c.w.-intracavity difference frequency generation (I.D.F.G.) by non-collinear mixing of three waves in two laser resonators

Tran-Ba-Chu, R. Fischer, A.J. Bouvier

J. Physique **46** (1985) 2083-2092 DÉCEMBRE **1985**, PAGE 2083

Classification
Physics Abstracts
42.65 — 42.65C

c.w.-intracavity difference frequency generation (I.D.F.G.) by non-collinear mixing of three waves in two laser resonators

Tran-Ba-Chu (*+), R. Fischer (**) and A. J. Bouvier (*)

(*) Laboratoire de Spectrométrie Ionique et Moléculaire (++), Université Lyon I,
Bât. 205, 43 Bd du 11 Novembre 1918, 69622 Villeurbanne Cedex, France
(**) Zentralinstitut für Optik und Spektroskopie, Akademie der Wissenschaften der D.D.R.,
Rudswer Chaussee 6, D.D.R. 1199 Berlin-Adlershof, G.D.R.

(*Reçu le 22 mai 1985, révisé le 30 juillet, accepté le 23 août 1985*)

Résumé. — Nous avons étudié la théorie de la génération intracavité d'onde par différence de fréquence en régime continu en effectuant le mélange de trois ondes non colinéaires dans deux lasers. On montre qu'en utilisant les méthodes de couplage et focalisation optimales, la puissance de sortie de la radiation infrarouge générée peut atteindre quelques dizaines de milliwatts, en raison de l'effet du champ laser dans les deux cavités et de celui de l'amplification paramétrique. Aux conditions optimales, le système d'I.D.F.G. donne une puissance de sortie de l'onde générée élevée et stable en comparaison avec les autres systèmes de D.F.G. opérant dans les mêmes conditions. Des résultats numériques ont été obtenus en se basant sur un système d'I.D.F.G. utilisant le cristal KDP (synchronisation de phase non colinéaire) placé dans les deux cavités du laser YAG continu et du laser à colorant continu en anneau. L'analyse a montré l'avantage de ce système pour étudier la spectroscopie à haute résolution.

Abstract. — The theory of intracavity c.w.-difference frequency generation (I.D.F.G.) has been studied by a non-collinear mixing of three waves in two laser resonators. It is shown that, using optimal coupling and focusing methods, the c.w.-I.R. output power can be of several tens of mW in practice, due to the high field intensities of the signal and pump waves within the cavities and the parametric amplification effect. Under optimal conditions, the I.D.F.G. gives a stable and large output power of the generated difference frequency wave in comparison with the other D.F.G. system working under the same conditions. Numerical results have been obtained for an I.D.F.G. using a non-collinear phase-matching K.D.P. crystal placed in c.w.-ring dye- and YAG-laser cavities. The analysis has shown its advantages for high resolution spectroscopy.

1. Introduction.

Continuous wave difference frequency generation (c.w.-D.F.G.) [1-3] has provided a tunable infrared source of narrow band coherent radiation, which is of particular interest in the high resolution spectroscopy of atoms and molecules. Using an extra-cavity non-critical phase matching $LiNbO_3$-crystal, Pine has obtained a c.w.-I.R. power of the order of 1 μW [1]. Since the non-linear effect is proportional to the laser input powers, Lahmann *et al.* [2] have placed the D.F.G. crystal inside the laser cavity. With the non-collinear three waves mixing, they have provided a c.w.-I.R. power of $10 \div 35$ μW and have also shown the advantages of their method : high input powers are possible so that the low conversion efficiency is more than compensated. The output beam is separated from two input beams.

In our previous work [3] we studied the theory of the intracavity c.w.-difference frequency generation using a collinear phase matching crystal placed in the signal laser cavity. Our results show that the enhancement of D.F.G. wave power of the above intracavity system is due to the high field intensity of the signal wave within the laser cavity and the parametric amplification effect. When the pump wave power is relatively high ($\simeq 50$ W) the D.F.G. output power becomes very large due to the resonant parametric amplification effect. However, in practice, the c.w.-dye laser output power (pump power) is not sufficiently high, the resonant regime is still difficult to obtain.

(+) Permanent address : Institut of Applied Physics, Hanoi, Vietnam.

(++) Associé au CNRS N° 171.

Recently, using the advantages of the high intracavity intensities of two lasers, in the difference and sum frequency generation experiments, Coherent has utilized the non-linear crystal common to two laser cavities [4]. The reported S.F.G. output power is around ten milliwatts.

Taking into account the advantages of the mentioned methods, we propose, in this paper, to study the c.w.-difference frequency generation by non-collinear three-waves mixing in two laser resonators and to treat the optimal conditions for which the output power of the generated difference frequency wave is maximized. We show also that, with the conventional c.w.-dye laser (in practice) and c.w.-YAG laser, the c.w.-generated difference frequency wave power can be of the order of thousand times greater than that obtained with an extracavity D.F.G. system.

Our analyses are performed with a Gaussian focusing eigenmode of two coupled lasers. Although the focusing is desirable for c.w. operation to obtain high conversion efficiency, however, with the focusing, a large number of different spatial field components appear, which violates the phase matching. To limit this effect, the optimal phase matching is considered in this paper.

Figure 1 shows the structure of our I.D.F.G. system. The mixing crystal NLC of length l is internal to both the c.w.-YAG- and the dye ring-laser cavities, which are of length L_s and L_p respectively. All the mirrors of the two cavities reflect the laser wave 100 %. The crystal forms the only common element in the otherwise independent resonators. The crystal faces are covered with antireflection coatings (to eliminate reflection losses) and the crystal acts therefore as a prism to disperse beams of different wavelengths which are collinear within the crystal.

The properties of the dye laser used have been described in [3-7], i.e. the dye is considered to be homogeneously broadened; the dye laser radiation is essentially monochromatic; the hole burning, mode-pulling-pushing effects are eliminated in the c.w.-ring dye laser by using a unidirectional device.

The method of derivation in this work is similar to that used in our previous paper [8]. Since the optic damage threshold of the KDP crystal is very large (> 400 MW/cm^2) and its absorption coefficient in the working domain is too small (2 % in comparison to that of LiIO$_3$-20 % [9]), our numerical results have been made on the KDP crystal.

2. Basic equations.

Adopting the same notation as in [3], we shall refer to the two incident waves as pump and signal waves (the pump wave corresponding to the higher frequency), while the wave generated at the difference will be named idler wave. The frequencies of these waves are ω_p, ω_s and ω_i satisfying the relation $\omega_p = \omega_s + \omega_i$ for energy conservation. The phase relation corresponding to maximum energy conversion, $\phi_p - \phi_s = \pi/2$ [10], has been also used.

The total electric field in the non-linear crystal may be expressed as

$$\mathbf{E}(\mathbf{r}, t) = \sum_{f=\mathrm{i,s,p}} \mathbf{e}_f \, E_f(t) \, \mathrm{e}^{j(\omega_f t + \phi_f)} \, U_f(r) \tag{1}$$

where $\mathbf{e}_f$ is the polarization unit vector, $U_f(r)$ the spatial mode and $E_f(t)$ the amplitude of the electric field. In this I.D.F.G. device, the signal and the pump waves are at resonance and if their TEMoo modes alone are at resonance, we can write $U_f(r)$ as a sum of a right and left travelling Gaussian beam, i.e.

$$U_f(r) = u_f(r) + u_f^*(r) \qquad (f = \mathrm{p, s}) . \tag{2}$$

In a negative uniaxial crystal, type I phase matching (eo-o) has to be used : the pump beam is extraordinary (e), the signal beam ordinary (o) and the idler will be ordinary too. To eliminate the double refraction effect, which reduces the effective interaction length and hence the efficiency of conversion, the non-collinear phase matching is used here. The two input beams enter in the crystal at appropriate angles so that their ray directions (given by the Poynting vectors) inside the crystal coincide. This means that the wave vectors of incoming beams will be no longer parallel, and the phase matching condition has to be written in vector form

$$\mathbf{k}_i = - \mathbf{k}_s + \mathbf{k}_p \tag{3}$$

with

$$k_f = \frac{n_f \, \omega_f}{C} \qquad (f = \mathrm{p, i, s}) \tag{4}$$

where n_f is the refractive index.

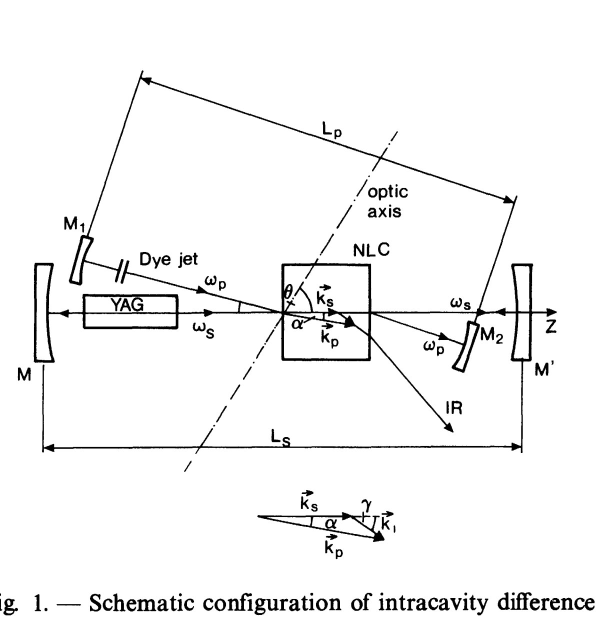

Fig. 1. — Schematic configuration of intracavity difference frequency in two laser resonators.

In figure 1, the signal wave (the ordinary ray) enters normally. The angle α (between the Poynting and pump wave vectors) is given by [11]

$$\operatorname{tg}(\theta - \alpha) = \frac{n_e^2}{n_0^2} \operatorname{tg} \theta \tag{5}$$

where θ is the phase matching angle.

The phase mismatching Δk is given by

$$\Delta k = k_p \cos \alpha - (k_s + k_i \cos \gamma) \tag{6}$$

where γ is the angle between $\mathbf{k}_i$ and Poynting vector.

If the centre of the non-linear crystal is located at the origin of a Cartesian coordinate system (X, Y, Z) which is centred in the signal (YAG) laser cavity with the Z-axis along the longitudinal signal laser cavity axis, the one-way travelling input waves take the Gaussian forms [8, 12]

$$u_p(r) = \frac{e^{(k_p \cos\alpha) Z}}{1 + j\tau_p} \exp\left\{ - \frac{X^2 + Y^2}{W_{0p}^2 (1 + j\tau_p)} \right\} \tag{7}$$

$$u_s(r) = \frac{e^{k_s Z}}{1 + j\tau_s} \exp\left\{ - \frac{X^2 + Y^2}{W_{0s}^2 (1 + j\tau_s)} \right\} \tag{7'}$$

$$\tau_{p,s} = \frac{2Z}{b_{p,s}}$$

$W_{0p,s}^2$ are the Gaussian beam waists of the input waves and $b_{p,s}$ ($= k_p \cos \alpha W_{0p}^2$; $= k_s W_{0s}^2$) are the confocal parameters. Using (1) and (7), (7′) and the results in our previous works [3, 8], we have the following formula for the field of the generated difference frequency wave :

$$E_i(X, Y, Z, t) = \frac{j\, 4\, \pi \omega_i\, d_{\text{eff}}}{C n_i}\, e^{j(k_i \cos\gamma) Z} . E_p(t)\, E_s^*(t) \times \int_{-l/2}^{Z} dZ' \frac{e^{j\Delta k Z'}\, e^{j(\omega_i + \phi_i)}}{1 + j\frac{1+\mu}{1-\mu}(\tau - \tau') + \tau\tau'} \exp\{\ \}_1 \tag{8}$$

where d_{eff} is the effective non-linear coefficient

$$\mu = \frac{k_s}{k_p} \tag{9}$$

is the degeneracy parameter, and

$$\{\ \}_1 = - \frac{j(1-\mu)}{W_{0p}^2(\tau - \tau')}(X^2 + Y^2) \left\{ -1 + \frac{1 + \tau'^2}{1 + j\frac{1+\mu}{1-\mu}(\tau - \tau') + \tau\tau'} \right\} . \tag{10}$$

Equation (8) was obtained assuming $b_p = b_s = b$. Then $\tau_p = \tau_s = \tau$; $\tau'_p = \tau_s = \tau'$. The non-linear interaction is maximized when the confocal parameters of the pump and signal waves are identical [13]. From (8) we have the spatial distribution of the idler wave field

$$U_i(r) = u_i(r) = j\, e^{j(k_i \cos\gamma) Z} \int_{-l/2}^{Z} dZ' \frac{e^{j\Delta k Z'} \exp\{\ \}_1}{1 + j\frac{1+\mu}{1-\mu}(\tau - \tau') + \tau\tau'} . \tag{11}$$

Taking into account polarizations in the laser medium and in the non-linear crystal, using the treatment and the results given in [3] and [8], together with expressions (1), (2), (7), (7′), (8), (11) and Lamb's equation [14] we have the following rate equation in the case of small gain lasers :

$$\frac{dP_p}{d\tau_p} = (-\alpha_p + \overline{N}_p(\tau_p)\, \sigma_p(\lambda_p)\, L_0 - \mathcal{C} P_s)\, P_p \tag{12}$$

$$\frac{dP_s}{d\tau_s} = \left(-\alpha_s + g_0 \left(1 - \frac{16\, \beta P_s}{C n_s W_{0s}^2} \right) + \frac{\omega_s}{\omega_p} \mathcal{C} P_p \right) P_s \tag{13}$$

$$\frac{d\phi_p}{d\tau_p} = - \frac{L_p\, \omega_p}{2\, C} \chi'_p - \frac{1}{2} \mathcal{C} P_s \frac{h'^{(1)}(\sigma, \mu, \xi)}{h^{(1)}(\sigma, \mu, \xi)} \tag{14}$$

$$\frac{d\phi_s}{d\tau_s} = - \frac{L_s\, \omega_s}{2\, C} \chi'_s + \frac{1}{2} \frac{\omega_s\, P_p}{\omega_p} \mathcal{C} \frac{h'^{(1)}(\sigma, \mu, \xi)}{h^{(1)}(\sigma, \mu, \xi)} \tag{15}$$

$$\frac{C}{L_p} \frac{d}{d\tau_p} \overline{N}_p(\tau_p) = - \frac{\overline{N}_p(\tau_p)}{\tau_R} - \frac{\overline{N}_p(\tau_p)\, \sigma_p(\lambda_p)}{h\nu_p\, CS} P_p + \frac{\sigma_0\, N_0\, P_0}{h\nu_p\, CS} \tag{16}$$

where

$$P_f = \frac{Cn_f}{8\pi} \int_{-\infty}^{+\infty} |E_f|^2 \, dX \, dY \qquad (f = \text{p, s}) \tag{17}$$

are the power of the interacting waves,

$$\alpha_f = \frac{L_f}{C} \frac{\omega_f}{Q_f}$$

are single-pass power losses for the pump- and signal modes,

$$\tau_f = \frac{Ct}{L_f}$$

are new time variables,

$$\mathcal{C} = \frac{128\,\pi^2\, d_{\text{eff}}^2}{C^3\, n_p\, n_s\, n_i} \cdot \frac{\mu}{1+\mu}\, \omega_i\, \omega_p\, lk_p \cos \alpha h^{(1)}(\sigma, \mu, \xi) \tag{18}$$

is the coupling parameter. Functions $h^{(1)}$ and $h'^{(1)}$ in (18) and in (14) are focusing functions determined by [8]

$$h^{(1)}(\sigma, \mu, \xi) = \mathfrak{Re}\,(2\,\xi)^{-1} \int_{-\xi}^{+\xi} d\tau \int_{-\xi}^{+\tau} d\tau' \frac{e^{-j\sigma(\tau-\tau')}}{1 - \frac{j}{2}\left(\frac{1+\mu}{1-\mu} + \frac{1-\mu}{1+\mu}\right)(\tau - \tau') + \tau\tau'} \tag{19}$$

and

$$h'^{(1)}(\sigma, \mu, \xi) = \mathfrak{Im}\,(2\,\xi)^{-1} \int_{-\xi}^{+\xi} d\tau \int_{-\xi}^{+\tau} d\tau' \frac{e^{-j\sigma(\tau-\tau')}}{1 - \frac{j}{2}\left(\frac{1+\mu}{1-\mu} + \frac{1-\mu}{1+\mu}\right)(\tau - \tau') + \tau\tau'}$$

with $\xi = l/b$ is the focusing parameter,

$$\sigma = -\frac{\Delta kb}{2} \tag{20}$$

is the phase matching parameter.

$\chi'_f(f = \text{p, s})$ results in frequency shifts mode pulling- and pushing- effects; g_0 is the single-pass unsaturated power gain of the YAG laser; β is a parameter accounting for saturation effects in the YAG laser [15].

$\overline{N}_p(\tau_p)$ is the populations inversion in the single excited state of the dye; τ_R is the fluorescence decay time; S is the area of the pump beam in the dye jet; h is Planck's constant; $\sigma_p(\lambda_p)$ is stimulated cross-section; σ_0 is the ground state absorption cross-section; L_0 is the optical path length in the dye jet; ν_0, P_0 are the wave-number and the power respectively of the second laser used to pump on the dye jet; and N is the number of unexcited molecules

$$N + \overline{N}_p(\tau_p) = N_0 \simeq N = \text{Const.} \tag{21}$$

Equations (12), (13), (14), (15) and (16) illustrate the time dependence of the powers and the phases of the pump and the signal waves. Equations (8), (13) determine the power of the generated difference frequency wave when all characteristics of the pump and the signal waves are known. In general case, these equations cannot be resolved analytically. They can be resolved only with the aid of a computer.

3. The optimal conditions for the non-collinear intracavity c.w. difference frequency generation.

3.1 THE POWERS OF THE INTERACTING WAVES. — In the steady-state regime of a c.w.-small gain dye laser, equations (12), (13), (14) (15) and (16) are equal to zero. Using equation (21) in conjunction with the above mentioned equations, we have following expressions for the intracavity power and the phase of the ring dye and the other small gain laser (c.w.-YAG laser) :

$$P_s = P_{0s} + \frac{P_{0s}}{g_0 - \alpha_s} \mu \mathcal{C} P_p \tag{22}$$

$$-\alpha_p + \frac{N_0\, L_0\, \sigma_0\, P_0\, \sigma_p\, \tau_R\, \nu_p}{(h\nu_p\, CS + \sigma_p\, \tau_R\, P_p)\, \nu_0} - \mathcal{C} P_{0s} - \frac{P_{0s}\, \mu \mathcal{C}^2}{g_0 - \alpha_s} P_p = 0 \tag{23}$$

$$\phi_p - \phi_s = \frac{\pi}{2} \tag{24}$$

where $P_{0s} = \frac{(g_0 - \alpha_s)}{16\, g_0\, \beta} C n_s W_{0s}^2$ is the intracavity power of the c.w.-YAG laser [15] in the absence of the parametric interaction effect.

Equation (24) shows that the spectral quality of the D.F.G. wave is the same as that of the input c.w.-laser beams. In practice, since $\mathcal{C}$ is small ($10^{-12} \div 10^{-10}$ u.e.s.) and the c.w.-YAG laser operates not close to the threshold, the terms which are proportional to $\mathcal{C}^2$ are also very small in comparison with the other terms in (23), one can ignore them. Using the results given in [3, 5] together with equations (22) and (23), we have the following pump and signal power formulae in the general case (small or large gain dye laser) :

$$P_p = \frac{1}{\alpha_p + P_{0s}\,\mathcal{C}} \times \left(\alpha_p P_{0p} - \frac{1 + v}{2\sqrt{1 + v^2}} \frac{h\nu_p CS}{\sigma_p \tau_R} P_{0s}\,\mathcal{C}\right) \quad (25)$$

$$P_s = P_{0s}\left(1 + \frac{\mu \alpha_p P_{0p}\,\mathcal{C}}{(g_0 - \alpha_s)(\alpha_p + P_{0s}\,\mathcal{C})}\right) \quad (26)$$

where P_{0p} is the intracavity power of the dye laser in the absence of the parametric interacting effect ($\mathcal{C} = 0$), which is determined by

$$P_{0p} = \frac{v}{\sqrt{1 + v^2}} (P_0 - P_{0th})(1 - e^{-N_0 L_0 \sigma_0}) \frac{\nu_p}{\nu_0\, \alpha_p} \quad (27)$$

where

$$P_{0th} = \frac{(1 + v)\, h\nu_0\, CS\alpha_p}{2\, v(1 - e^{-N_0 L_0 \sigma_0})\, \sigma_p\, \tau_R} \quad (28)$$

is the threshold of the dye laser only [3]; α_p is the total single pass loss at the dye laser wavelength [3, 5], v is defined as the ratio of the areas of the second pump laser and the dye laser beams at the jet [5].

Using (8), (17), (25) and (26) we have the following formula for the generated difference frequency wave :

$$P_i^{int} = \frac{2(1 - \mu)\, P_{0s}\,\mathcal{C}}{(\alpha_p + P_{0s}\,\mathcal{C})^2} \left[\alpha_p P_{0p} - \frac{1 + v}{2\sqrt{1 + v^2}} \frac{h\nu_p CS}{\sigma_p \tau_R} P_{0s}\,\mathcal{C}\right] \times \left[\alpha_p + P_{0s}\,\mathcal{C} + \frac{\mu \alpha_p P_{0p}\,\mathcal{C}}{(g_0 - \alpha_s)}\right] \frac{H^{(1)}(\sigma, \mu, \xi)}{\xi h^{(1)}(\sigma, \mu, \xi)} \quad (29)$$

where, as in [8] $H^{(1)}$ is a focusing function determined by :

$$H^{(1)}(\sigma, \mu, \xi) = \int_0^{\xi} d\tau' \int_0^{\xi} d\tau'' \frac{e^{-j\sigma(\tau' - \tau'')}}{1 - \frac{j}{2}\left(\frac{1 + \mu}{1 - \mu} + \frac{1 - \mu}{1 + \mu}\right)(\tau' - \tau'') + \tau' \tau''} . \quad (30)$$

3.2 OPTIMAL COUPLING PARAMETER. — Equations (25) and (26) show that the signal laser is amplified by the pump laser during the non-linear interacting process, and that the intracavity powers of the input waves are simple hyperbolic functions of the parameter $\mathcal{C}$. Since the parameter $\mathcal{C}$ depends only on the characteristics of the non-linear crystal and on the spatial structure of the input beams, equations (25), (26) also describe the dependence of the input powers on these characteristics. From (25) it is obvious that the parametric generation appears only when

$$\mathcal{C} < \mathcal{C}_1$$

with

$$\mathcal{C}_1 = \alpha_p A^{-1} \frac{P_{0p}}{P_{0s}} \quad (31)$$

where

$$A = \frac{1 + v}{2\sqrt{1 + v^2}} \cdot \frac{h\nu_p CS}{\sigma_p \tau_R} .$$

Using a rhodamine 6G c.w.-ring dye laser ($v = 1$, $S = 0.014$ mm; $P_{0p} = 5$ W; $\lambda_p = 0.58$ µm; $\sigma_p = 1.6 \times 10^{-16}$ cm^2; $\tau_R = 5 \times 10^{-9}$ s [7]; $A = 1.77 \times 10^7$ u.e.s.; $\alpha_p = 0.35$) and a c.w.-YAG laser, we have $\mathcal{C}_1 = 2 \times 10^{-9}$ u.e.s.

Figure 2 shows the dependence of the intracavity wave powers P_p, P_s and the D.F.G. output P_i^{int} on $\mathcal{C}$ as expressed in equations (25), (26), (29). We see that, if the coupling parameter $\mathcal{C}$ takes values which are greater than $\mathcal{C}_1$, the two lasers are not coupled, and each works independently of the other. Thus the term « coupling parameter » has a physical meaning only when $0 < \mathcal{C} < \mathcal{C}_1$. While the parameter $\mathcal{C}$ increases, pump power diminishes. The generated difference frequency power P_i^{int} increases to a maximal value, then decreases. This effect is the same as that in an intracavity single resonance optical parametric oscillator [8] : the total available power from the gain mechanism goes through a maximum and begins to decrease, in the same way that increasing output coupling losses to a laser can produce a similar maximum in output power [16].

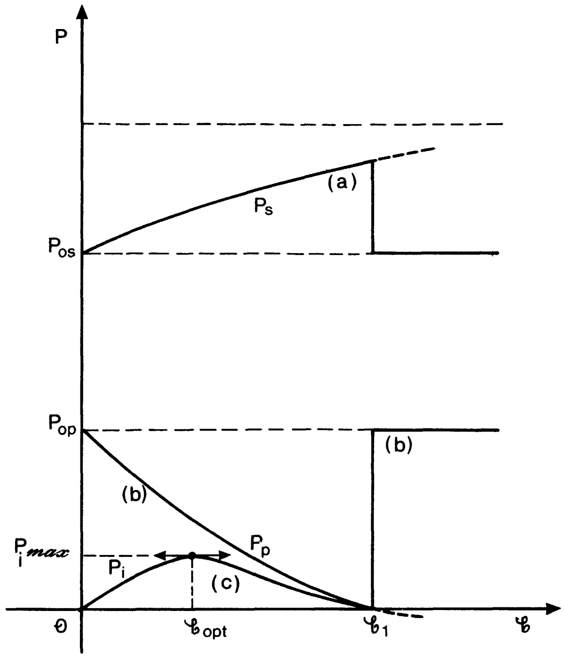

Fig. 2. — Plots of P_s, P_p and P_i as functions of $\mathcal{C}$. Curve (a) shows P_s as a function of $\mathcal{C}$; curve (b) shows P_p as a function of $\mathcal{C}$; curve (c) shows P_i as a function of $\mathcal{C}$.

The power of the generated difference frequency wave reaches a maximum when

$$\mathcal{C} = \mathcal{C}_{opt} = \alpha_p \left[\frac{\alpha_p \, \mu P_{0p}^2}{g_0 - \alpha_s} - (P_{0p} + A) P_{0s} \right] \times \times \left[\frac{\alpha_p \, \mu P_{0p} P_{0s}}{g_0 - \alpha_s} (2A + P_{0p}) + (P_{0p} + A) P_{0s}^2 \right]^{-1} . \tag{32}$$

Expression (32) gives the optimal condition to maximize the power of the generated difference frequency wave P_i^{int}. This is also the optimal coupling between two lasers, obtained by the non-linear interaction in the crystal, which enables us to deduce the requirements to be met in practice. It is obvious that, with a given non-linear crystal, we can choose the values of the intracavity laser powers P_{0s}, P_{0p} such that the condition (32) is satisfied to arrive at the maximal value of D.F.G. output power. It should be noted from (32) that the optimal coupling of an I.D.F.G. system exists only when the intracavity power of the c.w.-dye laser is greater than the threshold P_{0p}^{th} determined by

$$P_{0p}^{th} = \frac{P_{0s} + \sqrt{\frac{4 \mu \alpha_p A P_{0s}}{g_0 - \alpha_s} + P_{0s}^2}}{2 \mu \alpha_s} (g_0 - \alpha_s) . \tag{33}$$

To produce the maximal power of the generated difference frequency wave, the values of the optimal coupling parameter $\mathcal{C}_{opt}$ have been numerically estimated as a function of P_{0s} and P_{0p} as expressed in (32) and have been plotted in figure 3. The following cases are considered : an I.D.F.G. system bases on two conventional lasers : a c.w.-YAG laser ($\lambda_s = 1.06$ µm; $g_0 = 0.10$; $\alpha_s = 0.09$ for the case of the intracavity power value is 50 W) and a c.w.-ring rhodamine 6G dye laser ($\lambda_p = 0.58$ µm; $N_0 = 1.28 \times 10^{17}$ molecules/cm^3; $L_0 = 0.037$ cm; $\sigma_0 = 1.6 \times 10^{-16}$ cm^2 [7] and the values of the other necessary parameters are the same as those mentioned above). With P_{0s} = 40 W; 50 W; 60 W values of P_{0p}^{th} of 2.9 W; 4.1 W; 5.6 W have been found respectively.

The maximal value of the generated difference frequency wave in the case of weak focusing ($h^{(1)}(\xi \ll 1) \simeq \xi$; $H^{(1)}(\xi \ll 1) \simeq \xi^2$ [8]) is :

$$P_{i_{max}}^{int} = \frac{2(1-\mu) P_{0p} P_{0s} \mathcal{C}_{opt}}{(\alpha_p + P_{0s} \mathcal{C}_{opt})^2} \left(\alpha_p - \frac{A P_{0p} \mathcal{C}_{opt}}{P_{0p}} \right) \times \times \left(\alpha_p + P_{0s} \mathcal{C}_{opt} + \frac{\alpha_p \mu P_{0p} \mathcal{C}_{opt}}{g_0 - \alpha_s} \right) \tag{34}$$

where $\mathcal{C}_{opt}$ is determined by (32).

It is interesting to note that weak focusing conditions have often been used in practice, where the non-linear crystal is placed in the collimated arm of ring dye lasers. At this position the radiation fields are intense, but not strong enough to damage the crystal. The maximal I.R. output power can therefore be obtained over long periods of time by use of a long crystal and large interaction volume [4].

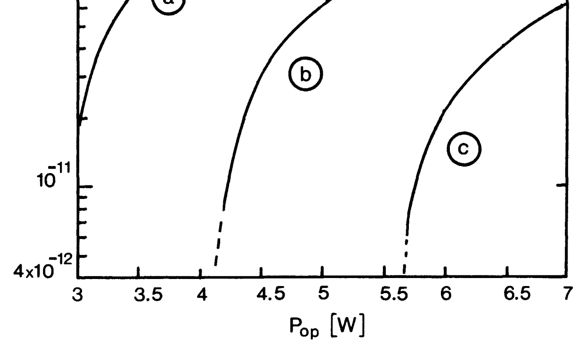

Fig. 3. — Dependence of the optimal coupling parameter $\mathcal{C}_{opt}$ on the intracavity laser powers. a) P_{0s} = 40 W; b) P_{0s} = 50 W; c) P_{0s} = 60 W; the value of the pump and signal wavelengths are $\lambda_p = 0.58$ µm, $\lambda_s = 1.06$ µm. The other parameter values are given in the text.

With a K.D.P. crystal of length $l = 5$ cm ($n_i = n_p = n_s \simeq 1.5$ [17]; $\lambda_s = 1.06$ μm; $\lambda_p = 0.58$ μm; $\theta = 42°56$; $\alpha = 1°58$, see (5); $d_{eff} = 1.02 \times 10^{-9} \sin(\theta + \alpha) = 8 \times 10^{-10}$ u.e.s. [17]), using weak focusing, equation (18) yields

$$\mathcal{C} = 8 \times 10^{-12} \text{ u.e.s.}.$$

A non-collinear phase matching I.D.F.G.-system based on this K.D.P. crystal works in the optimal condition when the P_{0p}, P_{0s} take the values 4.2 and 50 W respectively (see the curve b in the Fig. 3). Substituting the value of $\mathcal{C}_{opt}$, P_{0p}, P_{0s}, $g_0 - \alpha_s = 0.01$, $A = 1.77 \times 10^7$ u.e.s. into (34), one obtains the following maximal value of the generated difference frequency wave

$$P_{i_{max}}^{int}(\lambda_i = 1.315\ \mu\text{m}) = 13.01\ \text{mW}.$$

An extracavity D.F.G.-system operating in the same conditions (same crystal and focusing, same lasers) gives a D.F.G. output power determined by [3]

$$P_i^{ext} = \frac{2\,\omega_i}{\omega_p}\,\mathcal{C} P_{0p}\,P_{0s}\,T_s\,T_p \qquad (35)$$

where T_s, T_p are the transmission coefficients of the output mirrors of the c.w.-YAG and ring dye laser respectively. Using $T_s = 1\%$; $T_p = 10\%$; $\mathcal{C} = \mathcal{C}_{opt} = 8 \times 10^{-12}$ u.e.s.; $\lambda_p = 0.58$ μm; $\lambda_s = 1.06$ μm; $\lambda_i = 1.315$ μm, the extracavity D.F.G.-system yields on output power of 13 μW, which means that the I.D.F.G. gives an enhancement factor of about 1001 in the case of the weak focusing.

In the general case, using equation (29) and the result in [3] we obtain the following expression for the enhancement factor η :

$$\eta = \frac{P_i^{int}}{P_i^{ext}} = \frac{1}{T_s\,T_p(\alpha_p + \mathcal{C}P_{0s})}\left[\alpha_p - \frac{AP_{0s}}{P_{0p}}\,\mathcal{C}\right] \times \left[1 + \frac{\alpha_p\,\mu P_{0p}\,\mathcal{C}}{(g_0 - \alpha_s)(\alpha_p + \mathcal{C}P_{0s})}\right]. \qquad (36)$$

From (36) it is obvious that, when $\mathcal{C}$ is small (as is often the case with weak focusing), the enhancement factor becomes $1/T_s\,T_p$. Using the c.w. small gain- and c.w. dye lasers ($T_s \simeq 1\%$; $T_p \simeq 10\%$) the I.D.F.G. output power is of the order of one thousand time greater than P_i^{ext}. Since the term $(1 + \mu\alpha_p\,P_{0p}\,\mathcal{C}/(g_0 - \alpha_s)(\alpha_p + \mathcal{C}P_{0s}))$ explained by the parametric amplification effect appears in formula (36), the enhancement factor η increases as $\mathcal{C}$ at first, goes to a maximum and decreases to zero at the point where $\mathcal{C} = \mathcal{C}_1$. Figure 4 shows the dependence of η on the parameter $\mathcal{C}$ as expressed in (36). The enhancement has a maximal value η_{max} at $\mathcal{C} = \mathcal{C}_{opt}$ determined by (32). From (36) we have :

$$\eta_{max} = \frac{1}{T_s\,T_p(\alpha_p + \mathcal{C}_{opt} + P_{0s})}\left[\alpha_p - \frac{AP_{0s}}{P_{0p}}\,\mathcal{C}_{opt}\right] \times \left[1 + \frac{\alpha_p\,\mu P_{0p}\,\mathcal{C}_{opt}}{(g_0 - \alpha_s)(\alpha_p + \mathcal{C}_{opt}\,P_{0s})}\right]. \qquad (37)$$

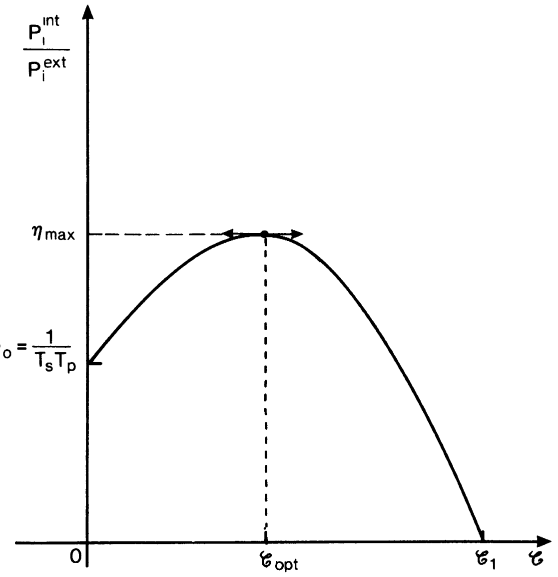

Fig. 4. — Plot of the enhancement factor $\eta = P_i^{int}/P_i^{ext}$ as a function of $\mathcal{C}$.

Since at the point $\mathcal{C} = \mathcal{C}_{opt}$ (i.e. $P_{i_{max}}^{int}$) both $dP^{int}/d\mathcal{C}$ and $d\eta/d\mathcal{C}$ are equal to zero, which means that the I.D.F.G. is very stable with respect to the fluctuation of $\mathcal{C}$. This is another advantage of an I.D.F.G. using two laser resonators.

For an I.D.F.G.-system based on two lasers (c.w.-YAG and ring dye lasers mentioned above) and on a crystal which has the optimal coupling parameter determined by (32), the dependence of the maximal enhancement factor η_{max}, on the pump and signal power is shown in figure 5. We see that, to obtain a high value of η_{max}, it is favourable to use a large value of $\mathcal{C}_{opt}$ and not very large value of P_{0s} (see the curve a in Fig. 3 and Fig. 5). In practice one can increase $\mathcal{C}_{opt}$ by satisfying the following conditions as shown in expression (18) :

— Choose a crystal with a large effective non-linear coefficient and small value of walk-off α.

— Increase $h^{(1)}$ by the optimal focusing [8] to obtain maximum value of $h^{(1)}$.

To carry out the second condition it is necessary to consider the optimal phase matching in the case of the input beams being non-collinear Gaussian focused.

3.3 Optimal phase matching in non-collinear mixing of three beams. — For the plane waves, the phase matching condition $\Delta k = 0$ gives maximal parametric conversion efficiency [18]. Using Gaussian focusing beams, a large number of different spatial field components appear, which violate the condition $\Delta k = 0$. To obtain high conversion efficiency in the D.F.G., the phase mismatching Δk – i.e. the

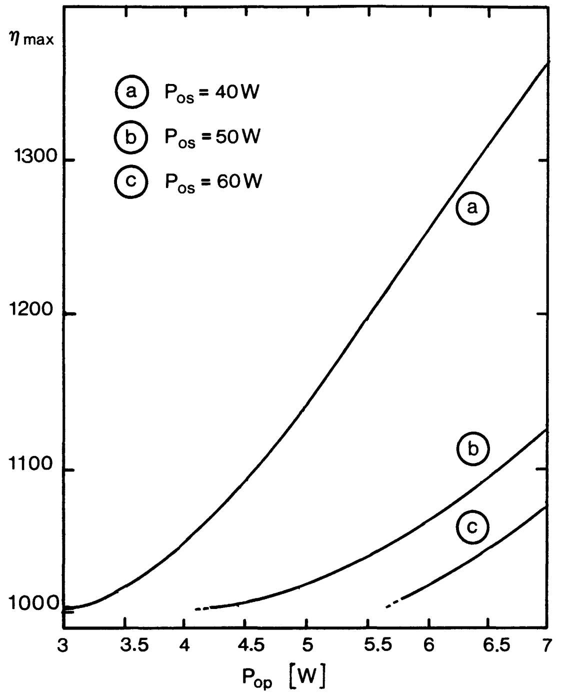

Fig. 5. — Dependence of the maximal enhancement factor $\eta_{\max}$ on the intracavity laser powers. a) $P_{0s} = 40$ W; b) $P_{0s} = 50$ W; c) $P_{0s} = 60$ W. The other parameter values are given in the text.

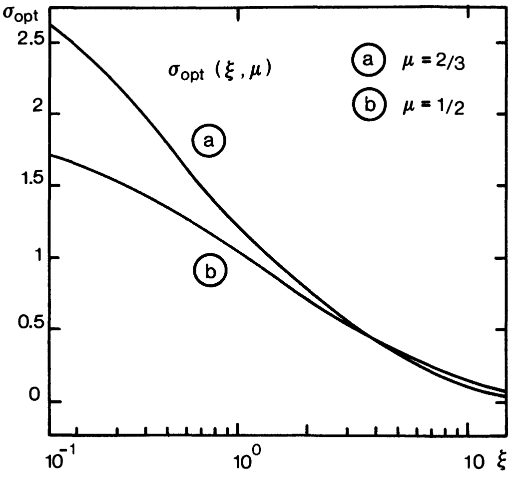

Fig. 6. — Plots of the optimal phase matching parameter σ_{opt} as a function of ξ : a) the case of $\mu = 2/3$; b) the case of $\mu = 1/2$.

phase matching parameter $\sigma = -\Delta k b/2$ — should take an optimal value which gives a maximal value of the function $h^{(1)}$. The optimal value $\sigma_{\text{opt}}(\xi, \mu)$ was calculated numerically by the computer and the result is given in figure 6. The function $h^{(1)}$ takes its optimal value when

$$\Delta k = -\frac{2\,\sigma_{\text{opt}}}{b}. \tag{38}$$

In the case of weak focusing ($b \gg 1$), equation (38) gives $\Delta k \simeq 0$, i.e. this is the same as that of plane wave.

Equations (6) and (38) yield

$$k_{\text{p}}.\cos\alpha - k_{\text{s}} - k_{\text{i}}.\cos\gamma = \frac{2\,\sigma_{\text{opt}}}{l}\,\xi. \tag{39}$$

With any given crystal, the phase matching angle θ is defined. Thus from (5), we see that with the focused Gaussian beams the walk-off α is unchanged. Equation (39) gives the consequential variation of the generated difference frequency wave beam direction

$$\delta = n_{\text{i}}\,\Delta\gamma = \frac{2\,\sigma_{\text{opt}}\,\xi}{l}\,\frac{n_{\text{i}}}{k_{\text{i}}.\sin\gamma}. \tag{40}$$

For an I.D.F.G. using a K.D.P. crystal of length 5 cm, placed at the second beam waist of the ring dye laser ($W_{0\text{p}} = 32$ μm), with the optimal phase matching focusing, we have $h^{(1)}(\xi = 2.8;\ \mu \simeq 1/2;\ \sigma_{\text{opt}} = 0.6) = h^{(1)}_{\max}$ (see [8] and Fig. 6 in this paper).

Taking into account $\gamma = 3°$; $\lambda_{\text{i}} = 1.315$ μm, (40) yields a beam shift δ of the order of one minute. This can be neglected since the generated difference frequency focused beam is large in practice.

From the energy conversion relation ($\omega_{\text{IR}} = \omega_{\text{dye}} - \omega_{\text{YAG}}$), it is interesting that, using non-collinear mixing I.D.F.G.-system, one can obtain a broad band of the tunable range by varying the dye laser wavelength without modifying the position of the crystal vis the axis of the YAG-laser resonator (θ = Const). However, to satisfy the phase matching condition, the direction of the input dye laser beam must change a little (see formula (5)).

Using the Rh 6G dye (working range 0.563 μm ÷ 0.607 μm), we obtain the following value of the I.R. tunable band : $\Delta\lambda_{\text{IR}} = 1.413$ μm $-$ 1.196 μm $=$ 0.217 μm. This numerical value shows that one dye can yield alone a large tunable band.

4. Conclusion.

The theory of c.w.-I.D.F.G. by non-collinear mixing of three waves in two laser resonators has been developed. This leads us to conclude that, using the optimal coupling and focusing, high values of the enhancement factor η may be obtained with the conventional c.w.-lasers available. The tunable c.w. infrared radiation obtained by non-collinear frequency mixing in two laser cavities compares favourably with other D.F.G. methods.

1) The output power of the generated difference frequency wave is stable with respect to the fluctuation of the laser beam orientations and the temperature of the crystal.

2) The I.R. output beam is separated from the two input beams.

3) The output power of c.w.-I.R. should in principle be of several tens of milliwatts.

References

[1] Pine, A. S., *J. Opt. Soc. Am.* **64** (1974) 1683.
[2] Lahmann, W., Tibulski, K. and Welling, H., *Opt. Commun.* **17** (1976) 18.
[3] Tran-Ba-Chu, Broyer, M., *J. Physique* **46** (1985) 523.
[4] Coherent Inc., Technology Report, Laser Focus/ Electro-Optics, July 1984, p. 30.
[5] Tran-Ba-Chu, Bouvier, A. J., Bouvier, A., Broyer, M., *J. Physique* **46** (1985) 551.
[6] Teschke, O., Dienes, A., Whinnery, J. R., *I.E.E.E. J. Quant. Electron.* **QE 12** (1976) 383.
[7] Snavely, B. B., in *Continuous Wave Dye Lasers, in Dye Lasers* (Schäfer F. P., Ed., N. Y., Springer-Verlag) 1973, p. 91.
[8] Tran-Ba-Chu, Broyer, M., *J. Physique* **45** (1984) 1599.
[9] a) Nath, G., Mehmanesh, H. and Gsänger, M., *Appl. Phys. Lett.* **17** (1970) 286.
b) Nash, F. R., Bergman, J. G., Boyd, G. D. and Turner, E. H., *J. Appl. Phys.* **40** (1969) 5201.
[10] Brunner, W., Paul, H. V. and Bandilla, A., *Annln Phys.* **27** (1971) 82.
[11] Zernike, F., Midwinter, J. E., in *Applied Non-Linear Optics* (N.Y.) 1973.
[12] Boyd, G. D., Kleinman, D. A., *J. Appl. Phys.* **39** (1968) 3597.
[13] Fisher, R., Tran-Ba-Chu, Wieckzorek, L. W., *Sov. J. Quant. Electron.* **7** (1977) 1455.
[14] Lamb, W. E., Jr., *Phys. Rev.* **A 134** (1964) 1429.
[15] Oshman, M. K., Harris, S. E., *I.E.E.E. J. Quant. Electron.* **QE 4** (1968) 491.
[16] Rigrod, W. W., *J. Appl. Phys.* **34** (1963) 2602.
[17] Kurtz, S. K., in *Laser Handbook* Ed. F. T. Arecchi and E. O. Shulz Dubois, vol. 1 (North-Holland), 1972.
[18] Fisher, R., Kulevskii, L. A., *Sov. J. Quant. Electron.* **7** (1977) 135.