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High amplitude wave propagation in collapsible tubes.

II. Forerunners and high amplitude waves

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Résumé. — Il a été montré qu'une onde de pression de grande amplitude qui se propage dans un fluide contenu dans un tube initialement partiellement collabé ou dilaté, à parois viscoélastiques, peut, dans certaines conditions, présenter des caractéristiques d'une onde de choc. Le temps et la distance de formation de cette onde de surpression peuvent être calculés à partir de la méthode des caractéristiques lorsque la loi de comportement dynamique du tube est connue. Il est d'autre part montré que la caractéristique essentielle du front d'onde de surpression, dans le cas d'un tuyau initialement partiellement collabé, est la présence d'ondelettes se propageant en précurseur du front d'onde principal, et dont la dynamique est dominée par les effets de tension longitudinale du tube. La propagation de ces ondes, déduites des mesures de déplacement des parois peut être caractérisée par une équation de dispersion dont on donne une interprétation théorique.

Abstract. — It is shown that, under certain circumstances, a pressure wave of large amplitude which propagates in a fluid, inside a deformable viscoelastic tube initially inflated or collapsed, can present the behaviour of a shock wave. The characteristic time and length of formation for such a shock like wave can be computed from the method of characteristics if the dynamic rheological law of the tube is known. The principal feature of such a shock wave propagation inside an initially collapsed tube is the presence of wavelets on the wave front. The dispersion relation of such wavelets, experimentally obtained from the wall displacements measurements, has been theoretically interpreted on the basis of dynamical effects dominated by the longitudinal tension of the tube.

1. Introduction.

In part one, we have reviewed and discussed the way to experimentally study and theoretically interpret the propagation of small amplitude pressure waves in a viscoelastic tube in a collapsed state. If one wants to study large amplitude wave propagation phenomena, and the generation of shock like waves, nonlinear effects in both the fluid dynamics and the wall mechanics must be taken into account. The phase velocity of each wavelet component has to be known for solving the basic equations of system dynamics. But the precise knowledge of the structure of the wave front requires the various dissipative mechanisms encountered in such a phenomenon to be taken into account. Among these, energy loss which takes place inside the wall due to the viscoelasticity of the tube and the generation of forerunning waves play an important role. Such effects have been experimentally demonstrated by using

hydro-mechanical models, and we present here, after a review of the main features characterizing high amplitude wave propagation, an analysis of the results related to the generation and the propagation of these forerunners.

2. Experimental results.

2.1 PROPAGATION OF WAVES FOR LARGE POSITIVE TRANSMURAL PRESSURE. — In order to generate, inside a deformable test section, transmural pressure waves of large amplitude ($> 10^4$ Pa), a suitable hydromechanical pressure generator was designed (Fig. 1). The measurements of pressure, external diameter and fluid velocity were then performed inside the elastic cylindrical tube. Such measurements occasionally have shown that a shock-like wave could occur; its formation time and thickness could, as a first step, be interpreted in terms of the gas dynamic analogy [1, 2].

Fig. 1. — Schematic representation of the hydrodynamical set up used to generate large amplitude waves. PFG : Pulsatile flow generator; OT : Optical displacement Transducer; PT : Pressure transducer; PDUV : Pulsed Doppler Ultrasonic Velocimeter; A : Amplifier; SP : Signal Processing which includes correlation, Fourier transform, digital calculations, and provides feedback loop to control the pulsatile flow generator. This set up allows the generation of pulsatile flow of the following characteristics (inside a 2 cm diameter tube) : steady flow Reynolds number $Re = 2RU/\nu$ up to 10; frequency parameter $\alpha = R(\omega/\nu)$ up to 10; amplitude parameter $\lambda = \bar{U}/U$ to 3 for $Re = 10^3$.

2.2 PROPAGATION OF PRESSURE WAVES INSIDE A PARTIALLY COLLAPSED TUBE. — Such an experimental set up was also used in order to propagate pressure waves of large amplitude (5×10^3 Pa) within an initially collapsed tube lying on a horizontal rigid plane, the external pressure within the test section being set up to atmospheric level. Typical results are presented in figure 2 where variations of the tube apparent diameter H measured on several sites, have been represented. The pressure ramp inside the pump reservoir is also shown as a function of time (Fig. 2b).

Two significant phenomena which occur while the main wave is propagating are worth considering : on one hand (Fig. 2b), a noticeable steepening of the wave front is observed, and on the other hand, a wave system, propagating as forerunning wavelets are generated at the base of that front. The number of periods within those wavelets greatly increases during the process, as can also be seen with figure 2a. Both wavelengths and phase velocities of such forerunners seems to be related to the characteristics of the main wave.

In these experiments the waves were generated at the entrance of the tube. In order to study more precisely the generation mechanism of the precursor waves, we have attempted to trigger them off at various tube sites. In so doing, we had to resort to a further experimental device (Fig. 3) which included a constant level tank feeding the tube which was locally compressed by means of an electromagnetic system. With such an experimental set-up, two different zones inside the test section have to be considered in regard to their different initial conditions : the first one is located upstream from the local constriction where the initial

Fig. 2. — Example of experimental results showing the apparent diameter H of the tube as a function of time and instantaneous shape of the tube at different times. These results show the wave front steepening and the generation of forerunner waves.

Fig. 3. — Schematic representation of the experimental device used to study the propagation of large amplitude waves on an initially collapsed tube, at any given location along the tube. P1, P2 : constant level tanks; OT : optical transducer allowing the apparent diameter measurements; SP : signal processor; A : amplifier; ED : electromagnetic device used to occlude the tube.

transmural pressure is determined by the water level inside the tank ($P_0 \sim 10-150 \times 10^2$ Pa). The second one, downstream from the constriction, has a transmural pressure ($P_0 \sim 0.5 \times 10^2$ Pa) defined as the difference between the internal pressure measured at the bottom of the tube and the atmospheric external pressure, such that this part is in an initial collapsed state.

A quick release of the compressed zone is obtained by means of the electromagnetic device, thus generating a high pressure wave propagation phenomenon inside the collapsed part of the test section. Measurements of the apparent diameter at different locations along the tube were carried out using an optical displacement transducer, the output signal of which was digitized and processed. Actually, the experiment does not only show the propagation of a single large amplitude compression wave in the initially collapsed part (cf. Fig. 4) : on opening, a rarefaction wave (b) is generated inside the high pressure zone of the tube. This wave is moving upstream with high velocity and is reflected at the entrance of the deformable test sec-

Fig. 4. — Description of the generation of forerunner waves. 1. Initial conditions; 2. After the sudden release of the occluding system : a) slow compression wave generating a forerunner waves system; b) fast rarefaction wave; 3. a) propagates downstream; b) is reflected and gives c; 4. a) still propagates far downstream; c) propagates faster than a and generates a second precursor wave system; 5. d) the two waves are mixed and form a unique precursor wave system.

tion. It thus appears in the collapsed region as a second large amplitude wave (c) propagating at a different velocity. Such a phenomenon allows us to study two different systems of forerunning wavelets : the first one is triggered in the vicinity of the closing system; the second is derived from the superposition of both pressure waves.

Such double wave systems can be distinguished both by the wave amplitudes as well as the velocities and wavelengths of their precursors. They enable us to gain a larger amount of measurements so as to study the propagation properties of these precursors. Some examples of experimental results have been shown in figure 5 which suggest the following remarks : the shock-like wave front seems to be established and be moving at constant speed ; such a point, however, is difficult to ascertain owing to the superposition of the two wave systems.

Further the precursors seem to move like a progressive wave at the same speed c as the wave-front does, with a wavelength depending upon both shock wave amplitude and induced curvature. Eventually, a further experimental observation has been implemented through photographic data allowing us to describe the instantaneous appearance of the tube (Fig. 6). One can establish that the precursor's system gives rise to a change of the apparent diameter of the tube which exhibits an axially symmetrical shape. During such a

Fig. 5. — Typical experimental results showing the two systems of waves before, during and after the mixing.

Fig. 6. — Instantaneous shape of the tube as observed from photographic data.

propagation, certain parts of the tube keeps pulling away from their stand. This observation suggests that gravity effects are negligible with regard to the other effects.

3. Theoretical interpretation.

3.1 LARGE AMPLITUDE WAVE DEFORMATION. — We have experimentally observed that for high amplitude positive transmural pressure a shock-like wave could be generated ; likewise the shape of a large amplitude wave propagating along an initially collapsed tube with negative transmural pressure is modified : during its propagation the wave front can steepen with the emergence of precursor wavelets.

When considering this large amplitude wave propagation, one can use the method of characteristics to compute the deformation of the wave front provided that it can be regarded as a superposition of small amplitude wavelets. The propagation of these wavelets is dependent on both transmural pressure P and fluid velocity v .

The part of the fluid velocity generated by each wavelet is assumed to be much smaller than its phase velocity, as defined by (cf. part I) :

$$c^2 = \frac{S}{\rho} \frac{E_d}{E_0} \left(\frac{dP}{dS} \right)_0. \quad (1)$$

This relation takes into account the viscoelastic properties of the tube wall material and the specific mass of the fluid. It shows that pressure-wavelet velocity can be deduced from the knowledge of both the static pressure (P) – area (S) law of the tube and the dynamic mechanical behaviour (dynamic E_d and static E_0 Young's modulus) of the wall material.

The characteristic flow parameters of the waves are such that they give rise to an unsteady viscous boundary layer the thickness of which is one order of magnitude lower than the apparent diameter H of the tube. Therefore, the flow induced by the wave can be assumed to be an inviscid fluid flow. Meanwhile, the tube at rest is assumed to be rectilinear (rest pressure P_0). Under such conditions and neglecting absorption effects due to wall viscoelasticity the quantity

$$v \pm \frac{1}{\rho} \int \frac{dP}{c(P)}$$

can be shown to be constant along characteristic curves expressed by

$$\frac{dx}{dt} = v \pm c(P) \quad (2)$$

where x stands for longitudinal coordinate along the tube [3].

Let us write

$$\phi(P) = \frac{1}{\rho} \int \frac{dP}{c(P)}. \quad (3)$$

Since $v < c$, the c^- characteristic lines (associated with the minus sign in (2)) have a slope which is always negative, and in a $t(x)$ diagram, are emerging from the x axis where $v = 0$ and $P = P_0$ at $t = 0$. We can thus write

$$v - \phi(P) = -\phi(P_0) = \text{Cst.}$$

Under such conditions using the classical equations of fluid dynamics, it can be shown [3] that P is constant along the c^+ characteristics given by :

$$\frac{dx}{dt} = \phi(P) - \phi(P_0) + c(P) \quad (4)$$

therefore, $\phi(P)$, $c(P)$ and v are constant too on such characteristics as well as dx/dt . These c^+ characteristics are consequently straight lines emanating from points $(0, \tau)$ on the positive t axis and equated :

$$x = [\phi(g(\tau)) - \phi(P_0) + c(g(\tau))] (t - \tau) = h(\tau) (t - \tau) \quad (5)$$

where $g(\tau)$ stands for the time variation of pressure at $x = x_0$, with $g(0) = P_0$.

Using such a characteristics method, one can evaluate the shape of the pressure wave during its propagation, granted that both the $c(P)$ relationship

and the time evolution of pressure (or that of apparent diameter H , with $P(H)$ given) are known. Given a $x = L$ position on the tube, one may infer the value of the prevailing pressure at that particular location.

Indeed the relation :

$$L = \left[\frac{1}{\rho} \int_0^\tau \frac{dg}{d\tau'} \frac{1}{c(g(\tau'))} d\tau' + c(g(\tau)) \right] (t - \tau) \quad (6)$$

enables us to relate a L value with a given τ value (and therefore a given pressure value) provided that t is known and conversely to know the shape of the pressure wave all along (and the subsequent shape of the tube as a function of time).

Some results illustrating this computation are given in figure 7. They show the calculated time evolution of the tube shape for a given inlet pressure which was experimentally recorded at the outlet of the hydro-mechanical pressure generator previously described (§ 2.1). However, this numerical method fails as soon as two characteristics converge in the $t.x$ diagram. This criterion (first crossing of two characteristics) is often invoked whenever one is computing the order of magnitude of the formation time of a shock-like wave. Such a calculation is exact if the effect of resistance to longitudinal curvature can be overlooked. As a matter of fact, if one wants to know the equation of the possible envelope of the characteristics, one

Fig. 7. — Example of numerical results obtained using characteristics method, giving the time evolution of the shape of the tube for a given pressure variation at the inlet of this tube. Depending on the criterion written in (9), one can distinguish a steepening and a flattening zone in the front wave. In the zone, referred to as undefined, a shock wave has been generated, corresponding to the crossing of two c^+ characteristics. It reveals a situation where the wavelets velocities inside the perturbed upstream part of the tube are greater than the wavelets velocities inside the unperturbed downstream part.

finds the following set of equations :

$$\left. \begin{aligned} t(\tau) &= \tau + \frac{h(\tau)}{\frac{dh}{d\tau}(\tau)} \\ x(\tau) &= \frac{h^2(\tau)}{\frac{dh}{d\tau}(\tau)} \end{aligned} \right\} \quad (7)$$

If $t(\tau)$ and $x(\tau)$ are both increasing monotonic functions, and if a time t_s and distance x_s for shocking are defined as the coordinate of the first critical point, we can infer :

$$t_s = \frac{h(0)}{\frac{dh}{d\tau}(0)}, \quad x_s = \frac{h^2(0)}{\frac{dh}{d\tau}(0)}. \quad (8)$$

If eventually the expression of the criterion of steepening is needed we may write $dh/d\tau \geq 0$, which, inserting (1), gives

$$\frac{1}{\rho c} \left(1 + \frac{E_d}{2 E_0} \left(1 - \frac{S \frac{d^2 S}{dP^2}}{\left(\frac{dS}{dP} \right)^2} \right) \right) \frac{dg}{d\tau} > 0. \quad (9)$$

If g is an increasing monotonic function of time, the pressure value and the mechanical behaviour of the wall will thereby control the steepening mechanism of the wave front.

These relations generalize, for a viscoelastic tube (under the hypothesis prevailing for (1)), the shock formation criterion in purely elastic ducts [4, 5]. An attempt was made to observe the wave form of this shock transition in the viscoelastic silicone rubber test section of the hydromechanical model. The change in tube diameter has been measured optically at various sites along its length during the passage of the shock front. From these measurements it was possible to obtain the instantaneous spatial variation in tube diameter throughout the shock. The results concerning the length and time required for shock formation were found to be in fair agreement with theoretical predictions derived from the above theory [1].

Such a theory takes into account wall viscoelastic effects upon large amplitude wave propagation and shock-like wave formation through their influence on the wavelet phase velocities. It is thus assumed that these effects give a negligible contribution to the dissipative mechanism which take place within the shock wave front. In fact, these dissipative effects can be attributed to viscous losses associated with forerunner wavelets which are generated downstream from the main wave front.

3.2 DISPERSION RELATION FOR THE PRECURSOR WAVES.

3.2.1 Hypothesis. — Let us consider pressure wave propagation in a collapsed horizontal tube stretched under longitudinal tension T . Such a propagation will be governed by the simultaneous action of both fluid inertial forces and restoring forces which are of different kinds :

- gravitational forces ;
- elastic forces due to wall rheological properties, whether it be resistance to curvature in a plane perpendicular to the tube axis or longitudinal tension in a plane including this axis.

It will be shown that the former is negligible with regard to the elastic forces, this assumption being furthermore confirmed by the observation of symmetrical deformation of the tube when the wave propagates. Each half of the tube (section by a horizontal plane including axis) behaves then as an open channel subjected to the joint action of gravitation and surface tension. An equivalent role to that of gravitation is achieved here thanks to the resistance to curvature forces and that of surface tension by longitudinal tension forces.

Such a formal analogy is reinforced when we consider the rheological behaviour of the tube within the relevant range of pressure : from the observation of the experimental results in this range (cf. Part I, Fig. 3b), where the tube is partially collapsed, a linear relationship applied to pressure variation vs. height H' (half apparent diameter of the tube) can be derived :

$$P = K(H' - H'_0) \quad (10)$$

where $\frac{dP}{dH'} = K = 3 \times 10^4 \text{ Nm}^{-3}$ and H'_0 is related to the zero value of the transmural pressure, (10) is then equivalent to the gravitational pressure law :

$$\Delta P = \rho g \Delta H'$$

As a matter of fact, such elastic restoring forces have the same magnitude as that of gravitational forces and can also be neglected. However, we have to observe that if the effects of these elastic forces were prevalent, they would control wave propagation phenomena whose phase speed would then be :

$$c_2 = \left(\frac{KH'_0}{\rho} \right)^{1/2} \quad (11)$$

or, with the experimental numerical values, $c_2 = 0.3 \text{ ms}^{-1}$. Those effects, as compared with those of longitudinal tensions help to introduce a characteristic length as defined by [6] :

$$\lambda_0 = 2\pi \left(\frac{T}{K} \right)^{1/2} \quad (12)$$

where T is the longitudinal stress. Taking into account the numerical values, with the experimentally imposed

longitudinal extension (20 %) and tube characteristics, the factor λ_0 has a value of 0.5 m. In fact, the wavelengths of the precursor waves, as experimentally observed are such that :

$$H'_0 \ll \lambda < \lambda_0 \quad (13)$$

which is relevant to the phenomenon of surface tension wave propagation in a shallow duct [7].

3.2.2 Basic equations. — Putting $H' - H'_0 = e(x, t)$, and neglecting the wall inertial, the equilibrium between the transmural pressure P and the elastic forces (stresses in the wall and equivalent longitudinal surface tension) can be written, if $H'_0 \ll \lambda_0$, as :

$$P(x, z = H', t) = Ke - T \frac{\partial^2 e}{\partial z^2} \quad (14)$$

where z represents the vertical coordinate.

In the same way, the velocity boundary condition will be written as

$$\left[\frac{\partial e}{\partial t} + (\mathbf{v} \cdot \nabla) e = v_z \right]_{z=H'} \quad (15)$$

where v is the fluid velocity.

While considering the fluid as a non-viscous medium as well as incompressible, and while assuming the precursors' amplitude to be small (which is generally true with regard to the main wave front amplitude), the fluid dynamic equations become :

$$\rho \frac{d\mathbf{v}}{dt} = -\nabla P \quad \text{with} \quad \nabla \cdot \mathbf{v} = 0. \quad (16)$$

Assuming an irrotational flow ($\nabla \times \mathbf{v} = 0$) one can introduce the velocity potential φ as defined by $\mathbf{v} = \nabla \varphi$ and write (16) as :

$$P = -\rho \frac{\partial \varphi}{\partial t}, \quad \Delta \varphi = 0 \quad (17)$$

with the following linearized boundary conditions :

$$\left. \begin{aligned} \rho \frac{\partial \varphi}{\partial t} \Big|_{z=H'_0} &= -Ke + T \frac{\partial^2 e}{\partial x^2} \\ \frac{\partial e}{\partial t} &= \frac{\partial \varphi}{\partial z} \Big|_{z=H'_0} \end{aligned} \right\} \quad (18)$$

One can then obtain the propagation equation for the velocity potential which is a standard feature in surface wave theory :

$$\rho \frac{\partial^2 \varphi}{\partial t^2} \Big|_{z=H'_0} = -k \frac{\partial \varphi}{\partial z} \Big|_{z=H'_0} + T \frac{\partial^3 \varphi}{\partial x^2 \partial z} \Big|_{z=H'_0}. \quad (19)$$

Starting with the assumption of a propagative solution like

$$\varphi(x, z, t) = \psi(z) \exp[i(\omega t - kx)] \quad (20)$$

taking into account the symmetry which requires that

$$v_z|_{z=0} = \frac{\partial \varphi}{\partial z} \Big|_{z=0} = 0 \quad (21)$$

and deriving from (17) it can be claimed that

$$\psi = \psi_0 \cosh(kz). \quad (22)$$

The dispersion equation can be written then :

$$\omega^2 = \left(\frac{K}{\rho} + \frac{T}{\rho} k^2 \right) k \tanh(kH'_0) \quad (23)$$

which is similar to the dispersion equation of gravity wave in open ducts, with prevailing surface tension forces [6]. This equation can be further expressed as $c(\lambda)$, in a non-dimensional form using c_2 and λ_0 as defined in (11) and (12) :

$$\left(\frac{c}{c_2} \right) = \left[1 + \left(\frac{\lambda_0}{H'_0} \right) \left(\frac{\lambda}{H'_0} \right)^{-2} \right] \times \left[\frac{1}{2\pi} \frac{\lambda}{H'_0} \tanh \left(2\pi \frac{H'_0}{\lambda} \right) \right]. \quad (24)$$

This relation is presented in figure 8 which shows the different shapes of the curve depending on the value of λ_0/H'_0 . If $\lambda \ll \lambda_0$ the phenomenon is controlled by longitudinal tension effects (part A); if $\lambda \gg \lambda_0$, the resistance effects due to curvature become predominant (part B). In our actual case (part C), λ_0/H'_0 is still large compared with unity (~ 50), while λ is still less than λ_0 . We are then faced with waves which are governed by longitudinal tension, (24)

Fig. 8. — Equation of dispersion of the waves in a partially collapsed tube as presented in a non-dimensional form (see text).

becoming now :

$$\frac{c}{c_2} = \left[1 + \left(\frac{\lambda_0}{\lambda} \right)^2 \right]^{1/2} \simeq \frac{\lambda_0}{\lambda}. \quad (25)$$

3.2.3 Comparison with experimental results. — It has been seen that precursors, although we were not able to explain in a quantitative way their generation mechanism, were created by the passing of a large amplitude wave. From experimental data, the phase speed and the wavelength can be derived. These data have been represented (Fig. 9) in relation to (25). Two kinds of tubes, with different elasticity moduli, have been used. The numerical values of the normalization factor c_2 is different for the second tube ($c_2 = 0.44 \text{ m s}^{-1}$).

A fair agreement between experimental results and the general theory must be pointed out here. Let us simply note that relative errors on wavelength measurements depend on the precursor's system used (Fig. 4), system (a) having larger wavelengths than system (d) relative errors must thus be accounted for (a) to 10 % and for (d) to 15 %; the relative error about speeds is assessed at 10 %.

4. Conclusion.

In this work we have attempted to obtain evidence of the tight relationship prevailing between large amplitude wave propagation in a collapsible tube and the mechanical properties of the tube wall. As far as the main wave is concerned, it turns out that, the behaviour of the dynamic $P(S)$ relation determines the change in wave shape throughout its propagation. Such a relation plays a predominant role with regard to the formation time and length of a shock-like wave.

On the other hand, both the viscous tube wall properties and the curvature resistance can be related to the thickness of that wave which, for the former, through a dissipative effect, withstands too serious steepening. In the same way, insofar as forerunning propagation waves are concerned, the longitudinal tension has chiefly to be taken into account.

At last, when the generation mechanism is considered, and with a reference system in uniform translation with the velocity of the main wave front, it will be fruitful to correct the above mentioned results with those of the study presented by Kececioğlu *et al.* [8] and MacClurken *et al.* [9] where such forerunning waves appear as a dissipative process on a stationary shock.

The dispersions relation that these authors have obtained, by taking into account circumferential bending stiffness, longitudinal membrane tension and

Fig. 9. — Experimental results in comparison with theoretical predictions (Eq. (25)). The open symbols are related to the first system of precursor (a) and the close symbols to the second system, after mixing (d).

longitudinal bending stiffness, give the phase velocity of forerunners relative to the fluid as a function of their wavelengths. Such a variation is quite similar to the relation shown here (Fig. 9). Nevertheless, the analysis of the propagation phenomenon, for wavelets generated from a stationary shock produced by a supercritical flow is more complex due to the effect of the fluid velocity. The propagation characteristics were thus analysed in terms of the variation of the Mach number associated with such a flow as a function of the wavelengths of the forerunners. Moreover, as in our case, the effects of longitudinal bending stiffness were shown to be of minor importance under the analysed experimental conditions. Then, if one assumes that the fluid velocity in the steady shock is equal to the wavelet phase velocity in the unsteady shock-like wave, our results are in complete agreement with those obtained in [9] (Fig. 5, p. 402).

The generation of the forerunners waves for unsteady flow can then be explained in the following way : among the waves generated by the shock transition, and propagating at different velocities, only one has the same velocity as the wave front ; as experimentally observed, it is this wave which propagates as a forerunner. We can then assert that the energy radiated by the shock transition as also observed on the steady state [10] induces the growth of the forerunning wave which, on the other hand, is damped by the viscous dissipative effects, which still remains to be clearly taken into account.

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