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Empirical relation for electronic stopping power of heavy ions in carbon

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Résumé. — Nous proposons une nouvelle relation empirique simple pour calculer le pouvoir d'arrêt électronique du carbone pour des ions lourds ($Z > 4$) d'énergie non relativiste. Cette relation est en bon accord avec les données expérimentales et peut être aisément codée sur un petit ordinateur.

Abstract. — We propose a new simple empirical relation to calculate the electronic stopping power of carbon for heavy ions ($Z > 4$) at non relativistic energies. This relation fits well the experimental data and can be easily coded on a small computer.

1. **Introduction.** — The growing interest in heavy ion beam physics emphasizes the need for reliable stopping power values. Northcliffe and Schilling [1] in 1970 made a semi-empirical tabulation of electronic stopping power values for heavy ions in different targets. These tables have been obtained from careful interpolations and extrapolations of the experimental results available at that time. New experimental results [2, 3] show significant discrepancies with the values tabulated by Northcliffe and Schilling for some targets and projectiles, specially at low energy (< 1 MeV/nucleon). However for heavy ions ($Z > 4$) in carbon targets Northcliffe and Schilling's tabulated values are good approximations as can be seen from recent experimental results [4-6]. Only for Cu and Kr projectiles in carbon, at about 5 MeV per nucleon, discrepancies as large as 30 % have been observed [6].

We propose an analytical expression for calculating easily the electronic stopping power of carbon for heavy ions ($Z > 4$) at all non relativistic energies. Other relations have been proposed previously [7-13] to describe the stopping power of ions in matter but none of these relations can be directly applied to heavy projectiles at all non relativistic energies. Most of these formulas have been established for proton projectiles and fail to reproduce the behaviour of heavy ion stopping power in the intermediate energy region ($0.1 < E/M$ (MeV/amu) < 1). As typical examples we have plotted in figures 1 and 2 the electronic stopping powers for carbon and oxygen projectiles in carbon as a function of their energy per nucleon. The electronic stopping power values are obtained from : *a*) the expression of Ziegler [11-13] ; *b*) the formula of Brice [10] ; *c*) the tabulated

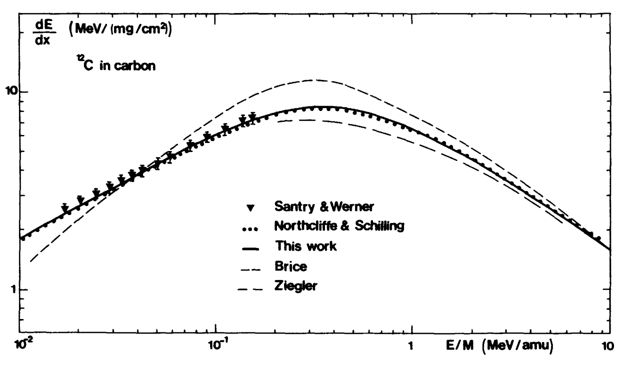

Fig. 1. — Electronic stopping power for ^{12}C in carbon as a function of the energy per nucleon of the projectile.

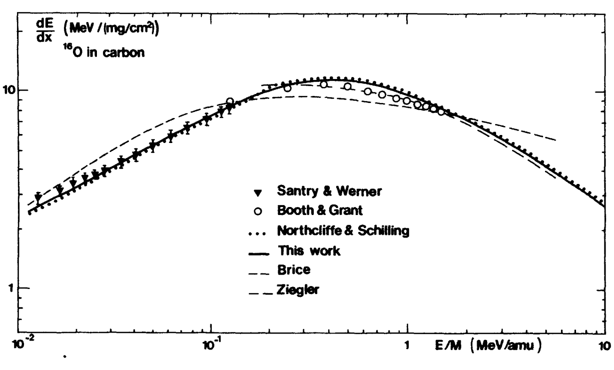

Fig. 2. — Electronic stopping power for ^{16}O in carbon as a function of the energy per nucleon of the projectile.

values of Northcliffe and Schilling [1] and *d*) the experimental results of Santry and Werner [4] for ^{12}C and ^{16}O in the energy range 0.2-2 MeV and those of Booth and Grant [14] for ^{16}O in the energy range 2-24 MeV. The nuclear stopping power (very small in the energy domain considered here) has been subtracted from the experimental stopping power data using the empirical relation given in reference [15].

The relation proposed by Ziegler [11-13] is calculated by the use of an expression for scaling from proton stopping powers and is based on data for incident energies greater than 0.2 MeV/amu and is only valid above this energy. The Ziegler's curves in figures 1 and 2 have been calculated using eqs. (6), (7) and (8) from reference [13] and table I from reference [12]. The Brice's formula is complicated and depends on three parameters (n, Z, a) which are not explicitly given in reference [10] for a carbon target. For carbon projectiles we determined the values of these parameters by equalling Brice's result at 0.04 MeV/amu to Santry and Werner's data [4] and Brice's result and slope at 10 MeV/amu to Northcliffe and Schilling's tabulated values [1] (at our knowledge there is no experimental result available above 0.17 MeV/amu for carbon projectiles in carbon). We obtained $n = 3.48$, $Z = 0.86$, and $a = 0.29$. For oxygen projectiles we calculated the Brice's parameters in order that the Brice's results are equal to the experimental values of Santry and Werner [4] at 0.0125 MeV/amu, and of Booth and Grant [14] at 0.125 and 1.5 MeV/amu; we obtained $n = 2.36$, $Z = 0.48$, and $a = 0.61$.

2. **Electronic stopping power formula.** — We propose the following analytical expression for describing the dependence on energy of the electronic stopping power of carbon for heavy ions ($Z > 4$) :

$$\frac{dE}{dx} = k_2 (E/M)^{k_1} \left(1 - \exp\left(-\frac{k_3}{(E/M)}\right)\right)^{k_4} \quad (1)$$

where dE/dx is the energy loss in MeV/(mg/cm^2), E/M is the projectile energy in MeV/amu, and k_1, k_2, k_3, k_4 are parameters depending only on the nature of the projectile.

The behaviour of dE/dx at low energy is determined by the parameters k_1 and k_2. The parameters k_3 and k_4 influence the slope and the amplitude of the curve at high energy. The analytical form of eq. (1) has been chosen to give a good fit to the experimental data in the intermediate energy region ($E \simeq 0.025\, Z^{4/3}$ where Z is the projectile atomic number). For each projectile the four parameters of eq. (1) have been adjusted by a least squares routine to fit the following data. For C, N and O projectiles we used the experimental data of Santry and Werner [4] available in the energy range 0.2-2 MeV (the nuclear stopping power has been subtracted from the total stopping power before the fit) and Northcliffe and Schilling's tabulated values [1] at higher energies. For heavier particles where only sparse data exist we used Northcliffe and Schilling's tabulated values [1].

We found that the dependence of the best values of the four parameters $k_1, \dots, k_4$ on the atomic number Z of the projectile nucleus is well described by the following relations [16] :

$$k_1 = 0.5 + 1.8 \times 10^{-3}\, Z \quad (2)$$

$$k_2 = 0.8\, Z + 7.8\, Z^{1/2} - 3.9 \quad (3)$$

$$k_3 = -\, 2.3 \times 10^{-2}\, Z + 0.42\, Z^{1/2} - 0.37 \quad (4)$$

$$k_4 = (2.4 \times 10^{-3}\, Z^2 + 1.12\, Z + 0.88)/Z\,. \quad (5)$$

Eqs. (1) to (5) allow direct calculations of dE/dx for any ion ($Z > 4$) at any non relativistic energy; the only *input* parameters being the atomic number and the energy per nucleon of the projectile. For $Z \leqslant 4$, the values of $k_1, \dots, k_4$ can no longer be given by formulas (2) to (5). However, it is still possible to determine $k_1, \dots, k_4$ such that formula (1) gives a good fit to the experimental data [16].

We have plotted in figures 1 and 2 the electronic stopping power values of carbon for carbon and oxygen projectiles calculated using eqs. (1) to (5). It appears that our relation gives a better fit to the experimental data or to Northcliffe and Schilling's tabulated values than other formulas [10-13], in the whole (non relativistic) energy domain.

In figure 3 we show the electronic stopping power values of carbon calculated by eqs. (1) to (5) for N, Ar, Br, I and U projectiles as well as the more recent experimental data for this electronic stopping power

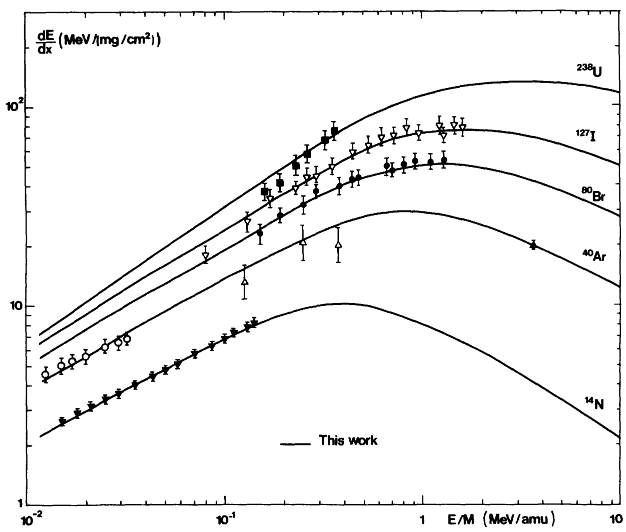

Fig. 3. — Electronic stopping power for N, Ar, Br, I and U in carbon as a function of the energy per nucleon of the projectile. The curves in full lines are calculated using eqs. (1) to (5). Experimental data : N : (▼) Santry and Werner, 1979 ; Ar : (○) Fastrup *et al.*, 1966 ; (△) Efken *et al.*, 1975 ; (▲) Bimbot, 1978 ; Br : (●) Moak and Brown, 1966 ; I : (▽) Bridwell *et al.*, 1967 ; U : (■) Brown and Moak, 1972.

[4, 17, 18, 6, 19, 20, 21] (the nuclear stopping power has been subtracted when necessary). We see that the agreement between experimental and calculated values is very good.

3. **Conclusions.** — We have proposed a simple empirical relation to calculate the electronic stopping power for projectiles in carbon as a function of their energy. This relation (eq. (1)) depends on four parameters which can be expressed as simple functions of the atomic number Z of the projectiles (when $Z > 4$) (eqs. (2)-(5)). These functions have been found using experimental data when available and Northcliffe and Schilling's tabulated values in the domain where no sufficient experimental results exist. Nevertheless other best values of the parameters may be adjusted to fit new experimental results when available. We want to emphasize that eqs. (1) to (5) can be easily coded on very small computers (in BASIC or FORTRAN the coding takes less than 5 instructions) so that $\mathrm{d}E/\mathrm{d}x$ can be computed on line, for example in beam-foil spectroscopy work.

The difference between the values calculated by our formula and the more recent experimental results or Northcliffe and Schilling's tabulated values is typically 3 %. Only for Cu and Kr projectiles at about 5 MeV/nucleon discrepancies as large as 30 % are observed [6]. Our relation gives in the whole non relativistic energy domain a better fit to the experimental data than previous formulas.

The analytical form of the relation proposed is expected to be valid for other targets than carbon but more experimental data are needed to verify this assumption.

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