\tau) \mu(\tau) d\tau$$

where $d\tau$ is an element of nuclear volume. Since the DNM can be different for different isotopes we express the interaction energy in the form

$$W_i = W_{\text{point}}(1 + \varepsilon_i)$$

where W_{point} is a fictitious point dipole interaction and ε_i an empirical parameter describing the deviation from it for a given isotope i .

2.1 THEORETICAL CALCULATION OF ε . — The problem was first formulated by Bohr and Weisskopf [3]. They derived a result of fundamental importance, namely that the interactions of the contact field with spin and orbital contributions to the nuclear dipole moment are essentially different. The element of nuclear spin magnetisation at radius r_0 from the origin interacts with the field at that radius $H(r_0)$ to give

$$W_s = - \int H(r_0) \mu_s(r_0) dr.$$

In contrast the nuclear orbital magnetisation acts like a current loop and its energy depends upon the *mean* field through the orbit $\bar{H}(r \leq r_0)$ to give

$$W_L = - \int \bar{H}(r \leq r_0) \mu_L(r_0) dr.$$

A non-relativistic derivation of these results has been given by Sorensen [4].

Thus since $H(r_0)$ varies as $|\psi(r)|^2$ and hence falls from a maximum at the origin, the deviation from a point nucleus interaction is larger for spin contributions than orbital contributions to the DNM at the same distance from the origin. Collective orbital contributions are treated in the same way as single particle orbital terms.

Contributions to ε are then written

$$\varepsilon = \varepsilon_S + \varepsilon_L$$

where

$$\varepsilon_S = - \langle K_S \rangle_{\text{av}} \alpha_S + \mathcal{G} \{ \langle K_S \rangle_{\text{av}} - \langle K_L \rangle_{\text{av}} \}$$

and

$$\varepsilon_L = - \langle K_L \rangle_{\text{av}} \alpha_L.$$

In these expressions α_S and α_L are the fractional spin and orbital contributions to the nuclear dipole moment, thus $\alpha_S + \alpha_L = 1$. $\langle K_S \rangle_{\text{av}}$ and $\langle K_L \rangle_{\text{av}}$ contain appropriate averages of powers of the electron density distribution over the nucleus. The

$\mathcal{G}$ term in ε_S allows for a non-spherical contribution to the spin interaction in aspherical nuclei and was introduced by Bohr [5].

Thus the calculation of ε depends upon the nuclear moment through the breakdown into spin and orbital contributions and their distributions within the nucleus, and also upon the electron wavefunctions. Tabulations to yield $\langle K_S \rangle_{\text{av}}$ and $\langle K_L \rangle_{\text{av}}$ for both $s_{1/2}$ and $p_{1/2}$ relativistic electron wavefunctions based on a uniform charge distribution are given by Eisinger and Jaccarino [6]. Stroke, Blin-Stoyle and Jaccarino [7] have shown the sensitivity of ε to the assumed charge distribution to be of order 30% by comparing calculations for uniform volume charge, uniform surface charge and diffuse or trapezoidal charge distributions.

More recent calculations of electron wavefunctions over the nuclear volume are available, the prime motivation being interpretation of the isomer shift measured by Mössbauer spectroscopy. The isomer shift is the electric monopole analogue of the magnetic hyperfine anomaly, its concern being the electrostatic interaction of nuclear and electronic charge distributions. A recent review of these improved wavefunction calculation techniques by Kalvius [8] serves only to emphasize the level of the demands such complex interactions make upon theory and how limited is its current success.

As regards the nuclear models, calculations have been made using pure shell model [3], shell model with weakly coupled core contributions [5], shell model with configuration mixing [7] and Nilsson [9] wavefunctions. It was hoped that measurements of hyperfine anomalies might assist the choice of appropriate nuclear models. This hope has not materialized since experimentally the effects are generally small, and they involve both the electronic and nuclear wavefunctions in a sophisticated way. Furthermore, since W_{point} is a notional quantity, the only measurable parameter, ${}^1\Delta^2$ involves the difference of ε for two nuclear states. Comparison with experiment is considered further in section 3.

2.2 THE MAGNITUDE OF ε AND OF ${}^1\Delta^2$. — Figure 1, due to Eisinger and Jaccarino [6], shows the radial dependence of the $s_{1/2}$ electron density for a point nucleus and for a uniform finite charge density distribution. An important result is that, to the order (nuclear radius/Bohr orbit radius)² the $s_{1/2}$ electron radial dependence is the same for all principal quantum numbers. This means that the anomaly will be independent of the shell in which s-electron interaction originates. For a given Z , $p_{1/2}$ electron anomalies are smaller than $s_{1/2}$ and they are only significant for $Z > 50$.

A calculation by Eisinger and Jaccarino [6] assuming uniform nuclear charge and DNM over a sphere of radius R_c gave

$$\varepsilon = \left(1 - \frac{|\psi(r)|^2}{|\psi(0)|^2} \right)_{\text{Ave}} = \frac{ZR_c}{a_0} \left\langle \frac{R^2}{R_c^2} \right\rangle_{\text{Ave}}$$

Variation of the S-electron density over the nuclear volume

FIG. 1. — Radial dependence of the $s_{1/2}$ electron density $a)$ for a point nucleus and $b)$ for a uniform charge density distribution.

where a_0 is the first Bohr orbit radius, R is the radial co-ordinate and the average is over the nuclear volume. Since, even for $Z = 50$, this gives ϵ less than 2% we can see why the hyperfine anomaly ${}^1\Delta^2 = \epsilon_1 - \epsilon_2$ is seldom more than 1% and for light nuclei generally less than 0.1%. A recent tabulation of anomaly measurements is by Fuller and Cohen [10].

However, there exist characteristic situations in which the anomaly can be much larger. These occur when the nuclear moment, which is always the vector sum of the spin and orbital contributions, is the small resultant of two large and opposing terms. Such cases arise in odd Z nuclei of low spin with the odd proton in «jackknife» $j = 1 - \frac{1}{2}$ orbitals. These are illustrated for the $p_{1/2}$ and $d_{3/2}$ cases in figure 2 where the unit on the vertical scale is the resultant moment in each case.

FIG. 2. — Cancellation of spin and orbital contributions to $p_{1/2}$ and $d_{3/2}$ odd proton nuclear dipole moments. The unit on the vertical scale in each case is the resultant nuclear moment.

As an extreme example, figure 3 illustrates schematically the case of exact cancellation of the nuclear moment contributions, i.e.

$$\mu = \mu_s + \mu_L = 0.$$

THE CASE OF INFINITE HYPERFINE ANOMALY

FIG. 3. — The case of infinite hyperfine anomaly. This schematic diagram shows $a)$ the exact cancellation of the nuclear moment contributions and $b)$ the different spin and orbital departures from a point nucleus hyperfine interaction leading to non-zero resultant.

The contact hyperfine interaction is given by

$$W = H_{pt}[\mu_s(1 + \epsilon_s) + \mu_L(1 + \epsilon_L)] \\ = H_{pt}[\mu_s \epsilon_s + \mu_L \epsilon_L]$$

which is non-zero since $\epsilon_s \neq \epsilon_L$. Such a nucleus thus has a non-zero contact hyperfine interaction despite having zero magnetic dipole moment. The hyperfine anomaly ${}^1\Delta^2$ between this isotope and any other would therefore be infinite.

The infinite anomaly is of course a singular case, but it is generally true that large departures from the point nucleus interaction (large ϵ) arise for $p_{1/2}$, $d_{3/2}$ and $f_{5/2}$ odd proton nuclei, the largest for those with high Z . Large anomalies then occur when comparisons are made between these states (of large ϵ_1) and other isomers or isotopes of the same element having very different DNM (and thus small ϵ_2) when

$$\Delta \approx \epsilon_1 - \epsilon_2 \approx \epsilon_1.$$

The largest measured anomalies (of order 5-10 %) involve $d_{3/2}$ proton states in gold and iridium isotopes (see below). For intermediate Z elements anomalies of 2-3 % have been measured between isotopes of silver (odd proton $p_{1/2}$).

3. Application to nuclear physics : Comparison of measured anomalies with nuclear model calculations.

— As mentioned at the end of section 2.1 various measurements of hyperfine anomalies have been made in the hope that comparison with calculations based on different model wavefunctions having different DNM would indicate a preferential choice of nuclear model. It must be emphasized that the measurement of a large Δ requires that the two isotopes have very different DNM, and that conversely a small Δ can occur between two isotopes, for both of which the

TABLE I
Comparison of theory and experiments for $^{3/2}\Delta^{1/2}$ ^{193}Ir for various electron wavefunctions and nuclear models (Perlow)

Model	$^{3/2}\Delta^{1/2}$	Comment
1) Bohr & Weisskopf model	10.0 %	
2) Pure single particle model with uniform nuclear charge	25.0 %	} Note effect of nuclear charge shape on electronic wavefunctions and hence on Δ
3) Pure single particle model with trapezoidal nuclear charge	14.0 %	
4) Weak coupling model with core excitation	27.0 %	
5) Nilsson model	20.0 %	
Measured value (Mössbauer)	7.2 %	Emphasizes that large Δ 's occur where strong cancellation makes moment theory difficult

TABLE II
Comparison of theory and experiment for $^{197}\Delta^{199}$ (Gold)
on various nuclear models

	$^{197}\Delta^{199}$	Comment
Bohr and Weisskopf	6.5 %	} All models give correct order of Δ , Measurement is not model selective
Weak coupling model (Bohr)	4.0 %	
Configuration mixing shell model	4.9-6.6 %	
Measured value (Atomic Beam)	3.7 %	

deviation ε_i from a point nucleus interaction may be large, if their DNM are closely similar.

Experimentally a reliable measurement of Δ for comparison with theory requires that the origin of the hyperfine interaction measured be well understood so that division into contact W_c and non-contact or orbital W_{nc} parts can be made. This is necessary since the non-contact interaction, being uniform over the nuclear volume to a high degree of accuracy, will not contribute to Δ and must be subtracted, thus in general

$$W_i = W_{ci} + W_{nci}$$

and

$$^1\Delta_c^2 = \frac{W_{c1}\mu_2}{W_{c2}\mu_1} - 1 \quad (3)$$

supercedes eq. (2). Furthermore, if both $s_{1/2}$ and $p_{1/2}$ terms contribute to the contact interaction their relative contributions f_s and f_p must be known, giving

$$^1\Delta_c^2 = f_s \cdot ^1\Delta^2(s) + f_p \cdot ^1\Delta^2(p).$$

As examples of the limited value of this procedure we consider briefly the cases of Ir and Au isotopes. Both show large anomalies which can be accurately measured.

3.1 Ir ISOTOPES. — The anomaly between the ground states of the stable isotopes ^{191}Ir and ^{193}Ir

deduced from NMR measurements in Ir metal [11], [12] and paramagnetic resonance is $-0.2(2)\%$. Since both are odd proton $d_{3/2}$ this small value confirms they that have closely similar DNM. Of much greater interest is the Mössbauer effect measurement of Δ between the $\frac{3}{2}^+$ ground state and $\frac{1}{2}^+$ excited state of ^{193}Ir by Perlow *et al.* [13]. The crucial hyperfine interaction experiments were done in IrF_6 and the magnetic moment ratio was obtained from measurements in an external magnetic field applied to non-magnetic iridium metal. After correction for the non-contact interaction in IrF_6 the value of $^{3/2}\Delta^{1/2}$ was 5.8 (6)%. Perlow [9] has reported calculations based on various nuclear models and electron wave-functions. These are summarised in table I.

3.2 Au ISOTOPES. — Atomic beam studies of Au yield useful Δ values directly since the electronic hyperfine interaction is due entirely to the outer 6s electron contact term. Measurements by Van den Bout *et al.* [14], [15] on $^{196,197,198,199}\text{Au}$ yielded large values of Δ . The value for $^{197}\Delta^{199}$ is compared with various model calculations in table II and calculations by Bacon [16] using the Eisinger and Jaccarino theory for this and other pairs of Au isotopes are given in table III. The latter shows clearly that the large anomalies arise from large ε_i ($\sim 10\%$) associated with ^{197}Au and ^{199}Au , both single proton

TABLE III
Calculated and measured hyperfine anomalies
for different Au isotopes

A	Cal. ε (%)	$^{197}\Delta^A$		$^{198}\Delta^A$	
	E & J	Cal.	Exp.	Cal.	Exp.
197	10.3	—	—	—9.0	—7.96 (8) (a)
198	0.4	+ 9.9	+ 8.53 (8) (a)	—	—
196	0.4	+ 9.9	+ 8.72 (24) (b)	0.0	+ 0.2 (3) (b)
196 m	—1.9	+ 12.5	—	+ 2.3	—
198 m	—1.9	+ 12.5	—	+ 2.3	—
200 m	—1.9	+ 12.5	—	+ 2.3	—
199	4.3	+ 5.8	+ 3.7 (2) (a)	—3.8	—4.5 (3) (a)

$d_{3/2}$ nuclei, contrasting with smaller ε_i ($\sim 1\%$) for the odd-odd isotopes and isomers with less cancellation in their DNM.

3.3 CONCLUSION. — For both the Au and Ir systems, while it is clear that calculations of ε_i can give qualitative agreement with experiment no strong indication of nuclear model results, not least because of the sensitivity of the calculation to the assumed electronic wavefunctions. This conclusion holds for other systems as discussed in previous reviews [6], [7]. More detailed calculations could be made, however, as Perlow has remarked [9], it is for just those cases of strong cancellation, which test the accuracy of theories of nuclear magnetism beyond their present level, that the larger and more precisely known anomalies arise.

4. The application of hyperfine anomaly measurements in solid state physics. — Although it is clearly difficult to make quantitative use of measured Δ 's to assist in questions of nuclear models, the hyperfine anomaly can be of value in elucidating the problem of the origin of a particular hyperfine interaction. This arises since only contact terms due to $s_{1/2}$ or relativistic $p_{1/2}$ electrons contribute to Δ . Provided some system, A, exists in which the proportion of a measured hyperfine interaction due to a single contact term and any combination of non-contact terms is known it follows that the anomaly characteristic of that contact hyperfine interaction Δ_c can be calculated. For simplicity we will proceed on the assumption that $p_{1/2}$ terms may be neglected unless specified. If the interaction for isotope 2 be written

$$(W_2)_A = -\mu_2(H_{\text{eff}2})_A$$

then one can show that the relation

$${}^1\Delta_A^2 = \frac{H_c^{(2)}(A)}{H_{\text{eff}}^{(2)}(A)} {}^1\Delta_c^2$$

holds between the measured anomaly ${}^1\Delta_A^2$, the effective field $H_{\text{eff}}^{(2)}(A)$ as defined above, the contact term contribution $H_c^{(2)}(A)$ and the pure s electron contact anomaly ${}^1\Delta_c^2$. Once ${}^1\Delta_c^2$ is known, then a measured anomaly

in any other environment ${}^1\Delta^2(B)$ can be used to separate the contact and non-contact contributions to $H_{\text{eff}}^{(2)}(B)$ through the relations

$$H_{\text{eff}}^{(2)}(B) = H_c^{(2)}(B) + H_{\text{nc}}(B)$$

and

$${}^1\Delta_B^2 H_{\text{eff}}^{(2)}(B) = {}^1\Delta_c^2 H_c^{(2)}.$$

Note that $H_{\text{nc}}(B)$ will be the same for all isotopes, but that both H_c and H_{eff} will be isotope dependent.

In contrast to the use of Δ in nuclear physics, here no *a priori* derivation of ${}^1\Delta_c^2$ is required. Rather it is used simply as an indicator and only ratios of measured anomalies in different environments are involved in the results. It is important also to re-emphasize the independence of ${}^1\Delta_c^2$ of the s electron shell involved, so that systems involving outer s electron interactions or contact interactions produced by inner core s electron polarization can be dealt with alike.

There has been little exploitation of this possible application of hyperfine anomaly measurements since measured Δ ratios are frequently too inaccurate to be of value, although the capability has been recognised for example in atomic beam [17] and ENDOR [18] studies. Apart from accuracy the fact that stable isotopes of an element often have similar DNM and the lack of known systems (A above) to «calibrate» the measured Δ 's also inhibit the prospect of general use of Δ in this way. Recent increases in accuracy of measurement of hyperfine interactions of excited states and radioactive isotopes by sensitive Mössbauer experiments and by nuclear magnetic resonance detected by radioactive methods have widened the range of nuclear states for which Δ measurements are possible and have led to a first concerted use of the anomaly in solid state physics. This has been in the field of hyperfine interactions in ferromagnetic materials for the elements Ir and Au for which, as described above, the anomalies are large and the value of ${}^1\Delta_c^2$ can be established. Related work on Eu isotopes has been done by Crecelius and Hufner [19] and by Aiga *et al.* [29] on Cu, Sb and Ir isotopes.

4.1 DILUTE ALLOYS OF Ir IN Fe, Co, AND Ni (MÖSSBAUER EFFECT). — Since Mössbauer spectra yield values of the hyperfine interaction for both nuclear states involved in the gamma transition under study a single measurement includes a value for ${}^*\Delta^{\text{ex}}$ between these states provided their moments are known. Thus Perlow *et al.* [13], having established the value of ${}^{3/2}\Delta_c^{1/2}$ for the ^{193}Ir transition, were able to interpret a spectrum published by Wagner *et al.* [20] on IrFe to yield H_c and H_{nc} . More recent results due to Wagner and Potzel [21] are given in table IV, where the values of H_{eff} and H_c are those for the spin $\frac{1}{2}$ excited state.

4.2 DILUTE ALLOYS OF Au IN Fe, Co (NMR/ON). — As a result of atomic beam studies values of the s electron contact anomalies between $^{196,197,198,199}\text{Au}$

TABLE IV
Interpretation of hyperfine fields at iridium
in ferromagnetic metals
using hyperfine anomaly data [21]

Alloy	$3/2\Delta^{1/2}$	H_{eff} (kOe)	H_c (kOe)	H_{nc} (kOe)
IrFe	+ 7.1 (4) %	— 1 403 (20)	— 1 558 (90)	+ 155 (90)
IrCo	7.3 (4)	— 1 038 (10)	— 1 162 (80)	+ 149 (80)
IrNi	7.2 (4)	— 468 (8)	— 520 (50)	+ 52 (50)

Contact anomaly $3/2\Delta_c^{1/2} = + 6.5$ (6) %.

are known (section 3.2). Figure 4 shows NMR spectra of ^{198}Au and ^{199}Au in dilute Fe alloys detected by the change in gamma ray distribution from nuclei polarised at 0.01 K [22]. The ^{199}Au resonance frequency predicted on the basis of the atomic beam anomaly value, $^{198}\Delta^{199} = - 4.5$ %, is marked by a vertical arrow. The discrepancy of 1.7 MHz is clearly seen and the current best value of the $^{198}\Delta^{199}$ in this system is $- 5.3$ (3) %. These data and similar results in cubic Co yield the values of H_{eff} , etc given in table V. Johnston [23], by further careful experimentation, has eliminated the possibility that the

FIG. 4. — NMR/ON resonances for ^{199}Au and ^{198}Au in dilute Fe alloys. The vertical arrow indicates the predicted ^{199}Au resonance frequency based on the ^{198}Au result and the pure contact term hyperfine anomaly.

resonance shift is caused by an electric quadrupole interaction [24]. Results on ^{196}Au in Fe [25], which closely agree with those on ^{198}Au , are also given in table V.

4.2.1 Interpretation of the results in ferromagnetic alloys. — The analysis of hyperfine anomaly measurements in ferromagnetic alloys of Ir and Au has experimentally confirmed that the hyperfine interactions are largely produced by contact interactions. This result was fully expected, being due to exchange polarisation of core s electrons of the impurity atoms by interaction with the polarised conduction electrons of the host. The magnitude and sign of the non-contact term is of more interest. Gehring and Williams [26] have analysed the results on both systems. They have shown first that the results cannot be explained in terms of a $p_{1/2}$ contribution to the contact interaction of reasonable magnitude. Secondly they find the explanation in terms of an orbital dipolar field of order + 200 kOe to be consistent both in sign and magnitude with the unquenched 5d orbital moment to be expected for these heavy impurity atoms which have strong spin-orbit coupling. Further evidence for this explanation comes from the observation of associated weak electric quadrupole interactions in the Ir alloys [21], [27], [28] which have been shown to be isotropic by an NMR/ON study of a single crystal IrNi sample [28].

4.3 CONCLUSION. — Recent work has shown that the hyperfine anomaly, while not amenable to accurate *a priori* calculation, can be a useful parameter in separating contact and non-contact terms in hyperfine interactions. For most elements anomalies will continue to be too small to be used in this way. However, the extension of precision hyperfine interaction techniques to measurements on unstable nuclear states with a wider variety of DNM, including « jack-knife » odd proton components should lead to more examples of large anomalies, which may be thus exploited.

Finally a note of warning. Applications of hyperfine interactions in solids to the measurement of unknown nuclear magnetic moments are always liable

TABLE V
 $^{198,199}\text{Au}$ NMR/ON results in ferromagnetic metals

Isotope	Host	ν_{res} (MHz)	$^A\Delta^{199}$ %	H_{eff} (kOe)	H_c (kOe)	H_{nc} (kOe)
199	Fe	167.7 (1)		— 1 233 (1)	— 1 453 (30)	+ 220 (30)
198	Fe	260.3 (1)	— 5.3 (2)	— 1 169 (1)	— 1 389 (30)	+ 220 (30)
196	Fe	259.6 (5)	— 5.25 (30)	— 1 170 (2)	— 1 335 (50)	+ 165 (50)
199	Co (cubic)	104.4 (1)		— 769 (1)	— 829 (30)	+ 60 (30)
198	Co (cubic)	162.95 (5)	— 4.8 (2)	— 732 (1)	— 792 (30)	+ 60 (30)
198	Ni	81.5 (2)		— 365.2 (8)		
Atomic beam results		$^{198}\Delta_c^{199} = - 4.5$ (3) %				
		$^{196}\Delta_c^{199} = - 4.6$ (3) %				

to error of order Δ where the technique involves the comparison of the interaction strength of two isotopes. This is obvious since the contact effective field is isotope dependent. Existing theories for Δ are sufficiently accurate to indicate where this error may become serious, and should be used to check the possibility especially where very different DNM is suspected. However, «correction» of measured moments for the anomaly, based on calculations of ϵ

is scarcely satisfactory since a residual uncertainty of order 30 % of the estimated anomaly correction is at present unavoidable.

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